



Upwash interference for wings in solid-liner wind tunnels using subsonic linearised-theory

Associated software: ESDUpac A9514

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UPWASH INTERFERENCE FOR WINGS IN SOLID-LINER WIND TUNNELS USING SUBSONIC LINEARISED-THEORY

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UPWASH INTERFERENCE FOR WINGS IN SOLID-LINER WIND TUNNELS USING SUBSONIC LINEARISED-THEORY

1. NOTATION

A	aspect ratio, $= 4s^2/S$
a_{ij}	influence coefficient (see Section 9)
a'_{pj}	influence coefficient excluding direct wing contributions (see Section 9)
b	wind-tunnel working-section dimension in y-direction
b'	$b/2$
C	working-section cross-sectional area ($C = 4b'h'$ for rectangular cross-section, $C = 4b'h'(1 - 0.5f^2)$ for cross-section with corner fillets)
C_D	drag coefficient
C_{Dcorr}	drag coefficient corrected for upwash interference
C_L	lift coefficient
C_{Lcorr}	lift coefficient corrected for upwash interference
C_{LL}	sectional lift coefficient ($= C_{LL}(\eta)$)
C_{L1}	limiting value of C_L for drag error of one count, associated with streamwise variation of upwash (see Section 6.3)
C_{L2}	limiting value of C_L for drag error of one count, associated with spanwise variation of upwash (see Section 6.3)
C_M	pitching moment coefficient (moment about y-axis)
C_{Mcorr}	pitching moment coefficient corrected for upwash interference
C_p	pressure coefficient
c	local wing chord ($= c(\eta)$)
c_0	centre-line chord
c_t	tip chord
$\bar{c}$	mean wing chord $= \left(\int_0^1 c(\eta) d\eta \right)$

d	arbitrary constant (see Section 7)
$e_{k,l}$	coefficient in polynomial representation of upwash (see Section 9.1)
f	fillet ratio (see Section 8)
f_1	factor related to f and s/b' (see Section 8)
h	wind-tunnel working-section dimension in z -direction
h'	$h/2$
i	integer denoting boundary point (see Section 9)
i_l	parameter related to number of wing images in y -direction (see Section 9)
j	integer denoting singularity (see Section 9)
j_l	parameter related to number of wing images in z -direction (see Section 9)
k	power of ξ in polynomial representation of δ (see Section 9.1)
L_p	lift on wing panel (see Section 9)
l	power of η in polynomial representation of δ (see Section 9.1)
M_∞	undisturbed stream Mach number (see Section 11 for comment on blockage correction to M_∞)
N	number of singularities (see Section 9)
n	local inward surface-normal coordinate
S	wing planform area
s	wing semi-span
s_p	span of wing vortex-panel
t	variation of δ over entire wing planform
U	undisturbed stream speed
v_{ni}	velocity component in n -direction at point denoted by i (see Section 9)
w	velocity in z -direction, in $z = 0$ plane, induced by wind-tunnel walls (upwash interference)
w_{rep}	representative value of w in region of wing (see Section 6)
X	longitudinal (streamwise) coordinate in transformed flow (see Section 3)
x	longitudinal (streamwise) coordinate, zero at leading edge in wing plane of symmetry

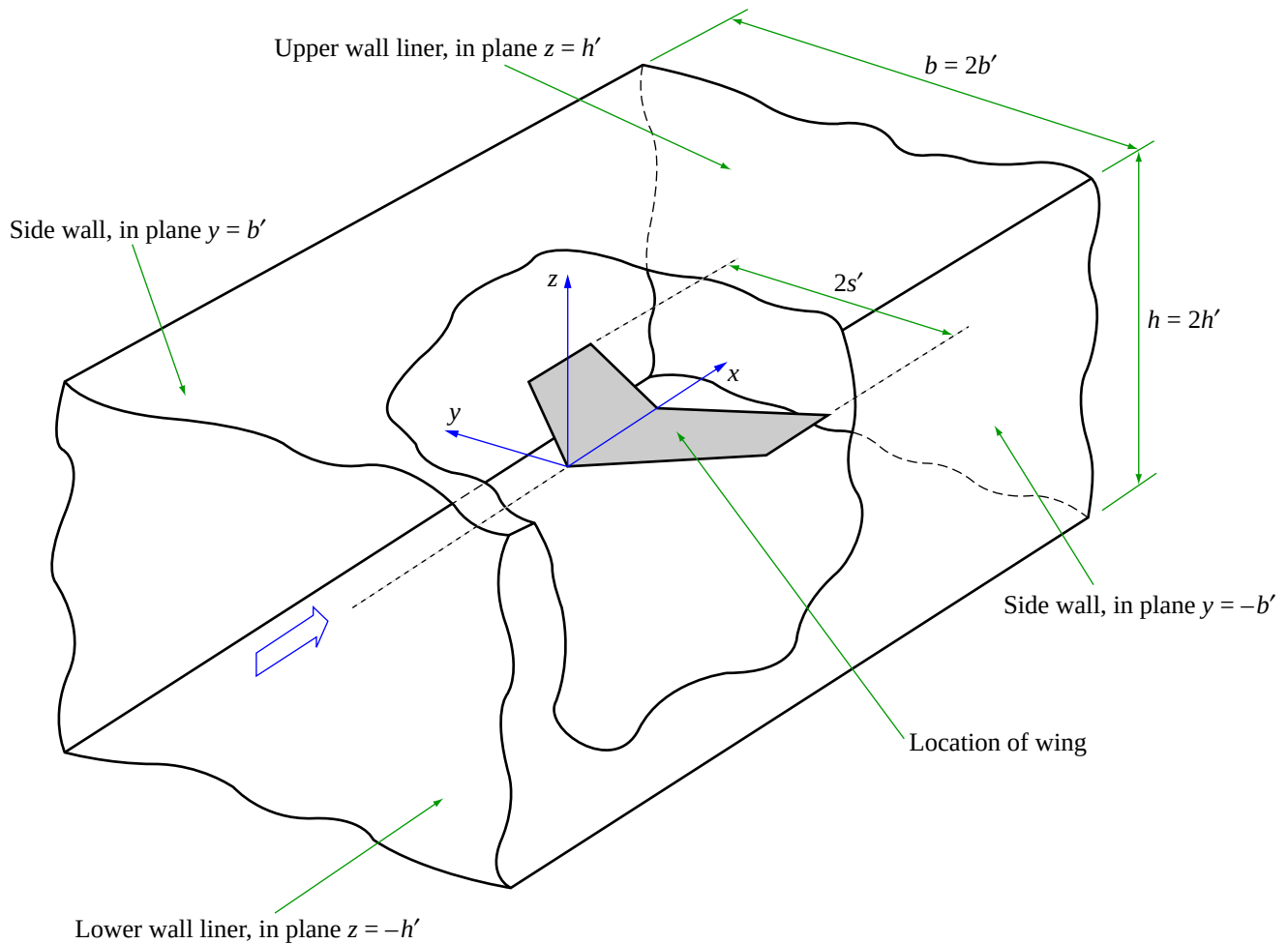
x_{ac}	x -coordinate of wing aerodynamic centre
x_{cp}	x -coordinate of wing centre of pressure
x_F	x -coordinate of trailing edge at wing tip
Y	spanwise coordinate in transformed flow (see Section 3)
y	spanwise coordinate, zero in wing plane of symmetry
Z	coordinate normal to working-section mid-plane in transformed flow (see Section 3)
z	coordinate normal to working-section mid-plane
z_c	local effective camber z -coordinate associated with upwash ($= z_c(\xi, \eta)$)
z_L	local z -coordinate of lower wing surface
z_m	local z -coordinate of mean wing surface ($= (z_U + z_L)/2$)
z_t	local wing semi-thickness ($= (z_U - z_L)/2$)
z_U	local z -coordinate of upper wing surface
α	local geometric incidence of wing mean surface ($= \alpha(\xi, \eta)$)
$\bar{\alpha}$	geometric incidence of wing datum-chord (<i>i.e.</i> incidence of wing centre-line chord)
$\bar{\alpha}_{corr}$	incidence of wing datum-chord corrected for upwash interference
α_{eff}	effective local incidence of wing mean surface, allowing for non-uniform upwash ($= \alpha_{eff}(\xi, \eta)$)
α_t	local effective wing twist relative to datum chord, allowing for non-uniform upwash ($= \alpha_t(\eta)$)
β	linearised-theory compressibility factor ($= (1 - M_\infty^2)^{1/2}$)
γ_j	strength of singularity denoted by j (see Section 9)
$\Delta\bar{\alpha}$	effective additional incidence of datum chord associated with upwash interference
$\Delta\alpha_D$	correction to incidence of datum chord, related to vortex drag (see Section 6.2)
$\Delta\alpha_L$	correction to incidence of datum chord, related to lift (see Section 6.2)
$\Delta\alpha_p$	effective upwash inclination angle averaged over bound-vortex segment
ΔC_p	difference between (notional) C_p values computed for uniform and non-uniform upwash interference

Δx_{ac}	aft shift of aerodynamic centre due to non-uniform upwash
Δ_1	maximum variation of δ over wing chord (see Section 6.3)
Δ_2	maximum variation of δ over wing span (see Section 6.3)
δ	upwash interference parameter ($= \delta(\xi, \eta) = (w/U)(C/SC_L)$)
δ_{av}	mean value of δ over wing planform (see Equation (6.7))
δ_D	value of δ related to $\Delta\alpha_D$
δ_L	value of δ related to $\Delta\alpha_L$
δ_R	upwash interference parameter for rectangular cross-section (see Section 8)
δ_{rep}	representative value of δ in region of wing (see Section 6)
δ_0, δ_1	parameters defined in Section 2
ε	change in wing incidence (see Section 3)
η	non-dimensional spanwise position ($= y/s$)
θ	local effective additional incidence associated with upwash interference ($= \theta(\xi, \eta) = w(\xi, \eta)/U$)
$\Lambda_{1/2}$	sweepback angle of mid-chord line (negative for forward-swept wings)
λ	taper ratio ($= c_t/c_0$)
ξ	local non-dimensional longitudinal coordinate ($= (x - x_{le})/c$)
ξ'	integration variable (see Section 9.2)
ρ	undisturbed stream density
ϕ	disturbance-velocity potential ($= \phi(x, y, z)$)
ϕ_a	antisymmetric part of ϕ (see Section 3)
ϕ_s	symmetric part of ϕ (see Section 3)

Suffices

le	denotes leading edge
T	denotes transformed quantity (see Section 3.3)

1.1 Axis System



Sketch 1.1 Typical wing in wind-tunnel working-section, indicating axis system and geometry nomenclature

2. INTRODUCTION

This Item gives data, based on linearised theory, for wall-induced upwash interference relevant to tests at subsonic speed on wings in wind tunnels with test sections having solid wall-liners and rectangular cross-section. Use of the data for test sections with corner fillets is also dealt with.

The wall-liners of a wind-tunnel test-section constrain the flow around a model, so making it different from that around the model in free air. The constraining effects are due to aerodynamic interaction (interference) between the wall-liners and the model flow, which may be represented as an interference flow field, considered to be superimposed on a free-air (*i.e.* unconstrained) flow field. Considerable efforts have been made, and are continuing, to develop methods of assessing the wall-induced interference flow field produced in various situations. The present Item makes use of research (Ref. 22) indicating the value of utilising three-dimensional linearised flow-theory. Much effort has also been given to procedures for ‘correcting’ the measured test parameters, such as incidence, lift, drag and stream speed, to take into account the effects of the wall interference flow field, and also to the development of methods of accounting for other features of the test situation, such as interference from the model support system, thereby finally providing the desired aerodynamic characteristics for the ‘model-alone’ in free air. A widely-used source of interference estimation and correction procedures is Ref. 5. A short bibliography of more recent work is given in Ref. 20. Other major relevant sources are Refs 15, 24 and 25.

It is usual to consider the wall-induced interference on models of lifting configurations such as wings and aircraft in terms of two, largely independent, effects, attributable respectively to the x -component of the interference velocity field (see Sketch 1.1) termed blockage, and the z -component, termed upwash interference (more simply, upwash) or lift interference. Effects of the y -component of interference are usually negligible. Provided the model is positioned close to the mid-plane of the working-section, denoted by $z = 0$, as shown in Sketch 1.1, blockage in this plane is determined by the model thickness and its distribution, and by the displacement effects of the model boundary-layer and viscous wake, and is therefore only indirectly affected by the model lift and its distribution. Similarly, upwash in the $z = 0$ plane is determined by the model lift and its distribution and is not directly affected by the model thickness or viscous wake – only indirectly by them so far as they influence the model lift and its distribution. In simple terms a positive upwash component in the model region increases the effective wing incidence, the amount of the increase depending on the model lift, whilst a positive blockage component increases the effective wind speed, with consequent increases in the effective kinetic pressure, Mach number and Reynolds number of the test. For high quality testing, interference levels are necessarily small. In such cases changes to stream speed due to upwash, and to flow angle due to blockage, are both very small and may be neglected.

Where the interference flow field is more or less uniform in the region of the model, corrections to the wind speed and incidence are well-justified and are simple to apply. Where there are significant variations in blockage and/or upwash over the region occupied by the model the corrections are less soundly based and more complex to apply. Approximate methods are often used to account for a streamwise gradient in the blockage component (in terms of a streamwise ‘buoyancy’ force), and for a streamwise gradient in the upwash component, which is considered as an effective additional camber in two-dimensional flow, but which generally also introduces an effective additional twist across the model span in three-dimensional flow. Methods for correcting for upwash are considered further in Section 6, both for ‘nearly’ uniform and for non-uniform upwash. The purpose of these corrections is to yield estimates for the aerodynamic forces and moments experienced by the actual test model in free air, rather than for a geometry which differs from the actual one by having an additional camber and twist. Corrections to the aerodynamic coefficients to account for the effects of blockage (which are not dealt with in this Item), are normally applied subsequent to those accounting for upwash, see, for example, Ref. 13. Studies and data relating to blockage corrections are given in Refs 5, 14, 15, 22, 24, 25 and 30.

The situation where the interference flow field has significant non-uniformity, making correction to free-air conditions unreliable, has been considered in Ref. 18, where criteria are given for levels of flow non-uniformity which should not be exceeded if a specified tolerance on corrected (free-air) drag coefficient is to be met. The implications of these criteria for upwash non-uniformity and corresponding maximum acceptable values of lift coefficient for wind-tunnel tests are considered in Section 6.3. Results for the range of cases considered in the Item are given in Table 13.5.

Work by Firmin and McDonald (Ref. 22) on a swept half-wing model mounted in a solid-liner test section revealed that the upwash interference could be determined with good accuracy, at a given lift coefficient, by means of subsonic linearised-theory at test Mach numbers even as high as 0.85. It was found that calculations of the upwash using the linearised theory may be adequate even in cases where shock waves and boundary-layer separation occur in the flow near the model. The fact that the linearised theory can be used to determine the interference flow, at such Mach numbers and conditions where it would not be adequate for dealing with the flow about the model, may be ascribed to the nature of the wall-induced interference flow field, in which disturbances are everywhere small, even at relatively high subsonic speeds, as opposed to the model flow field, in which large disturbances arise. Simply because the linearised theory can provide the upwash interference adequately in certain cases it cannot be inferred that the same theory necessarily gives adequate corrections to the wind tunnel data for those cases, since such corrections rely on the linearised theory providing the aerodynamic effects of the interference flow on the model. Deficiencies in the correction procedure are likely to be most severe when the model flow is complex and the interference flow has significant non-uniformity in the region of the model.

The somewhat surprising range of applicability of linearised theory (which is considered in more detail in Section 11) justifies its adoption for the present Item, where it is used to generate upwash data for a range of wing and wind-tunnel configurations.

The data may be applied both to full-wing and half-wing tests. For the latter, the wind-tunnel wall-liner on which the half-wing is mounted is assumed to act as a reflection plane. For convenience the presentation of data is given with notation appropriate to full-wing tests. Symmetry about the $y = 0$ plane is assumed, with the wing located near the mid-plane ($z = 0$) of the test section.

Adoption of the subsonic linearised-theory leads to the use of a coordinate transformation to take account of compressibility (Section 3), in which the y - and z -coordinates are multiplied by $\beta (= (1 - M_\infty^2)^{1/2})$ and the x -coordinate is left unchanged. Generally, the wings considered have zero twist and camber, and according to the linearised theory, their lifting characteristics and interference upwash are then defined, at a given value of M_∞ , by planform alone, see Section 3. Effects of wing twist and camber are considered in Section 7. Aerodynamic characteristics and upwash for the actual wing planform may be derived from data for the corresponding transformed planform, the shape of which is obtained by reducing the spanwise dimensions of the actual planform by factoring them by β . In the Item, data for wall-induced upwash are presented for a range of 66 transformed 'swept-tapered' wing planforms, see Table 13.1, each for three values of s/b' and three values of h'/b' (i.e. nine cases for each planform). Use of the equivalent wing concept enables the results to be applied to wings having more complex planforms than those in Table 13.1. (e.g. to wings with leading- and trailing-edge cranks), see Section 4.3.

Upwash interference data over the region of the wing planform are presented in terms of distributions of the upwash interference parameter δ . Adopting δ as the relevant parameter links the non-dimensional upwash velocity, w/U , to the lift coefficient rather than to wing incidence. This is beneficial since it is the lift (and its distribution), rather than the wing incidence, which determines the upwash. Accurate values of δ can thus still be derived from linearised theory, even where the theory poorly predicts the overall lift at a given incidence, provided it gives the distribution of lift adequately. Since the effects of changes in the lift distribution on the upwash are relatively weak (Section 7), it is not even important to the calculation of upwash for the prediction of the lift distribution by the linearised theory to be particularly good.

In dealing with upwash interference and its effects on wings of small span relative to the working-section dimensions, and also for treating upwash interference in two-dimensions (*i.e.* for aerofoils), it is valuable to utilise the parameters δ_0 and δ_1 (see References 5 and 13 for example). The parameter δ_0 is the value of δ at the wing centre of pressure, and δ_1 is directly proportional to the streamwise gradient of δ (at the same point). In these situations, the influence of upwash interference on the linearised-theory corrections to incidence, lift, drag and pitching moment involves only δ_0 and δ_1 . For wings which are not small relative to the working section dimensions, other features of the distribution of upwash, besides δ_0 and δ_1 , influence the linearised-theory corrections. In the present Item, consideration is given to these corrections directly, without reference to the parameters δ_0 and δ_1 .

In Figures 1 to 66, spanwise variations of δ along the leading edge, mid-chord line and trailing edge are shown. In addition, Table 13.2 gives mean values of δ (where the mean is taken over the planform area), and Table 13.3 gives values of δ_L and δ_D , the values of δ which, according to the linearised theory, provide corrections to incidence such that values of lift and (vortex) drag respectively are unchanged between the wind tunnel and free-air (*i.e.* there are then zero 'residual' corrections to be applied to those particular force components). The aerodynamic centre positions of the transformed wings and the aft shifts in these positions produced by non-uniformity in upwash interference, as calculated using the linearised theory, are given in Table 13.4.

In addition to the data in the Figures and Tables, interference data for the range of cases considered are provided by the computer program A9514, which is described in Appendix A. The program also provides the effective additional incidence, twist and camber associated with upwash. Use of this program should be particularly valuable should it be desired to interpolate (or extrapolate) interference data for the cases provided to other values of A_T , $A_{\tan\Lambda_{1/2}}$, λ , s/b' or h'/b' .

3. BASIC RELATIONS OF LINEARISED FLOW THEORY

3.1 Velocity Potential and Boundary Conditions

The governing equation for the disturbance-velocity potential in linearised, three-dimensional, ideal compressible flow is

$$\beta^2 \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0. \quad (3.1)$$

The undisturbed stream, of speed U , is in the x -direction.

The velocity components in the x , y , z directions are respectively $U + \partial\phi/\partial x$, $\partial\phi/\partial y$ and $\partial\phi/\partial z$.

The pressure coefficient in linearised flow theory is given by the first-order expression

$$C_p = -\frac{2}{U} \frac{\partial \phi}{\partial x}.$$

The boundary condition that the flow is tangential to the wing surface is transferred to the plane $z = 0$. Denoting the wing surface by

$$\begin{aligned} z &= z_U(x, y), & (\text{upper surface}), \\ z &= z_L(x, y), & (\text{lower surface}), \end{aligned}$$

(see Sketch 3.1), the linearised boundary condition is given by the equations

$$\left. \begin{aligned} \frac{\partial \phi}{\partial z}(x, y, 0+) &= U \frac{\partial z_U}{\partial x}(x, y) \\ \frac{\partial \phi}{\partial z}(x, y, 0-) &= U \frac{\partial z_L}{\partial x}(x, y) \end{aligned} \right\} \quad (3.2)$$

This condition is applied over the region defined by the projection of the wing on to the $z = 0$ plane. Equations (3.2) indicate that $\partial \phi / \partial z$ is discontinuous across the plane $z = 0$. In Equations (3.2), $0+$ and $0-$ denote conditions on either side of the plane $z = 0$ (i.e. they refer to limits as $z = 0$ is approached through positive and negative values).

For a wing in a wind tunnel, the condition that the flow is tangential to the solid wall-liners also has to be applied. Denoting the general inward normal-coordinate on the test-section boundary by n , this boundary condition is

$$\frac{\partial \phi}{\partial n}(x, y, z) = 0 \quad (3.3)$$

Equation (3.3) has to be applied over each of the wall liners, which are taken in this Item as extending to infinity upstream and downstream.

In the linearised theory of wings the velocity potential is split into two components, ϕ_s and ϕ_a , respectively symmetric and antisymmetric with respect to $z = 0$. Both ϕ_s and ϕ_a satisfy Equation (3.1). Thus

$$\left. \begin{aligned} \phi(x, y, z) &= \phi_s(x, y, z) + \phi_a(x, y, z), \\ \text{where } \phi_s(x, y, z) &= \phi_s(x, y, -z), \\ \text{and } \phi_a(x, y, z) &= -\phi_a(x, y, -z). \end{aligned} \right\} \quad (3.4)$$

The symmetric component, ϕ_s , satisfies the boundary conditions

$$\frac{\partial \phi_s}{\partial z}(x, y, 0 \pm) = \pm U \frac{\partial}{\partial x} \frac{(z_U - z_L)}{2} = \pm U \frac{\partial z_t}{\partial x}, \quad (3.5)$$

where z_t is the wing semi-thickness ordinate.

The antisymmetric component, ϕ_a , satisfies the boundary conditions

$$\frac{\partial \phi_a}{\partial z}(x, y, 0 \pm) = U \frac{\partial}{\partial x} \frac{(z_U + z_L)}{2} = U \frac{\partial z_m}{\partial x}, \quad (3.6)$$

where z_m denotes the wing mean-surface ordinate.

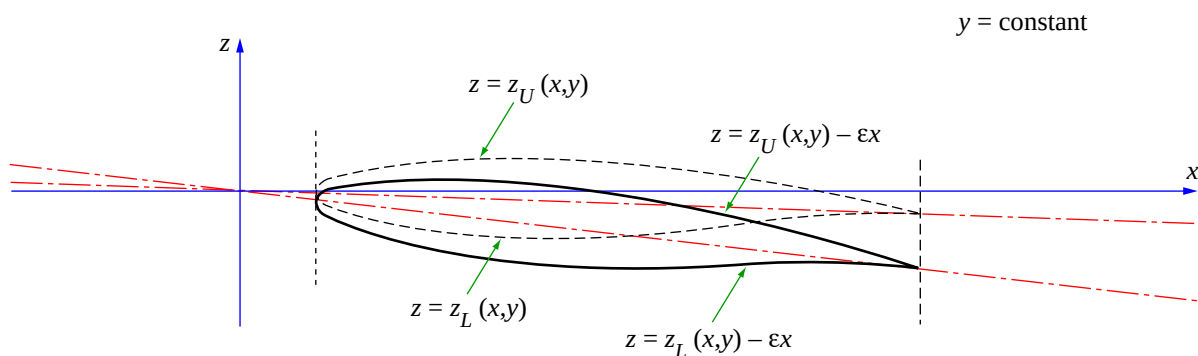
The sum of ϕ_s and ϕ_a then satisfies Equations (3.1) and (3.2). Only ϕ_a can contribute a pressure difference across the $z = 0$ plane, and so, according to linearised theory, wing lift is independent of wing thickness.

When the x -axis is aligned with the undisturbed stream, as in this Item, a change in wing angle of incidence

is introduced by changing the wing surface coordinates. Since incidence is assumed to be small, an incidence change of ε is equivalent to changing both upper and lower surfaces (and therefore z_m) by $-\varepsilon x$ (see Sketch 3.1), leaving z_t and the region over which the boundary conditions are applied unaffected.

It is only the *slope* of the mean surface that enters the boundary condition Equation (3.6) and affects the wing lift. For a wing, with zero twist and camber, inclined at incidence $\bar{\alpha}$, the boundary condition for ϕ_a is

$$\frac{\partial \phi_a}{\partial z} (x, y, 0 \pm) = -U\bar{\alpha} . \quad (3.7)$$



Sketch 3.1 Approximate representation of incidence change – undisturbed stream aligned with x-axis

3.2 Upwash Interference

For a wing symmetrically positioned in a wind tunnel (as shown in Sketch 1.1), a velocity potential ϕ_s satisfying the wing boundary condition of Equation (3.5) and the wind tunnel wall-liner boundary condition of Equation (3.3) still satisfies the symmetry condition in Equation (3.4). Similarly a potential ϕ_a which satisfies the wing and wall-liner boundary conditions still satisfies the antisymmetry condition. These results apply *only* when the wing is located near the mean z -plane of the test section.

It follows that for wings symmetrically located in the test section, the contribution to ϕ_s from the walls (*i.e.* the wall interference) must also be symmetrical about $z = 0$. Since the wall-liner contribution to ϕ_s must be non-singular in the plane $z = 0$, its z -derivative (*i.e.* upwash component) must be zero in that plane. Thus for wings symmetrically located in the test section, as in this Item, the wall-induced upwash in the $z = 0$ plane is independent of the wing thickness. It should be noted that wall-induced upwash away from the $z = 0$ plane does have a contribution from the wing thickness.

The solution procedure for determining the flow field corresponding to ϕ_a , which is described more fully in Section 9, entails representing the antisymmetric wing flow contribution by a vortex-lattice, and the wall influence by a combination of wall source-panels and reflections of the wing vortex-lattice. Once the flow solution is determined, the wall-induced interference flow is derived by adding the effects of the wall source-panels and the vortex-lattice reflections – direct contributions from the wing vortex-lattice are not included.

3.3 Transformed Flow; Relations Between Actual and Transformed Quantities

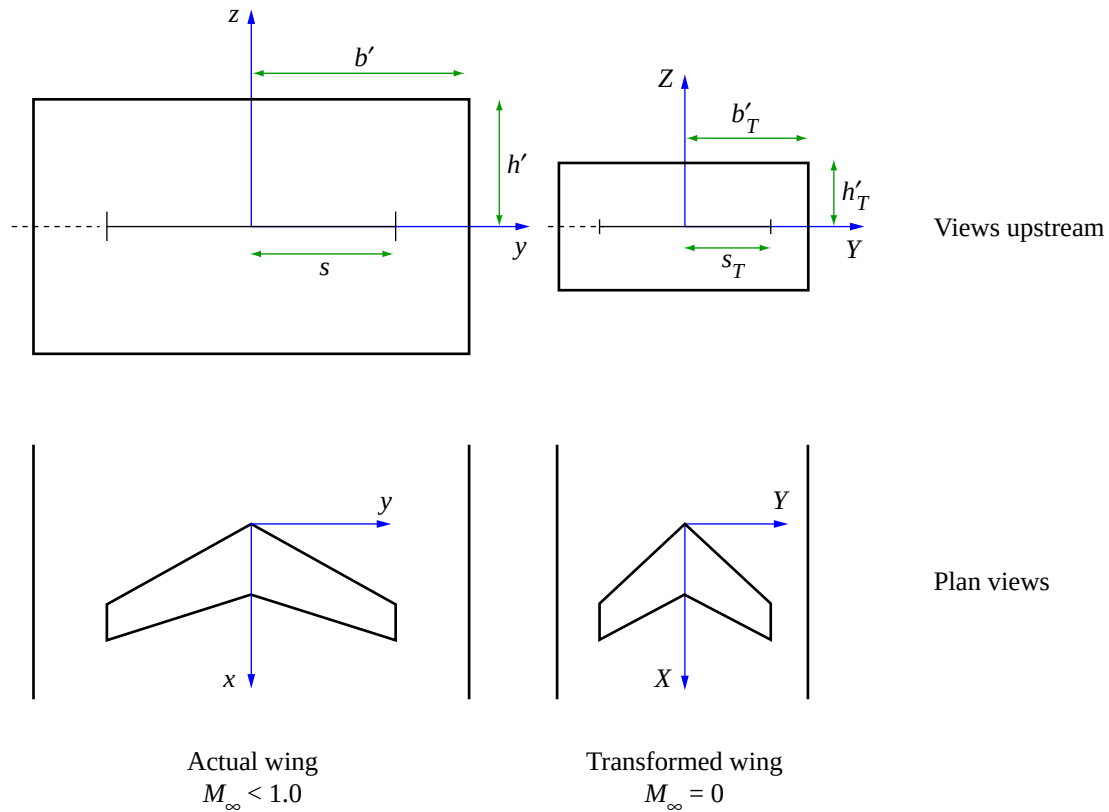
For subsonic flows it is convenient to transform the system of equations in Section 3.1 to an equivalent

system of equations relating to a flow with $M_\infty = 0$, (i.e. $\beta = 1$ in Equation (3.1)) to which the methods of solving the incompressible flow equations can be applied. This transformation may be done in various ways; the following procedure is both common and convenient. It is the one utilised and explained in detail in Ref. 11. The wing shape obtained in the transformed flow is referred to here as the transformed wing; it is also commonly known as the ‘analogous wing’ (as in Ref. 5, for example). Variables without a suffix relate to actual (or physical) quantities; those with suffix T denote quantities in the transformed flow, with the exception of the coordinates, where (X, Y, Z) are used for the transformed flow. Thus

$$\begin{aligned} X &= x, \\ Y &= \beta y, \\ Z &= \beta z, \\ U_T &= U, \\ \phi_T &= \beta^2 \phi. \end{aligned}$$

Applying these transformations to a wing in a wind tunnel yields the following geometrical relations (see Sketch 3.2).

$$\begin{aligned} s_T &= \beta s, & b'_T &= \beta b', \\ Y/s_T &= y/s, & h'_T &= \beta h', \\ c_{0T} &= c_0, & C_T &= \beta^2 C, \\ C_{iT} &= c_i, & s_T/b'_T &= s/b', \\ \lambda_T &= \lambda, & h'_T/b'_T &= h'/b', \\ S_T &= \beta S, \\ A_T &= \beta A, \\ \tan \Lambda_{\frac{1}{2}T} &= (1/\beta) \tan \Lambda_{\frac{1}{2}}, \\ A_T \tan \Lambda_{\frac{1}{2}T} &= A \tan \Lambda_{\frac{1}{2}}. \end{aligned}$$



Sketch 3.2 Relation between actual and transformed wing planforms and wind-tunnel cross-sections

The following relations between actual and transformed aerodynamic quantities also apply.

$$\begin{aligned} C_{pT} &= \beta^2 C_p, \\ C_{LT} &= \beta^2 C_L, \\ C_{DT} &= \beta^3 C_D, \\ \alpha_T &= \beta \alpha, \\ \delta_T &= \delta. \end{aligned}$$

In Sections 4 to 11, where a transformed quantity is equal to the corresponding actual quantity, as, for example, $A_T \tan \Lambda_{\frac{1}{2}T} = A \tan \Lambda_{\frac{1}{2}}$, the quantity is generally referred to *without* the use of the suffix *T*.

4. WING PLANFORM AND WIND TUNNEL PARAMETERS CONSIDERED

4.1 Unswept and Rearward-Swept Wing Planforms

The selected values of the transformed wing planform parameters, A_T , $A \tan \Lambda_{1/2}$, λ , covering unswept and rearward-swept cases, are given in Table 4.1. The resulting range of 60 transformed planforms is shown in Table 13.1, where the numbers assigned to the wing planforms are given. Corresponding results for these transformed planforms are presented in Figures 1 – 60.

A_T	1.5	3	5	8	12
$A \tan \Lambda_{1/2}$	0	2	4	6	
λ	0.25	0.5	1		

TABLE 4.1 Parameter values of transformed wing planforms

The range of transformed planforms adopted should cover most cases of interest for values of M_∞ up to about 0.9. Choosing fixed values of the parameter $A \tan \Lambda_{1/2}$ ($= A_T \tan \Lambda_{1/2}$) rather than fixed values of $\Lambda_{1/2}$ excludes highly swept wings of high aspect ratio which are rarely of interest. For each planform, results are presented for nine relative sizes of wind-tunnel cross-section, corresponding to $s/b' = 0.4, 0.6$ and 0.8 , and $h'/b' = 7/10, 1$ and $10/7$. Values of s/b' below 0.4 would normally be considered unnecessarily small (*i.e.* a relatively larger model yielding higher test Reynolds numbers would be preferred) whilst values above 0.8 would normally be considered likely to give rise to too much variation of wall-induced interference over the outer part of the wing. The values of h'/b' adopted cover the range of test-section proportions in common use for full-wing tests but for half-wing tests values of h'/b' as low as 0.5 may be employed. In Section 5.3 some effects associated with low values of h'/b' are presented.

4.2 Forward-Swept Wing Planforms

Results for six forward-swept transformed wing planforms, numbered as indicated in Table 13.1 are presented in Figures 61 – 66. For each of these wings $\lambda = 0.5$. Values of A_T selected are $1.5, 3$ and 5 , and values of $A \tan \Lambda_{1/2}$ selected are -2 and -6 .

4.3 Extension of Upwash Data to Wings of More General Planform

In this Section an approach is considered for extending to wings of more general planform the utility of the upwash data presented for the swept-tapered wings detailed in Sections 4.1 and 4.2.

The spanwise load distribution, due to an increment of incidence[†], of a swept wing having a leading edge and/or trailing edge which is cranked or curved is treated in Ref. 3, where a rapid estimation method is presented. A somewhat simpler approach, utilising the concept of an equivalent swept-tapered wing planform, is presented in Refs 8 and 12 and is adequate for estimating overall wing properties. The latter approach may be adopted to extend the present method of determining upwash interference for wings of swept-tapered planform so as to provide a method of estimating upwash for wings of more general planform. The basis of this method is the observation that the distribution of upwash parameter over the wing planform is relatively insensitive to the distribution of wing loading, both chordwise and spanwise. To be applicable, therefore, load distribution changes, resulting from any twist or camber present, should not be so large as to affect the upwash significantly. Effects of load distribution changes produced by twist and camber are considered further in Section 7.

[†] This corresponds to the load distribution of a wing with zero twist and camber.

Replacing an actual wing by an equivalent wing has a similar effect, at a moderate distance from the wing, to redistributing the aerodynamic loading in the chordwise and spanwise directions. Section 4.3.1 gives examples of estimating the upwash by use of an equivalent wing, together with comparisons of calculations in which the planform effects are correctly dealt with.

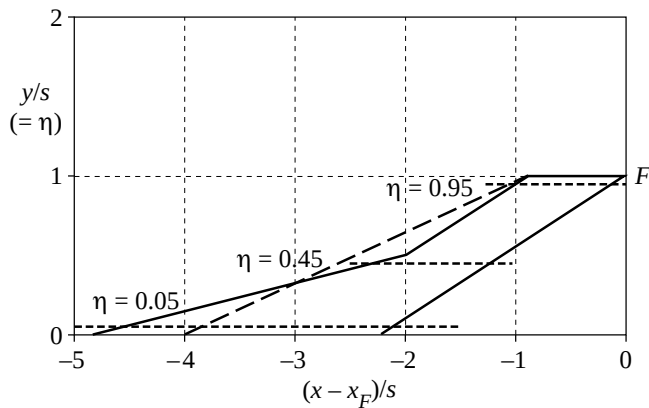
4.3.1 Examples of estimation of upwash by use of the equivalent wing concept

In this Section two examples of cranked planforms are considered, and the distribution of upwash interference for each wing is compared with that of its equivalent swept-tapered planform. In one example the leading edge has a crank, and in the other the trailing edge has a crank.

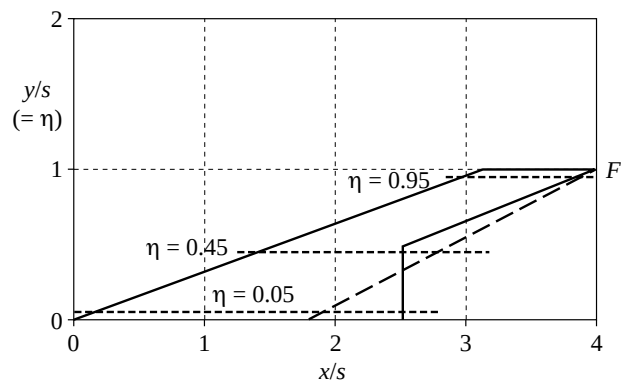
In each case the crank is at mid semi-span, and the equivalent wing is transformed planform 31 ($A_T = 1.5$, $A \tan \Lambda_{1/2} = 4$, $\lambda = 0.5$). The rule for obtaining the equivalent wing amounts to replacing the cranked edge by a straight one that gives the same planform area and the same tip chord as those of the cranked-edge planform – see Reference 12.

Both cranked planforms have untapered outer wing panels. The cranked trailing-edge planform has an unswept trailing edge on the inner wing panel. The planforms are shown in Sketches 4.1 and 4.2, in each case in relation to transformed planform 31. Note that the orientation of the planforms in Sketches 4.1 and 4.2 corresponds to a stream direction across the page, not down the page as in Figures 1 to 66. Transformed planform 31 is chosen because it yields relatively large chordwise and spanwise variations of the upwash interference parameter, δ , and is therefore likely to be more sensitive to changes in wing load distribution and edge geometry than most of the transformed planforms.

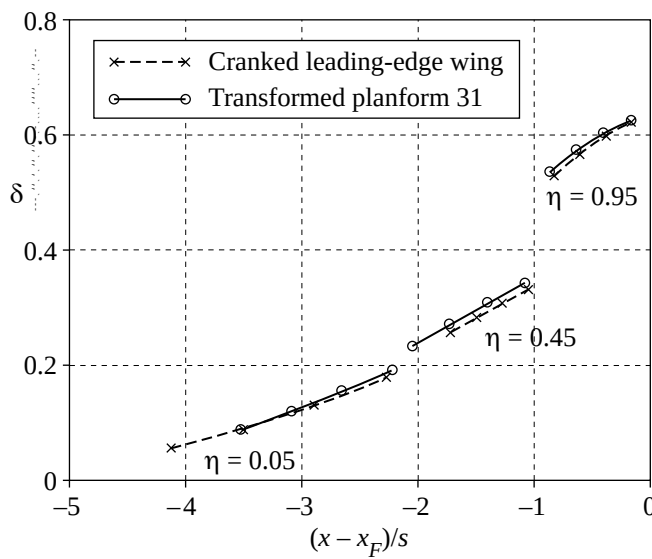
Results for δ are shown for $\eta = 0.05$, 0.45 and 0.5 for the two cranked planform wings in Sketches 4.3 and 4.4, where results for transformed planform 31 at the same values of η are also given. (These η values correspond to mid-span locations of panels utilised in the method of determining upwash interference – see Section 9). When plotted relative to a fixed tip chord and in terms of x rather than ξ , the cranked wing results are seen to differ little from those of transformed planform 31, despite the large differences between the cranked wing planforms and the equivalent wing, which amount to differences in local chord (at fixed η) of up to $0.67 \bar{c}$.



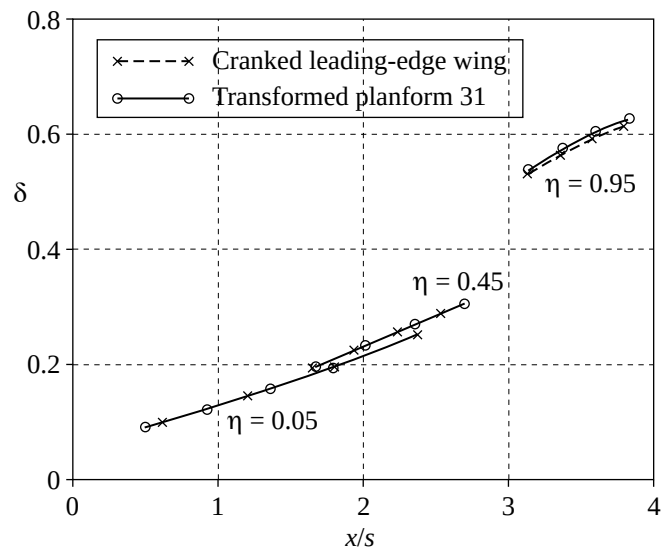
Sketch 4.1 Cranked leading-edge wing and transformed planform 31



Sketch 4.2 Cranked trailing-edge wing and transformed planform 31



Sketch 4.3 Comparison of upwash interference parameter for cranked leading-edge wing and equivalent wing



Sketch 4.4 Comparison of upwash interference parameter for cranked trailing-edge wing and equivalent wing

5. RESULTS FOR FLAT WINGS

5.1 Presentation and Interpretation

Data for $\delta (= \delta_T)$ for each case of transformed planform and relative size of test section are presented graphically in Figures 1 to 66 in the form of curves showing the variation of δ along three straight lines, namely the leading edge, the mid-chord line and the trailing edge. Each Figure contains nine graphs, arranged with s/b' increasing across the page, and h'/b' increasing up the page. These results relate to flat[†] wings, *i.e.* the geometric incidence of the wing mean surface has been taken to be uniform over the planform, so the wings are without twist or camber. In each case, δ , at any spanwise position, increases in the streamwise direction, generally in a nearly linear manner. In Section 7, effects on δ of changes in the load distribution from that produced on a flat wing are presented, and shown to be generally small or readily taken into account.

Appendix A describes the computer program ESDUpac A9514 which provides detailed results for upwash (and the other interference-related parameters considered in this Item) for any case (or cases) selected from the range presented in Section 4. An example program output is given in Section A4, and comments on that example are given in Section A5.

The curves of δ provide a means of assessing the interference upwash distribution over the wing planform, and of comparing the effects of changes of various parameters, such as s/b' , h'/b' , and test Mach number, M_∞ . Consideration of effects of changing M_∞ for a particular wing is facilitated as a result of selecting fixed values of $A \tan \Lambda_{1/2}$, since changing M_∞ leaves this parameter (as well as λ) unchanged. The range of M_∞ to which the results are applicable is considered in Section 11. As an example, to see what effect increasing M_∞ has on δ for a wing with $A = 12$, $\tan \Lambda_{1/2} = 0.5$ ($\Lambda_{1/2} = 26.6^\circ$), $\lambda = 0.5$ (which corresponds to transformed planform number 40 at $M_\infty = 0$), the transformed planforms to be considered are those on the same horizontal line as transformed planform 40 in Table 13.1, with the correspondence with M_∞ as shown in Table 5.1.

Transformed planform number	β	M_∞	$\Lambda_{1/2}, \text{deg}$
40	1	0	26.6
39	0.667	0.745	36.9
38	0.417	0.909	50.2
37	0.250	0.968	63.4
36	0.125	0.992	76.0

TABLE 5.1 Example of correspondence between transformed wing planforms and M_∞

In this example, only the first two values of M_∞ fall within the range for which results may be reliably used in practice, see Section 11. Use of the results for the other values of M_∞ in Table 5.1 can be valuable for interpolation to intermediate values of M_∞ .

5.2 Trends In Upwash Parameter for Unswept and Rearward-Swept Planforms

A number of trends are revealed by the data in Figures 1 to 60. These are considered in Sections 5.2.1 to 5.2.5.

[†] It is sufficient for the wings to have zero twist and camber. It is not strictly necessary for them to have planar mean surfaces, *i.e.* they may be bent spanwise and have dihedral. Nevertheless, it is convenient simply to refer to the wings as flat.

5.2.1 Effect of span to cross-section dimension in spanwise direction, s/b'

The clearest of the trends revealed is the effect of s/b' . Reference to any of the figures presented shows that δ tends to increase more rapidly with y/s as s/b' increases. This trend is associated with the progressively more pronounced effects on upwash as the wing tips are approached for the cases where the walls normal to the wing plane are relatively close to the wing tips. The effect may be understood by considering these walls as acting as reflection planes, with the consequential appropriately placed 'image' wings producing the (wall-induced) upwash. The closer the image wings (*i.e.* the greater s/b') the greater the spanwise variation in upwash they produce. Another noticeable effect of increasing s/b' is to increase the streamwise variation of δ , *i.e.* to increase the difference between the three curves denoting leading edge, mid-chord and trailing-edge data. This effect is associated with the streamwise variation of the loading of the image wings (which is partly the cause of the streamwise variation of δ) having a greater influence for the closer image wings.

5.2.2 Effect of cross section proportion, h'/b'

A clear trend shown by all the graphs of δ is that for a given transformed wing planform, increasing h'/b' results in an increase in δ . For s/b' values of 0.4 and 0.6 this increase is fairly uniform with y/s and roughly in proportion to the value of h'/b' particularly for transformed wings with high A_T . For $s/b' = 0.8$, the increase with h'/b' becomes more pronounced as y/s approaches 1. The first of these effects can be explained partly in terms of the way δ is defined, namely with the wind tunnel cross sectional area C in the numerator. If the wind tunnel walls parallel to the wing plane had no effect on upwash (as would be the case for very large h'/b' values) then increasing h'/b' would result in a proportional increase in δ . Even for the values of h'/b' adopted, the tendency for the walls parallel to the wing plane to have a minor influence on upwash is seen. In the case of $s/b' = 0.8$ the walls parallel to the wing plane have the most effect (for any given h'/b') and the closer they are to the wing (*i.e.* the lower the value of h'/b') the more they tend to suppress the increase of δ towards the wing tips. Again this effect can be understood in terms of the walls acting as reflection planes. The image wings 'producing' the effects of the walls parallel to the wing plane are negative images (unlike those producing the effects of the walls normal to the wing plane which are positive images), and thus their effect is to reduce the wall-induced upwash.

5.2.3 Effects of transformed wing aspect ratio, A_T

For given values of $A \tan \Lambda_{1/2}$, λ , s/b' and h'/b' , changes in A_T are equivalent to changes in M_∞ for a particular *actual* wing planform, with decreases in A_T corresponding to increases to M_∞ . Decreases in A_T result in increases in the streamwise variation of δ . A further effect of an increase in M_∞ is to increase the amount by which δ rises across the span. Both of these effects can be seen by comparing, for example, Figures 15 and 11. The former effect is attributable to the increases of the transformed wing chord lengths (relative to span) as A_T decreases, since although the rate of change of w/U with x decreases slightly with increase in chord length, the net result of an increase in chord is to increase δ over the chord. The effect on the variation of δ across the span is largely attributable to the fact that, for given $A \tan \Lambda_{1/2}$ ($= A_T \tan \Lambda_{1/2}^T$) as A_T decreases, $\tan \Lambda_{1/2}^T$ (and hence sweep angle) increases. This results in the spanwise load distribution changing, with the outboard region tending to become more highly loaded as A_T decreases. The consequence is that the outboard values of δ tend to increase relative to those inboard.

5.2.4 Effects of parameter $A \tan \Lambda_{1/2}$

For given values of A_T , λ , s/b' and h'/b' , increasing $A \tan \Lambda_{1/2}$ has the effect of significantly increasing the spanwise variation of δ and slightly decreasing the streamwise variation of δ . Both of these effects can be seen by comparing, for example, Figures 44 and 59. The former effect can be attributed to the tendency for a greater proportion of the spanwise loading to be carried outboard with increasing $A \tan \Lambda_{1/2}$. The latter effect can be explained in terms of the influence of the image wings. As $A \tan \Lambda_{1/2}$ increases, the

load distribution of each image wing extends over a longer overall streamwise distance, with the consequence that the net variation of loading over a given length decreases. It is this more gradual streamwise variation of loading that produces the decreasing streamwise variation of δ with increasing $Atan\Lambda_{1/2}$.

5.2.5 Effects of taper ratio, λ

For given values of A , $Atan\Lambda_{1/2}$, s/b' and h'/b' , increasing λ results in very little change in the values of δ apart from an increase in the streamwise variations of δ towards the tip. This is seen by comparing, for example, Figures 3 and 43. The effect is linked to the fact that increases in λ correspond to greater chord lengths over the more outboard regions of the wing, and consequently to an increased variation in upwash over the chords of these outboard regions for a given streamwise rate of increase of upwash.

5.3 Trends in Upwash Parameter for Forward-Swept Planforms

Trends in the interference upwash parameter, δ , for forward-swept planforms, revealed in Figures 61 to 66, are influenced by three main effects: the tendency of δ to increase both with increasing X (at fixed Y), and with increasing (positive) Y (at fixed X), and the tendency for an increasing proportion of spanwise loading to be carried inboard with increasing forward sweep.

The first two of these effects apply also to rearward-swept planforms, for which they combine to result in δ increasing outboard at fixed ξ (*i.e.* for rearward-swept planforms, δ increases outboard along the leading edge, mid-chord line and trailing edge). For forward-swept planforms the first two effects oppose each other (in terms of spanwise changes in δ along a line of fixed ξ) so that, compared to the corresponding rearward-swept planform, the spanwise variations of δ are much reduced. For the more extreme forward-sweep angles (see Figures 64 – 66), values of δ at fixed ξ actually decrease outboard.

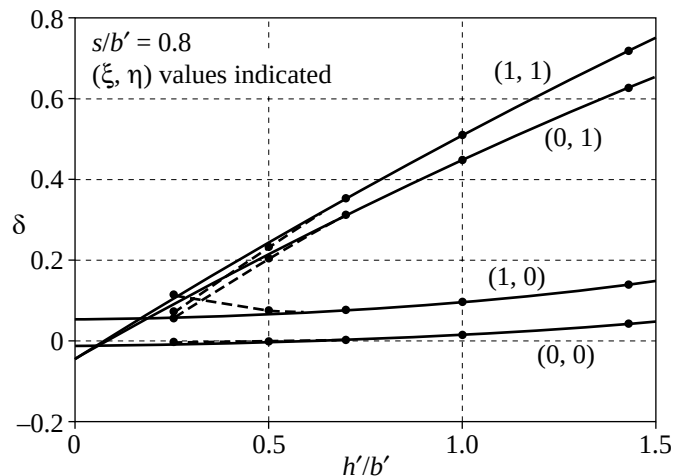
As a result of such differences in the influence of the first two effects, changes in the behaviour of δ resulting from changes in s/b' , h'/b' , A_T and $Atan\Lambda_{1/2}$ are either much reduced or reversed for forward-swept planforms in comparison with rearward-swept planforms. For example, for forward-swept planforms, increasing s/b' does not invariably result in an increased variation of δ (at fixed ξ) over the span – see Figure 66.

5.4 Effects of Low Values of h'/b'

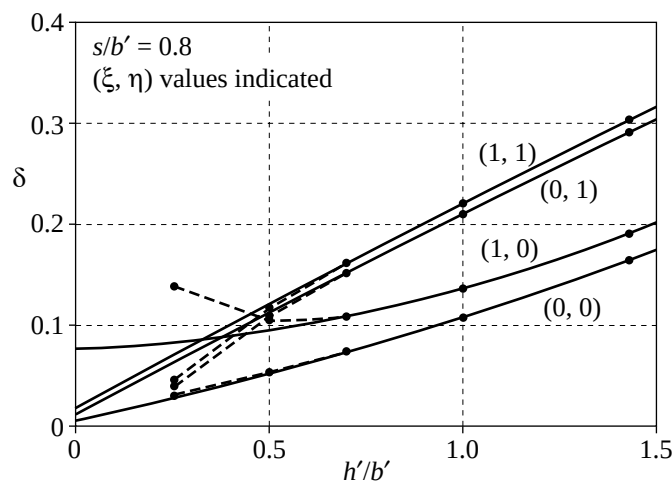
Data for δ are presented in Figures 1 to 66 for three values of h'/b' : 0.7, 1.0 and 10/7. For values of h'/b' intermediate between these, linear interpolation is adequate. Occasionally values of h'/b' lower than 0.7 are of interest. For such lower values the character of the wall interference, as revealed by the variations of δ with ξ and η , tends to change, with the spanwise variation decreasing, and the chordwise variation generally increasing inboard, but decreasing outboard. Although there is also a marked decrease in the average level of δ as h'/b' decreases, this is largely a consequence of the definition of δ involving C , the working-section cross-sectional area. It does not imply a corresponding decrease in the wall-induced upwash angle (w/U) for given C_L .

Provided δ is required for values of h'/b' no lower than about 0.5, it appears that the data presented may be used to provide results with fair accuracy by means of extrapolation. As an example, in Sketch 5.1 data for δ are shown corresponding to transformed planform 36 for $s/b' = 0.8$ and for values of h'/b' in the range 0.25 to 10/7 (= 1.429..). Dashed lines connect the calculated points. The solid lines are parabolic fits of the data for $h'/b' = 0.7$, 1 and 10/7. It is seen that errors in δ resulting from extrapolation to $h'/b' = 0.5$ are about 0.01. This is an indication of the levels of error in δ that can be expected from similar extrapolation for other transformed planforms and values of s/b' . Extrapolation to $h'/b' = 0.25$ leads to large errors in δ and is clearly unsatisfactory.

For cases where there is less variation in δ across the span than in the example in Sketch 5.1 (i.e. for transformed planforms with relatively high values of A_T , and for the lower values of s/b') the errors in δ resulting from extrapolating in h'/b' are generally decreased, although the inboard increase in the chordwise variation of δ (with decreasing h'/b') means that local errors in δ of about 0.01 still occur for $h'/b' = 0.5$. Sketch 5.2 gives the example of transformed planform 19 for $s/b' = 0.6$. Note that the scales for δ in Sketches 5.1 and 5.2 differ. Again the solid lines are parabolic fits of the data for $h'/b' = 0.7, 1$ and $10/7$ and dashed lines connect the calculated points. Extrapolation of δ to values of h'/b' less than 0.5 again gives rise to unsatisfactorily large errors.



Sketch 5.1 Effect of h'/b' on δ for transformed planform 36 with $s/b' = 0.8$



Sketch 5.2 Effect of h'/b' on δ for transformed planform 19 with $s/b' = 0.6$

6. CORRECTION FOR UPWASH INTERFERENCE

6.1 Nearly-Uniform Upwash Interference

If the wall-induced upwash, w , is uniform, or nearly so, in the region occupied by the wing, it effectively changes the local wing incidence, α , by a constant amount over the planform, and is then equivalent to a change in $\bar{\alpha}$ without influencing the effective wing twist or camber. So far as the upwash interference is concerned, the flow about the test wing in the wind tunnel is then the same as that about the test wing in free air at the 'corrected' value of incidence. If w_{rep} denotes, the selected 'representative' value of w in the region of the test wing, then the corrected incidence is given by

$$\bar{\alpha}_{corr} = \bar{\alpha} + \Delta\bar{\alpha}, \quad (6.1)$$

where

$$\Delta\bar{\alpha} = w_{rep}/U. \quad (6.2)$$

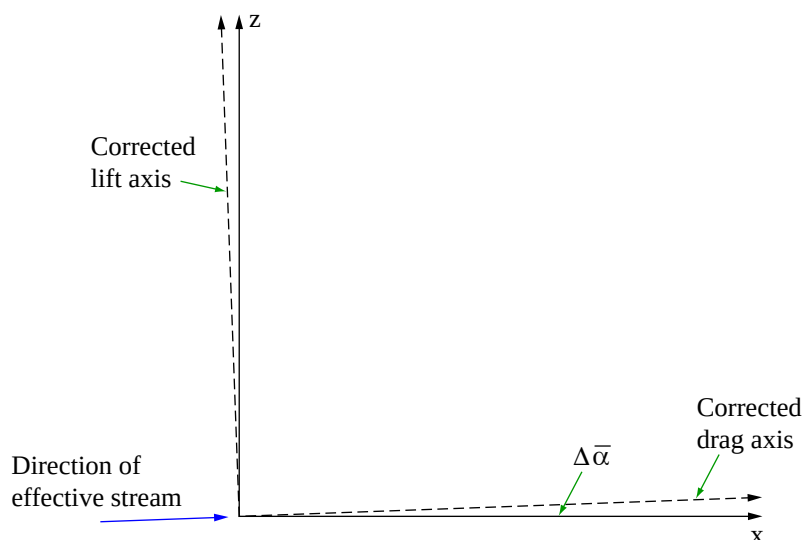
Lift, drag and moment coefficients, corrected for upwash interference, are obtained by taking the equivalent free-air undisturbed stream to be inclined upwards at the angle $\Delta\bar{\alpha}$ to the x -axis. The relation between the corrected lift and drag axes and the corresponding wind-tunnel axes, namely the z - and x -axes, is shown in Sketch 6.1.

Since $\Delta\bar{\alpha}$ is small

$$C_{Lcorr} \approx C_L - C_D \Delta\bar{\alpha}, \quad (6.3)$$

$$C_{Dcorr} \approx C_D + C_L \Delta\bar{\alpha}, \quad (6.4)$$

$$C_{Mcorr} = C_M. \quad (6.5)$$



Sketch 6.1 Relation between wind-tunnel axes and corrected axes

Here C_L and C_D are the coefficients of lift and drag (*i.e.* the aerodynamic force components in the z and x directions) of the test wing, derived from balance measurements, while C_{Lcorr} and C_{Dcorr} are the lift and drag coefficients that would be experienced by the test wing in free air at incidence $\bar{\alpha}_{corr}$.

Generally $C_D \Delta \bar{\alpha}$ may be neglected in Equation (6.3), so

$$C_{Lcorr} \approx C_L. \quad (6.6)$$

The choice of w_{rep} and the corresponding value of the upwash interference parameter, δ , denoted by δ_{rep} , may be made in a number of ways. An obvious choice for δ_{rep} is as the mean value of δ over the planform area, δ_{av} , *i.e.*

$$\delta_{rep} = \delta_{av} = \int_0^1 \int_0^1 \delta(\xi, \eta) \frac{c(\eta)}{\bar{c}} d\xi d\eta, \quad (6.7)$$

so the incidence correction is then

$$\Delta \bar{\alpha} = \delta_{av} \frac{SC_L}{C}.$$

Two other choices of δ_{rep} are presented in Sections 6.2.1 and 6.2.2.

Results for δ_{av} are given in Table 13.2 for the range of cases considered in Section 4. As an indication of the degree of non-uniformity of δ , Table 13.2 also gives values of t , the maximum variation of δ over the entire wing planform.

6.2 Allowance for Non-Uniform Upwash using Linearised Theory

As indicated in Section 6.1, where the upwash interference is nearly uniform in the region of the test wing, the wind-tunnel lift and drag are applicable to the wing in free air – all that is needed is a rotation of the lift and drag axes through the angle $\Delta \bar{\alpha}$, the correction to the angle of incidence. When the upwash interference is non-uniform, it is possible to determine, using linearised theory, incidence corrections, $\Delta \alpha_L$ and $\Delta \alpha_D$, say, such that the wind-tunnel lift and drag[†] are, respectively, still applicable to the wing in free air, *i.e.*

$$C_{Lcorr} \approx C_L - C_D \Delta \alpha_L, \quad (6.8)$$

and

$$C_{Dcorr} \approx C_D + C_L \Delta \alpha_D. \quad (6.9)$$

Here C_L and C_D are the coefficients of lift and drag (*i.e.* the aerodynamic force components in the z and x directions) of the test wing, C_{Lcorr} is the lift coefficient that would be experienced by the test wing in free air at incidence $\bar{\alpha} + \Delta \alpha_L$, and C_{Dcorr} is the (vortex) drag coefficient that would be experienced by the test wing in free air at incidence $\bar{\alpha} + \Delta \alpha_D$. These corrections are to allow for the effect of the upwash interference component only. The incidence corrections $\Delta \alpha_L$ and $\Delta \alpha_D$ are dealt with in detail in Sections 6.2.1 and 6.2.2, respectively. In a similar way it is possible to determine a correction to the pitching moment characteristics using linearised theory – see Section 6.2.3.

[†] In linearised theory, the drag is due to the vortex wake and is related to the overall lift and lift distribution. The observed drag depends to a significant extent on viscous effects and on any shock waves present. Their contributions also vary with the local effective incidence (and thus with the lift and its distribution) but in a different way from that of the vortex-related drag. Care must therefore be exercised in applying $\Delta \alpha_D$ derived using linearised theory.

6.2.1 Incidence correction related to lift, $\Delta\alpha_L$

For the range of cases presented in Section 4, linearised theory has been used to yield the lift coefficient experienced by a wing set at incidence $\bar{\alpha}$ in a wind tunnel. By applying the linearised theory to the equivalent free-air situation, for each case the incidence of the wing in free air giving the same lift coefficient as experienced in the wind tunnel has been found. Denoting this incidence by $\bar{\alpha} + \Delta\alpha_L$, the parameter $\Delta\alpha_L$ can be considered as the incidence correction to be applied to the wind-tunnel test to give the correct free-air lift coefficient. The lift distributions over the wing planform will in general differ in the test and free-air flows as a result of non-uniformity in the upwash interference.

Since the linearised theory is not as reliable at predicting the lift coefficient of a wing at given incidence as at yielding 'far-field' flow angles induced by a wing of known lift, it is better to obtain incidence corrections indirectly by using the parameter δ . Thus, linearised theory is used to obtain δ_L , where

$$\delta_L = \Delta\alpha_L \frac{C}{SC_L}, \quad (6.10)$$

and linearised theory values of $\Delta\alpha_L$ and C_L are employed. To determine the incidence correction, $\Delta\alpha_L$, to be used in practice, Equation (6.10) is re-written as

$$\Delta\alpha_L = \delta_L \frac{SC_L}{C}, \quad (6.11)$$

and in this relation wind-tunnel test values of C_L are utilised.

For an aerofoil with a parabolic camber line, the lift at subsonic speed is proportional to the local incidence of the mean line at the 3/4-chord point, according to linearised theory. Similarly for simple twisted and cambered wings, the 'effective' incidence corresponds to the local incidence of a point near the 3/4-chord point of the mean-chord line. Since non-uniform upwash interference corresponds, according to linearised theory, to an effective twist and camber, it is to be expected that δ_L would correspond approximately to the value of δ at the 3/4-chord point of the mean-chord line (*i.e.* at $\xi = 0.75$, $\eta = 0.5$ for the wings considered). Values of δ_L for the range of cases of Section 4 are given in Table 13.3. These values are higher than the corresponding values of δ_{av} (the mean values of upwash parameter) which appear in Table 13.2, and are indeed found to be close to the values of $\delta(\xi, \eta)$ with $\xi = 0.75$, $\eta = 0.5$.

6.2.2 Incidence correction related to drag, $\Delta\alpha_D$

Linearised theory may be utilised to obtain an incidence correction, $\Delta\alpha_D$, which, when applied to the wind-tunnel test, results in the correct relationship between incidence and drag coefficient for free air (according to the theory). This may be done in a similar way to the determination of $\Delta\alpha_L$, by comparing linearised-theory evaluations of the drag coefficient of a wing in constrained and free-air conditions, and 'adjusting' the incidence of the latter until the drag coefficients in the two conditions are equal. In order to carry out this procedure it is necessary to have a reliable numerical method of determining the linearised-theory drag coefficient in the two conditions. There are two possible approaches to this, either by considering the momentum flux across a transverse plane in the distant wake (the Trefftz plane) or from integrating the effects of normal pressures and a suction force distributed along the leading edge (see, for example, Ref. 1 and 7). The former approach requires the spanwise distribution of lift to be determined to a good accuracy, and the latter requires accurate evaluation of the distribution of normal pressures in both chordwise and spanwise directions.

It was found, however, that adopting the following simpler approach provided results for $\Delta\alpha_p$ to an adequate accuracy.

Due to the upwash interference the 'lift' vector, of magnitude L_p , acting on a vortex panel (see Sketch 9.4), is effectively inclined forward from the z-direction, by rotation through an angle $\Delta\alpha_p$ (say). Here $\Delta\alpha_p$ is the effective interference upwash angle over the span of the bound vortex segment (see Sketch 9.2) which may reasonably be taken as the interference upwash angle at the mid point of the quarter-chord line of the wing panel, *i.e.* the point C in Sketch 9.4. Thus the drag experienced by the wind-tunnel model, associated with that panel, is *reduced* by $L_p \Delta\alpha_p$ relative to the drag that would be experienced in free air. The effective average of $\Delta\alpha_p$ yielding the correct adjustment to the overall drag for the wing, namely $\Delta\alpha_D$, is found from summing the reductions $L_p \Delta\alpha_p$ over all the vortex panels representing the wing, and equating that sum to $L\Delta\alpha_D$, where L is the wing lift, *i.e.* the sum of the lift over the panels. Thus

$$\Delta\alpha_D = \left(\sum_{\text{panels}} L_p \Delta\alpha_p \right) / L, \quad (6.12)$$

where
$$L = \sum_{\text{panels}} L_p.$$

The drag coefficient in free air, $C_{D_{corr}}$, is then given by Equation (6.9), *i.e.*

$$C_{D_{corr}} = C_D + C_L \Delta\alpha_D.$$

As in the case of determining $\Delta\alpha_L$, it is preferable to determine δ_D from linearised theory, where

$$\delta_D = \Delta\alpha_D \frac{C}{SC_L}, \quad (6.13)$$

in which linearised-theory values of $\Delta\alpha_D$ and C_L are used, and subsequently to determine the incidence correction, $\Delta\alpha_D$, to be used in practice by re-writing Equation (6.13) as

$$\Delta\alpha_D = \delta_D \frac{SC_L}{C}, \quad (6.14)$$

and in this relation using observed values of C_L .

Values of δ_D for the range of cases of Section 4 are given in Table 13.3. Values of δ_D are lower than the corresponding values of δ_{av} (the mean values of upwash parameter) which appear in Table 13.2, and are close to the values of $\delta(\xi, \eta)$ at $\xi = 0.25$, $\eta = 0.5$, *i.e.* a point close to the corresponding wing aerodynamic centre. This is to be expected from the way in which δ_D is derived. It is noted from Table 13.3 that values of δ_D for given s/b' and h'/b' are almost the same for all the wing planforms. As might be expected, this is not fortuitous, and is related to the fact that the bound vortex segments have virtually no influence on δ_D . Values for δ_D for an unswept wing of infinite aspect ratio[†] are given, for reference, in Table 6.1. Corresponding values of δ_D in Tables 13.3 and 6.1 are always close.

[†] Here the limit of infinite aspect ratio is approached by keeping s_T (and thus s) finite and making $\bar{c}$ approach zero.

TABLE 6.1 Values of δ_D for unswept wing of infinite aspect ratio

s/b'	0.4			0.6			0.8		
h'/b'	0.7	1	10/7	0.7	1	10/7	0.7	1	10/7
δ_D	0.115	0.139	0.193	0.111	0.142	0.199	0.109	0.149	0.210

6.2.3 Correction to aerodynamic centre

For any particular case of wing planform and wind-tunnel cross-section presented in Section 4, the magnitude of the interference upwash at any fixed point of the planform is proportional to the lift coefficient. Each wing is taken to have zero twist and camber, and so the centre of pressure (centre of lift) is the same as the aerodynamic centre, i.e. $x_{cp} = x_{ac}$. According to linearised theory, therefore, non-uniformity in upwash interference, in any of these cases, results in a change in pitching moment coefficient proportional to the lift coefficient of the wing in the wind tunnel, C_L , and therefore to a shift, Δx_{ac} in aerodynamic centre, which is independent of C_L . Because of the tendency for the upwash interference to increase in the streamwise direction, the shift in aerodynamic centre is always aft, i.e. Δx_{ac} is always positive. Results for $\Delta x_{ac}/\bar{c}$ are given in Table 13.4. The results in Table 13.4 are presented to four decimal places, even though the accuracy in absolute terms for C_L , C_M and x_{ac} is only about 1 per cent (see Section 10). This is done so as not to obscure trends.

The generally small shift in aerodynamic centre is noteworthy. The largest shift calculated (for transformed planform 56) is slightly less than 3.5 per cent of the mean chord.

For wings which have non-zero twist and/or camber, and therefore load distributions differing from that of the corresponding wing without twist and camber, the change in centre of lift due to non-uniform upwash interference differs, in general, from the result given in Table 13.4. However, since the *non-uniformity* in upwash is almost unaffected by quite significant changes in load distribution (see Section 7), it is likely that for many twisted and cambered wings, the aft shifts in the centre of lift are close to the values of $\Delta x_{ac}/\bar{c}$ given in Table 13.4.

6.3 Limits on C_L Related to C_D Errors Due to Upwash Non-Uniformity

As stated in Section 6.1, where the wall-induced upwash is nearly uniform in the region of the test wing, its effect is considered to be equivalent simply to a change in the (datum) wing incidence, and a corrected incidence (as in Section 6.1) is sufficient to permit the test data of lift, drag and pitching moment to be applied to the model wing in free air[†]. Correction for non-uniform upwash within the assumptions of linearised theory, where it is equivalent to an effective (additional) twist and camber, is considered in Section 6.2. In Section 6.2 it is shown that, according to the theory, corrections to incidence can be applied to yield the free-air relationship between lift coefficient and incidence and (with different corrections) between drag coefficient and incidence. However, uncertainties arise in any such procedure, particularly at transonic speeds, because of inaccuracies associated with linearised theory.

Reference 18 considers uncertainties in the determination of free-air aerodynamic characteristics for a test wing when the interference flow is non-uniform, along with more general effects of test-section flow quality. The important aspect of error in C_D is given particular attention in Ref. 18, and flow non-uniformities likely to produce a *remaining* error in C_D of 0.0001 (one drag count) are presented. Here, 'remaining' refers to the error after a wind-tunnel correction scheme incorporating an allowance for the *average level of upwash* (only) has been applied. This choice of error magnitude is considered appropriate for the cruise conditions of transport aircraft configurations, but less stringent requirements on drag coefficient may be

[†] The speed of the free-air flow should be corrected for blockage – see Sections 2 and 11.

appropriate for tests relating to other conditions and other types of aircraft. The basis of one drag count error contributions leads to the following estimated limits on the magnitudes of streamwise flow curvature and spanwise variation of flow angle[†], according to Ref. 18.

$$\text{Streamwise flow curvature, } \frac{\partial(w/U)}{\partial(x/c)} < 0.0005. \quad (6.15)$$

$$\text{Spanwise variation of flow angle, } \frac{\partial(w/U)}{\partial(y/s)} < 0.0017. \quad (6.16)$$

Values of the limits on streamwise flow curvature and spanwise variation of flow angle for other values of drag coefficient error may be obtained by assuming a linear relation between the drag error and the magnitude of flow non-uniformity.

Expressing the upwash in terms of the parameter $\delta(\xi, \eta)$ results in Expressions (6.15) and (6.16) being transformed into corresponding limiting values of C_L . Let the maximum streamwise variation of δ over the chord for a particular case, *i.e.* $|\delta(1, \eta) - \delta(0, \eta)|_{max}$, be denoted by Δ_1 , and the maximum variation of δ over the span (for any fixed ξ value) be denoted by Δ_2 . Then Expressions (6.15) and (6.16) become, respectively

$$C_{L1} < \frac{0.0005 C}{\Delta_1 S}, \quad (6.17)$$

and

$$C_{L2} < \frac{0.0017 C}{\Delta_2 S}. \quad (6.18)$$

Then, so long as the test C_L values do not exceed the values given by the Expressions (6.17) and (6.18), the likely C_D errors due to upwash non-uniformity should not exceed one drag count.

If some transformed quantities are introduced, Expressions (6.17) and (6.18) become (for wind tunnels with rectangular cross-sections)

$$\beta C_{L1} < \frac{0.0005}{\Delta_1} A_T \left(\frac{b'_T}{s_T} \right)^2 \left(\frac{h'_T}{b'_T} \right), \quad (6.19)$$

and

$$\beta C_{L2} < \frac{0.0017}{\Delta_2} A_T \left(\frac{b'_T}{s_T} \right)^2 \left(\frac{h'_T}{b'_T} \right). \quad (6.20)$$

Values of Δ_1 and Δ_2 have been found for the range of cases presented in Section 4, and the corresponding limits βC_{L1} and βC_{L2} given by Expressions (6.19) and (6.20) determined. These values are given in Table 13.5 and are also provided by ESDUpac A9514. For the cases in Table 13.5 values for βC_{L1} and βC_{L2} lie between relatively small numbers (*i.e.* less than 0.1) and relatively high numbers (*i.e.* one or above). The former indicate cases where Expressions (6.19) and (6.20) are unlikely to be met in practice while the latter indicate cases where they would be readily met and there would be no restriction on the range of test C_L for which an error of one drag count is likely due to upwash non-uniformity.

[†] The limits are given in Ref. 18 in degrees, and the numbers given are 0.03 deg and 0.1 deg.

6.4 Non-Uniform Upwash Considered as Additional Incidence, Twist and Camber

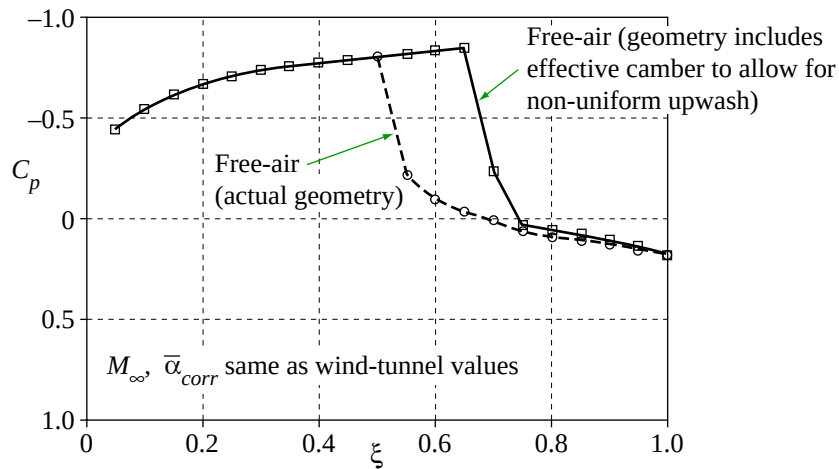
A non-uniform distribution of interference upwash may be considered as producing a change of incidence which varies over the wing planform. The same local incidence as occurs in the wind tunnel may be generated in free-air by applying an increment to the datum chord incidence, together with specific distributions of additional twist and camber. Equations giving the relations between the non-uniform upwash and the 'equivalent' additional incidence, camber and twist are given in Section 9.

The computer program, ESDUpac A9514, described in Appendix A, provides upwash distributions and the equivalent additional incidence, camber and twist for the ranges of transformed planforms and wind tunnel cross-sections of Section 4.

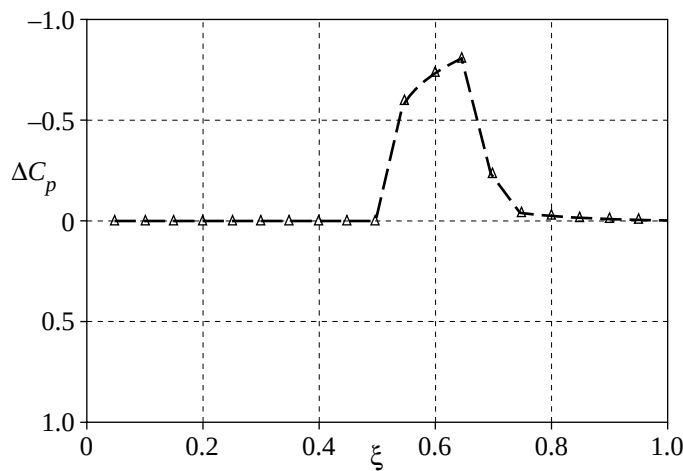
The aerodynamic effects on the wing of non-uniform wall-induced upwash and those of the equivalent additional incidence, twist and camber are the same according to linearised theory, but are *not* the same according to any more generally applicable theory such as non-linear potential theory or the inviscid theory embodied in the Euler flow equations. Corrections to allow for the effects of non-uniform upwash, based on linearised theory, such as those for C_L , C_D and C_M presented in Section 6.2 tend to become increasingly unreliable with increases in the extent of any supersonic regions in the test flow, increases in the strength of any shock waves in the test flow and increases in the degree of upwash non-uniformity.

There is evidence for two-dimensional flows that allowing for the streamwise variation of upwash by use of an equivalent additional camber derived from linearised theory can provide good agreement with observed pressure distributions, even where supersonic flow regions and shocks of moderate strength occur (Refs 19, 21 and 29). In these cases the aerofoil flows, including the effects of the additional camber, are evaluated by computational methods, such as those described in Refs 19 and 23, that are capable of giving at least fair predictions of the type of mixed subsonic-supersonic flows occurring in the tests. Such computational methods have to be capable of accounting for the boundary-layer development around the aerofoil. So long as the predicted effects of the upwash non-uniformity are not unduly large, predicted differences between the aerodynamic characteristics for the wind-tunnel and free-air cases are close to those observed between the actual wind-tunnel characteristics and the characteristics for the aerofoil in free air. In this way, corrected free-air characteristics can be obtained which should be more reliable than those obtained from corrections based on linearised theory. When shock waves are present, corresponding corrections to the surface pressure distributions to allow for non-uniform upwash are more difficult to estimate, because of the nature of the surface pressure distributions.

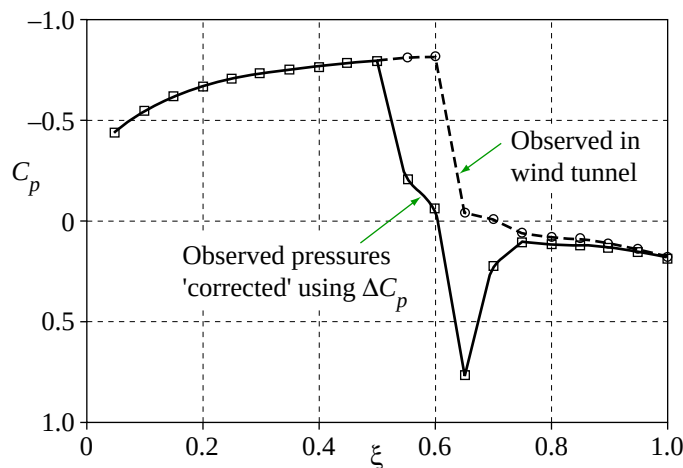
Sketch 6.2 illustrates this by means of idealised upper-surface pressure distributions. It is not adequate to evaluate a change in pressure coefficient due to upwash non-uniformity, and apply it without regard to the calculated shock position.



(a) Notional calculated upper-surface pressure distributions



(b) Notional calculated upper-surface pressure difference due to non-uniform upwash



(c) Application of correction to pressure distribution leading to unsatisfactory result

Sketch 6.2 Demonstration that simple allowance for effective camber may be unsatisfactory

Evidence is lacking of the effectiveness for three-dimensional flows of similar schemes of allowing for non-uniformity in upwash interference. However, there is no reason to suppose that such schemes would not be both applicable and useful in three-dimensional flows. Some data have been presented in Ref. 26 which should permit the accuracy of the approaches to be assessed for certain transonic wing flows. It is

expected that applying the equivalent twist and camber, in a three-dimensional computational method capable of predicting the type of flow observed to fair accuracy, would permit quite a good estimate of the effects of non-uniform upwash interference to be made.

7. INFLUENCE OF CHANGES IN WING LOAD DISTRIBUTION

Results for δ in Section 5 are presented for wings without twist or camber, but since wings are generally twisted and cambered it is important to consider how the results are influenced by such geometrical changes. To do this, the effects of twist and camber are considered for one of the cases presented in Figure 31, namely $A_T = 1.5$, $A_{\tan\Lambda_{1/2}} = 4$, $\lambda = 0.5$, with $s/b' = 0.8$ and $h'/b' = 10/7$. This case is selected since it presents marked variations in δ over both the span and the chord, and is therefore relatively susceptible to changes in the aerodynamic load distribution. The influences of one particular twist and two camber distributions are considered.

The twist considered corresponds to 'washout' which is linear across the span. This washout is combined with an incidence of the datum chord that makes the tip chord incidence zero. The wing sections are taken to be uncambered, so the local incidence considered is of the form

$$\alpha(\xi, \eta) = d(1 - \eta), \quad (7.1)$$

where here, and in Equations (7.2) and (7.3), d is an arbitrary constant.

This twist results in a marked change in the aerodynamic load distribution, as shown in Sketch 7.1, where curves of $C_{LL}(\eta)/C_L$ are given for the flat wing and twisted wing. For both the flat and twisted wing cases these are the distributions that are determined for the wing in the wind tunnel. In Sketch 7.2 the distributions of δ across the span (at $\xi = 0, 0.5$ and 1.0) are given for the same cases. It is seen that the change in aerodynamic load distribution has a remarkably small effect on the δ distributions. The small changes in δ produced are consistent with what would be expected on the basis of the change in load distribution: values of δ inboard tend to be increased and those outboard decreased. On the basis of these results, the effects of wing twist on δ is very small.

The first of the two cambers considered for transformed wing planform 31 has parabolic arc camber lines with maximum camber ordinate proportional to local chord. The wing datum-chord incidence and wing twist are taken as zero, so the local incidence is of the form

$$\alpha(\xi, \eta) = d(2\xi - 1). \quad (7.2)$$

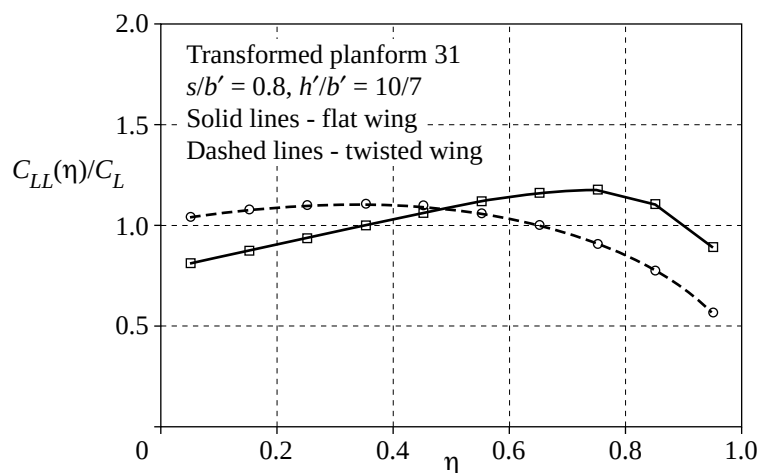
This camber distribution results in a moderate change in the calculated aerodynamic load distribution across the span, as shown in Sketch 7.3. The chordwise distributions of aerodynamic load are changed considerably, however, and result in the centre of pressure for the cambered wing being determined as $x_{cp}/\bar{c} = 1.75$, compared with $x_{cp}/\bar{c} (= x_{ac}/\bar{c}) = 1.39$ for the flat wing, an aft shift of $0.36\bar{c}$. The distributions of δ at $\xi = 0, 0.5$ and 1 for the flat and cambered wings are shown in Sketch 7.4. The rearward shift of aerodynamic load produced by the camber can be thought of as shifting a more or less unchanged distribution of δ bodily in the rearward direction, so that the value of δ at any fixed point decreases. The values of δ (at a fixed η value) at $\xi = 0, 0.5$ and 1 for this cambered wing case are seen from Sketch 7.5 to be close to the values of δ at $\xi = -0.36, 0.14$ and 0.64 (i.e. 36 per cent of the *local chord* more forward) for the flat wing.

The second camber distribution considered has quartic camber lines, with a local incidence of the form

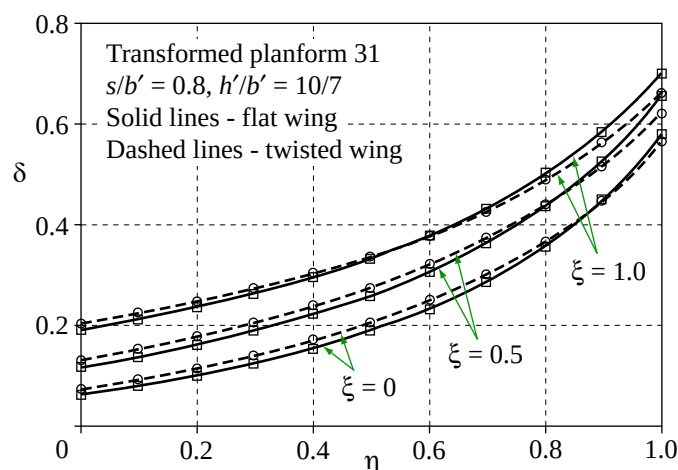
$$\alpha(\xi, \eta) = d(3\xi^2 - 4\xi^3). \quad (7.3)$$

This has the maximum camber ordinate at each spanwise position located at the three-quarter chord point. The curve of $C_{LL}(\eta)/C_L$ for this camber is compared with that for the flat wing in Sketch 7.6. With this 'quartic' camber, the chordwise distributions of aerodynamic load are changed to a greater degree than by the parabolic camber, and give $x_{cp}/\bar{c} = 1.85$, *i.e.* an aft shift in the centre of pressure of $0.46\bar{c}$ relative to the flat wing. Again the effects on the distribution of δ , resulting from the rearward shift in aerodynamic load produced by this camber, are closely matched by shifting the values of ξ of the distribution of δ for the flat wing. The values of δ (at a fixed η value) at $\xi = 0, 0.5$ and 1 for the quartic camber are shown in Sketch 7.7 to be close to the values at $\xi = -0.46, 0.04$ and 0.54 (*i.e.* 46 per cent of the local chord more forward) for the flat wing.

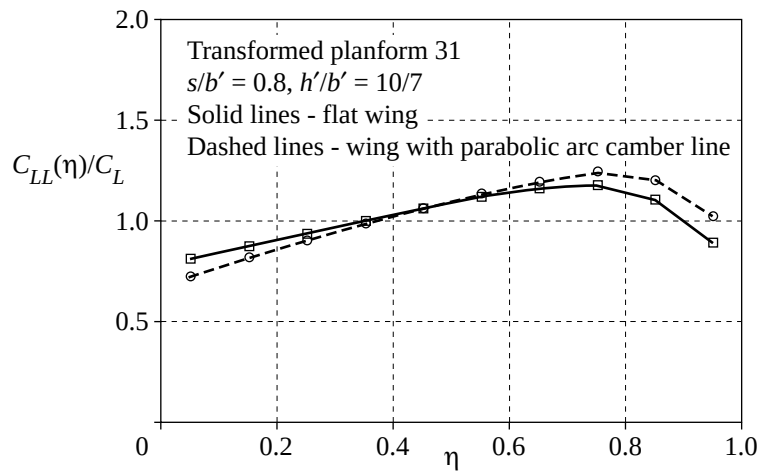
On the basis of these two camber distributions, it is suggested that the effects (at a particular value of $\bar{\alpha}$) on upwash interference of a particular applied camber may be adequately assessed from the shift in centre of pressure that it produces (at the particular value of $\bar{\alpha}$) relative to the flat wing centre of pressure. By applying this shift, expressed as a proportion of $\bar{c}$, and appropriately interpolating (or extrapolating) the flat wing values of δ (given in Figures 1 to 66 and by ESDUpac A9514), as illustrated in Sketches 7.5 and 7.7, good approximations to the values of δ for the cambered wing case should be obtained.



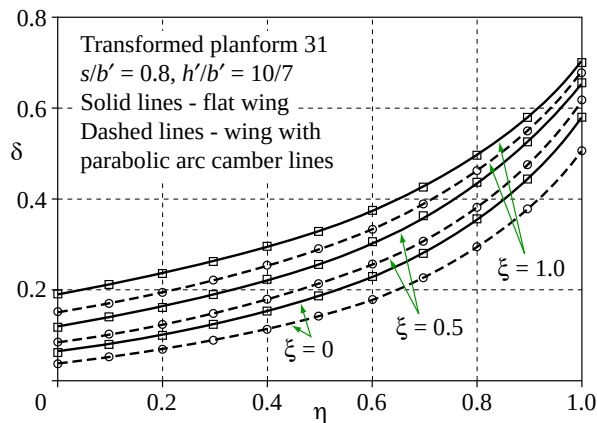
Sketch 7.1 Spanwise load distributions for flat wing and twisted wing



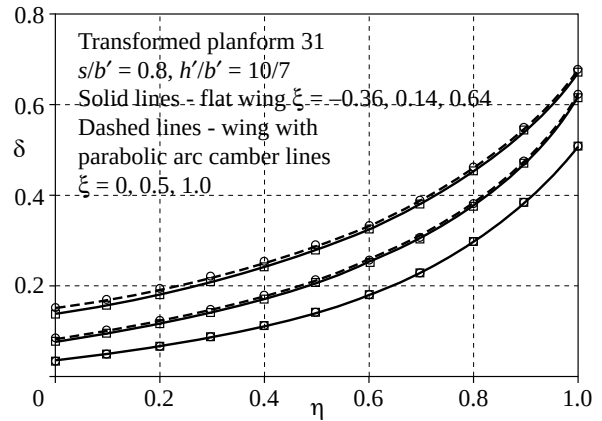
Sketch 7.2 Distributions of δ for flat wing and twisted wing



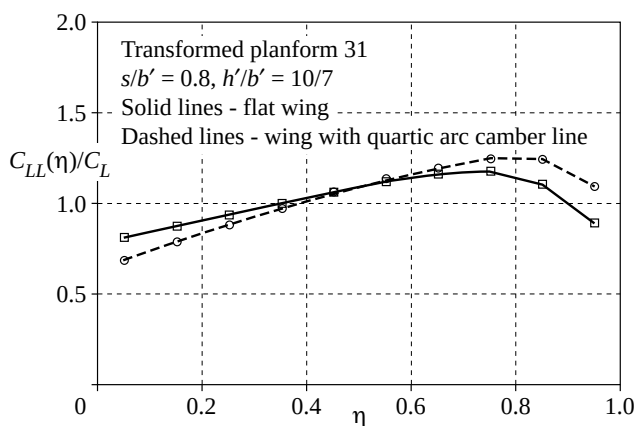
Sketch 7.3 Spanwise load distributions for flat wing and wing with parabolic arc camber lines



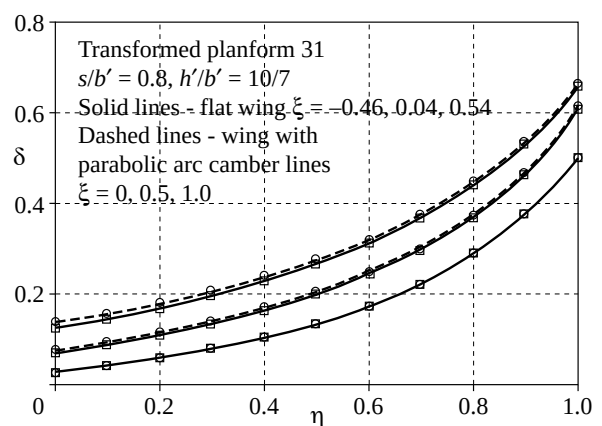
Sketch 7.4 Distributions of δ for flat wing and wing with parabolic arc camber lines



Sketch 7.5 Distributions of δ for flat wing and wing with parabolic arc camber lines – differing ξ values



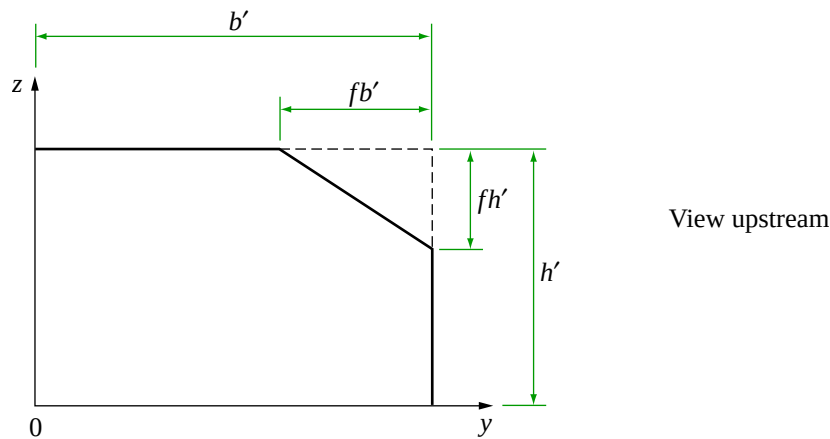
Sketch 7.6 Spanwise load distributions for flat wing and wing with quartic camber lines



Sketch 7.7 Distributions of δ for flat wing and wing with quartic camber lines – differing ξ values

8. INFLUENCE OF CORNER FILLETS ON UPWASH INTERFERENCE

In this Section the influence of the presence of corner fillets on δ , as determined by the linearised theory, is presented. Corner fillets are often a feature of the working sections of wind tunnels and, by decreasing the cross-sectional area (for given values of h and b), their use may result in a decrease in the power required to operate the tunnel. Fillets can decrease the severity of the troublesome corner flows which may be produced in the boundary layers formed on the wall liners. The fillets considered are characterised by the factor, f , defined in Sketch 8.1. Calculations using the method of Section 9 have been made for values of f from 0 to 0.8. The upper limit for values of f of practical interest would be about 0.5.



Sketch 8.1 Definition of factor f for cross sections with fillets

Studies on the effects of corner fillets on wall-induced upwash for the simple case of an elliptically loaded lifting line are reported in Ref. 5. Adopting the partly-empirical relation presented for this simple case (Equation 3.107 in Ref. 5), for the general situation, results in the following approximation for the upwash in a test section with corner fillets,

$$\delta(\xi, \eta) \approx \delta_R(\xi, \eta) \left[\left(\frac{1}{2} + \frac{C}{8b'h'} \right) - \left(\frac{1}{2} - \frac{C}{8b'h'} \right) \left(\frac{s}{b'} \right)^2 \right]. \quad (8.1)$$

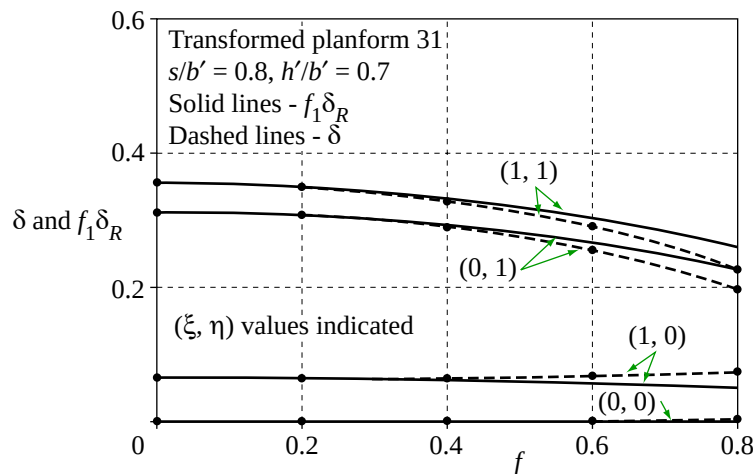
Here, and elsewhere in this Section, suffix R denotes a value corresponding to the rectangular test section having the same values of s/b' and h'/b' as the test section with corner fillets. With the fillet ratio defined as in Sketch 8.1, Equation (8.1) may be written as

$$\delta(\xi, \eta) \approx f_1 \delta_R(\xi, \eta), \quad (8.2)$$

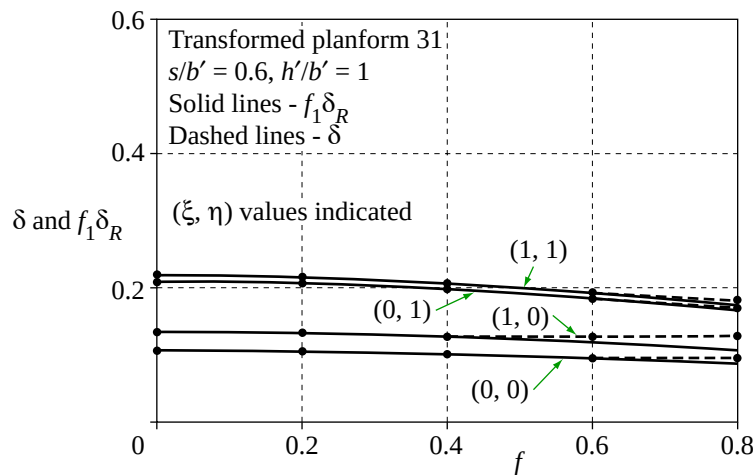
with

$$f_1 = 1 - \frac{f^2}{4} \left[1 + \left(\frac{s}{b'} \right)^2 \right]. \quad (8.3)$$

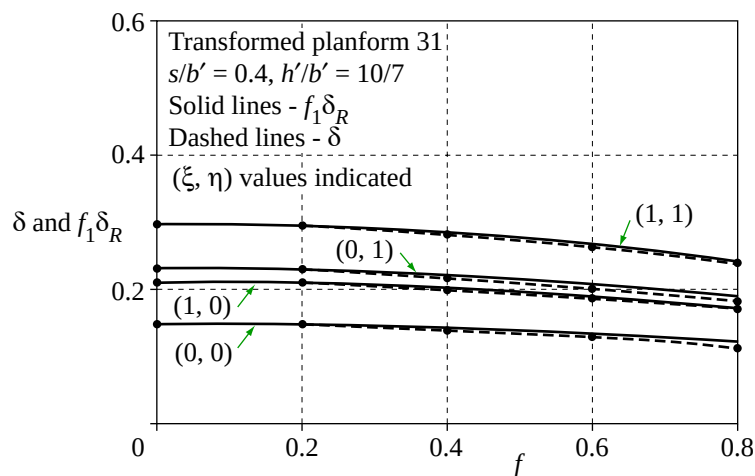
Examination of a number of cases suggests that use of Equation (8.2) yields $\delta(\xi, \eta)$ within an error of about 0.01 provided f is less than 0.5. In Sketches 8.2 to 8.4, $\delta(\xi, \eta)$ and $f_1 \delta_R(\xi, \eta)$ are compared for particular cases of transformed planform, s/b' and h'/b' for a range of values of f , for (ξ, η) values of (0, 0), (0, 1), (1, 0) and (1, 1). The good agreement between $\delta(\xi, \eta)$ and $f_1 \delta_R(\xi, \eta)$ for values of f up to about 0.5 is apparent.



Sketch 8.2 Variations of $f_1 \delta_R$ and δ with f for transformed planform 36



Sketch 8.3 Variations of $f_1 \delta_R$ and δ with f for transformed planform 19

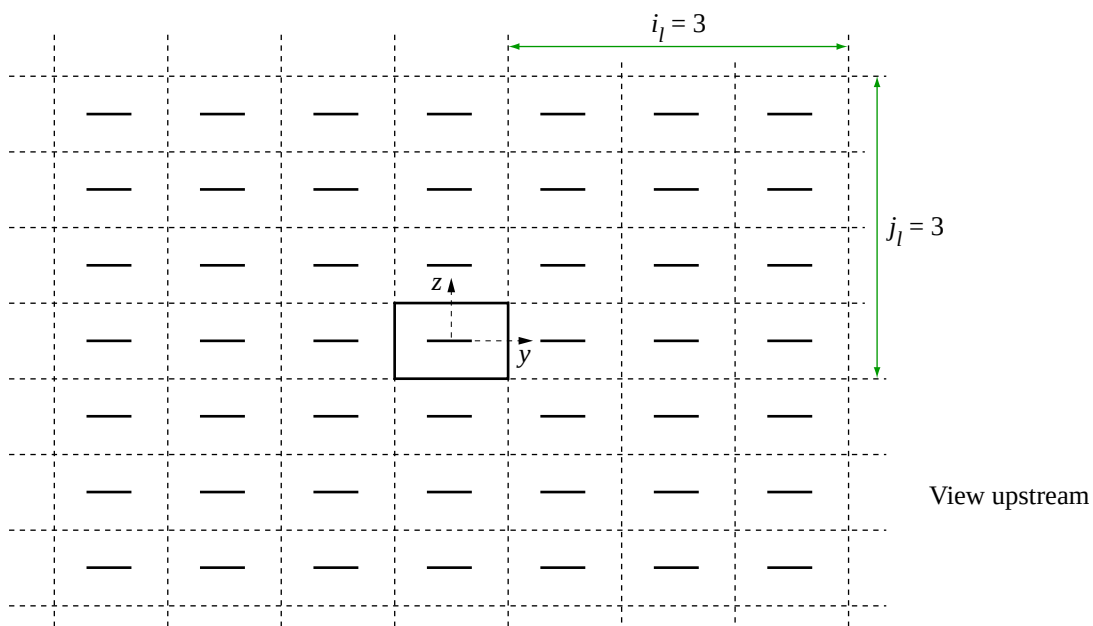


Sketch 8.4 Variations of $f_1 \delta_R$ and δ with f for transformed planform 46

9. DESCRIPTION OF METHOD FOR DETERMINATION OF UPWASH INTERFERENCE

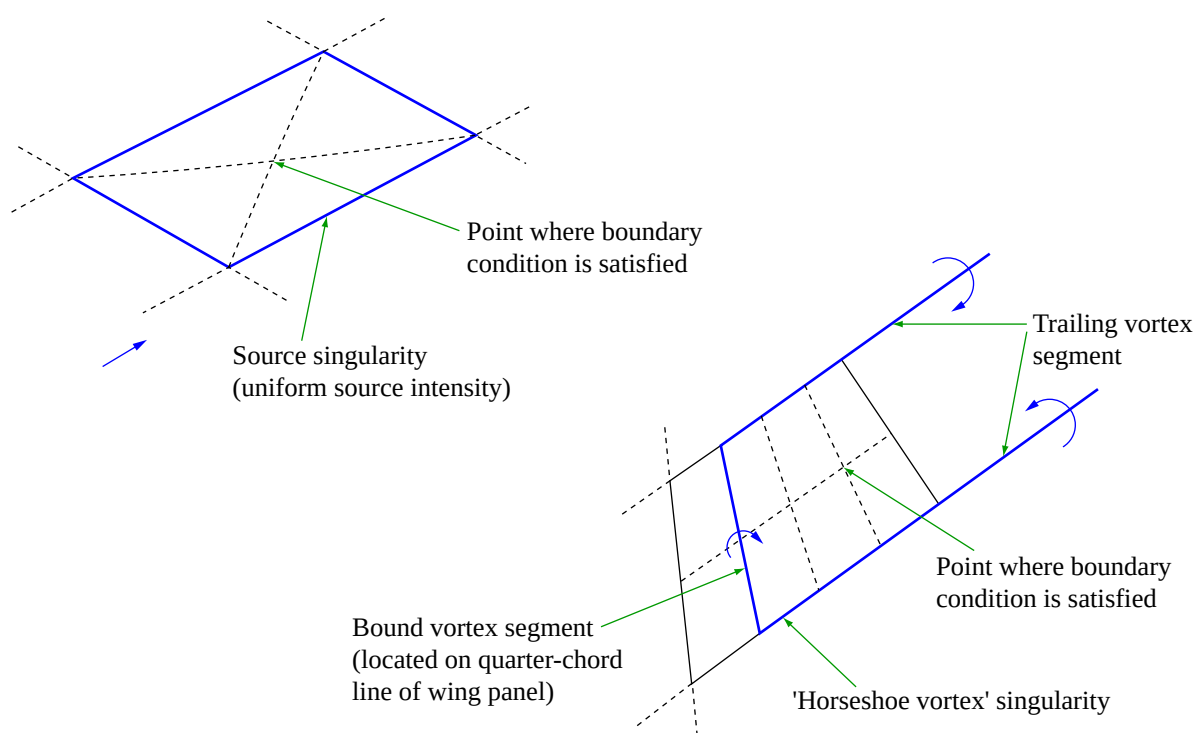
In Ref. 27, Fiddes and Gaydon describe an efficient numerical method for determining wall-induced interference in solid-liner wind tunnels with subsonic streams. This method achieves its efficiency by combining features of the image method (see Ref. 5) and panel methods (Refs 6 and 17). The flow about the model is produced by panel singularities, either as in a surface panel method (Ref. 4), or, where only lifting effects are to be considered, by a vortex lattice (see, for example Refs 9 and 10). In the panel method, in order to satisfy the boundary condition of zero normal velocity component over the test section walls, a detailed panel representation of those walls is necessary, as in Ref. 17. In the image method an extensive array of images of the model is necessary to achieve a similar accuracy, see Sketch 9.1. The hybrid method of Ref. 27 uses a small number of images only, retaining a single reflection of the model in each of the four walls (where the model is arranged as in Sketch 9.1), and combines these with a coarse wall panel representation. The images ensure that the wall boundary condition is roughly satisfied, since, for any wall, the normal velocity disturbances due to the model and those of the image in that wall cancel, but normal velocity disturbances resulting from other wall images are not cancelled. A coarse panelling of each wall is all that is necessary to cancel the disturbances from these relatively distant singularities. The hybrid method of Ref. 27 shows to greatest advantage when a detailed panel representation of the model is utilised with, say, several hundred panel singularities. Adopting a wall panelling with about a hundred further unknown singularity strengths, together with four model images, then adds relatively little to the computation in comparison with that involved in predicting the flow about the model in free air. Methods relying solely on images to represent the walls would entail a great deal of additional computational effort, as would methods relying solely on wall panels. Further benefits of the hybrid method are its ability to deal simply with test sections with corner fillets, and its use in studies of model support interference (Ref. 27).

In the present Item, the hybrid method has been used with a vortex-lattice representation of the wing and rectangular panels of uniform source intensity for the wall singularities. The number of images used has been more than the four, one for each wall, employed in Ref. 27. Referring to Sketch 9.1, the parameters i_l and j_l have generally been set to 3, giving 48 images. This enables particularly simple wall panel representations to be used, and proves beneficial when simple vortex lattice layouts are employed to represent the wing.



Sketch 9.1 Array of images introduced to satisfy zero normal velocity on walls of wind tunnel of rectangular cross-section

The method follows a standard panel-type procedure, (see, for example, Ref. 4). Each panel, whether on the wing (or, more precisely, in the plane $z = 0$) or a wall, is denoted by an index number j , where j runs from 1 to N , and is assigned a singularity, initially of unknown strength γ_j . Each singularity (either a source panel or a horseshoe vortex) is a basic solution of the governing field equation, which in this case is the transformed potential equation (*i.e.* Equation (3.1) with $\beta = 1$). The singularity strengths are determined by satisfying the boundary condition that the flow is tangential to the surface (*i.e.* the normal component is zero) at N points. In order to achieve a good representation of the flow these 'boundary' points must be suitably located, with one point associated with each singularity. The positions of the boundary points associated with the vortex and source singularities are shown in Sketch 9.2.



Sketch 9.2 Positions of points where boundary condition is satisfied

The flow induced at the boundary point denoted by i is affected by all the singularities, $j = 1, 2, \dots, N$, although the strongest effects generally occur when $j = i$. The boundary condition at point i is written as

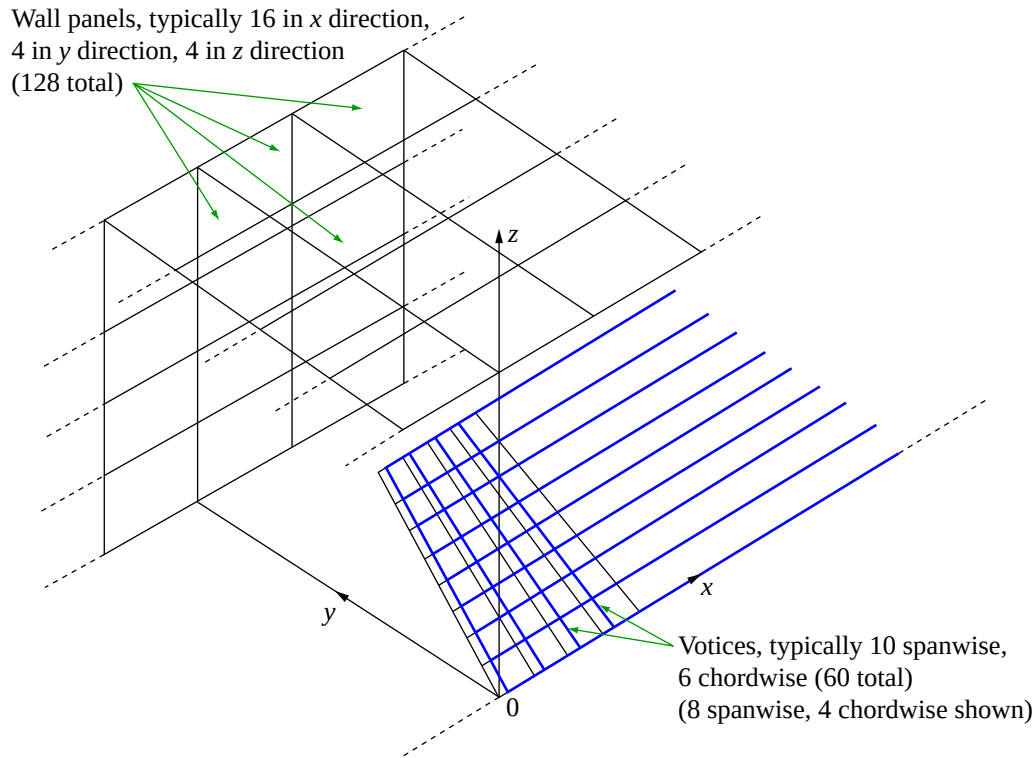
$$\sum_{j=1}^N a_{ij} \gamma_j = -v_{ni}, \quad (9.1)$$

where the left-hand side gives the sum of the contributions to the velocity component normal to the surface due to the singularities, and the right-hand side represents the (negative of) the component of the free stream velocity (U in the x -direction) along the surface normal. Where i refers to a point on a wall, $v_{ni} = 0$. Where i refers to a point on the wing, $v_{ni} = U\alpha$, since the mean wing surface is inclined at a (small) angle of incidence, α , to the free stream[†]. Considered as a system of linear equations, Equation (9.1) may be solved to give the singularity strengths γ_j . The term a_{ij} is called an influence coefficient, and represents the effect (or influence) on the velocity component along the surface normal at the point i due to the unit strength singularity denoted by j . The number of unknowns, N , to be determined can be reduced by invoking any symmetry or anti-symmetry relations that apply to the singularities. In calculations for this Item, the wing

[†] For wings without twist or camber, α is constant, and equal to $\bar{\alpha}$.

is symmetrical about the plane $y = 0$, and the potential is antisymmetric about the plane $z = 0$. This effectively halves the number of unknown vortex singularity strengths needed to represent the wing, and reduces to a quarter the number of source panels needed to represent the walls. Calculation of the influence coefficients must include appropriate contributions from symmetry and antisymmetry effects.

In Sketch 9.3 the layout of wing vortices and wall panels typically used in the Item is shown. Generally, the (half) wing vortex lattice adopted divided the semi-span into ten equal intervals and the chord into six, thus giving 60 vortex singularities. The number of source panels generally used was 128.



Sketch 9.3 Layout of vortices and wall panels utilised

Once Equation (9.1) has been solved for γ_j , the interference flow may be determined as the sum of the effects of the wall singularities and the image wings. The interference required is the upwash (z -component of velocity) in the plane $z = 0$. Denoting a point in this plane by P , the upwash interference at P is evaluated as

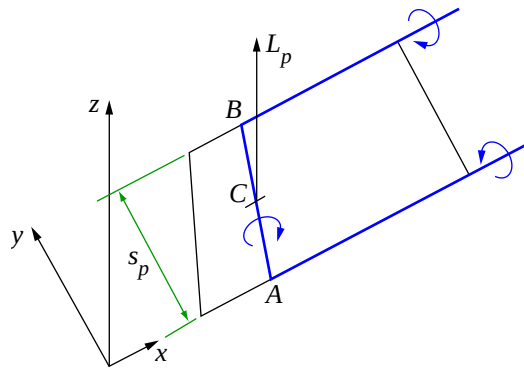
$$\frac{w}{U} = \frac{1}{U} \sum_{j=1}^N a'_{pj} \gamma_j, \quad (9.2)$$

where a'_{pj} is the z -component of velocity induced at P due to the unit strength singularity j , *excluding* any direct wing contribution.

In order to find δ , the wing lift coefficient is needed while the pitching moment coefficient is also of interest. These parameters are found by considering the lift, L_p , acting on a wing panel, which is given by

$$L_p = \rho U \gamma_j s_p, \quad (9.3)$$

and by assuming that the line of action of this lift force passes through C , the mid-point of AB – see Sketch 9.4.



Sketch 9.4 Lift associated with a vortex panel

9.1 Power Series Representation of Upwash Distribution

When determining the mean upwash value, and the effective additional incidence, twist and camber associated with a particular upwash distribution, it is convenient to represent the distribution of $\delta(\xi, \eta)$, approximately, by an analytic expression. The method adopted for this was to evaluate δ (for each particular case) at the 11×21 points (in the transformed flow $Z = 0$ plane) given by the following ξ, η values

$$\xi \ 0(0.1)1.0, \quad \eta \ 0(0.05)1.0,$$

and fit the values of δ at these 231 points by the method of least squares with a polynomial of the form

$$\delta(\xi, \eta) = \sum_{k,l}^{0 \leq k+l \leq 4} e_{k,l} \xi^k \eta^l. \quad (9.4)$$

In each case considered this fitting procedure gave a satisfactory approximation to δ over the transformed wing planform.

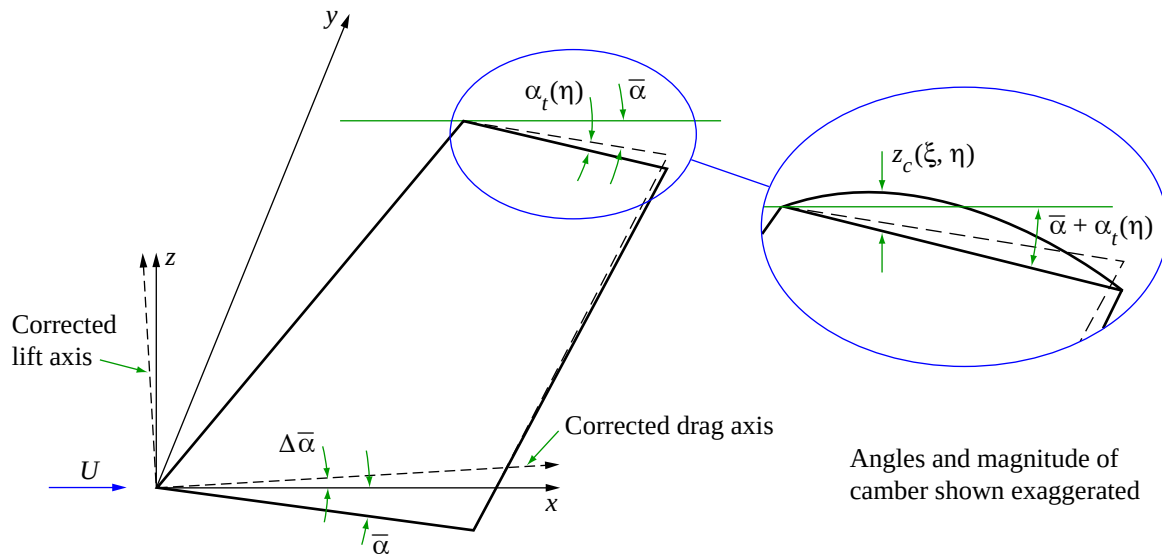
9.2 Determination of Effective Additional Incidence, Twist and Camber

In Section 6.4 wall-induced upwash is considered in terms of an ‘equivalent’ additional incidence, twist and camber, *i.e.* those changes which, if applied to the wing in free air, would result in the same local incidence as occurs in the wind tunnel after allowance is made for (local) wall-induced upwash.

Considering cases where the wing has zero (geometric) twist and camber, the local effective angle of incidence, α_{eff} , of the mean wing surface is given by

$$\alpha_{eff}(\xi, \eta) = \bar{\alpha} + \Delta\bar{\alpha} + \alpha_t(\eta) - \frac{\partial z_c(\xi, \eta)}{\partial x}, \quad (9.5)$$

where the last three terms on the right hand side are, respectively, the effective additional incidence (of the datum chord), the local effective twist, and the local effective camber slope, associated with wall-induced upwash – see Sketch 9.5.



Sketch 9.5 Effective additional incidence, twist and camber contributions to α_{eff}

Then

$$\alpha_{eff}(\xi, \eta) = \bar{\alpha} + \frac{w(\xi, \eta)}{U}. \quad (9.6)$$

Denoting w/U by θ ,

$$\alpha_{eff}(\xi, \eta) = \bar{\alpha} + \theta(\xi, \eta). \quad (9.7)$$

The relationship between θ and δ is therefore

$$\theta(\xi, \eta) = \delta(\xi, \eta) \frac{C_L S}{C}. \quad (9.8)$$

From Equations (9.5) and (9.7),

$$\Delta \bar{\alpha} + \alpha_t(\eta) - \frac{\partial z_c(\xi, \eta)}{\partial x} = \theta(\xi, \eta). \quad (9.9)$$

Using Equation (9.9), $\Delta \bar{\alpha}$, $\alpha_t(\eta)$ and $z_c(\xi, \eta)$ may all be found individually in terms of $\theta(\xi, \eta)$ by noting the following,

$$\alpha_t(0) = 0, \quad (9.10)$$

$$z_c(0, \eta) = z_c(1, \eta) = 0. \quad (9.11)$$

The resulting expressions are

$$\Delta \bar{\alpha} = \int_0^1 \theta(\xi, 0) d\xi, \quad (9.12)$$

$$\alpha_t(\eta) = \int_0^1 \theta(\xi, \eta) d\xi - \int_0^1 \theta(\xi, 0) d\xi, \quad (9.13)$$

$$z_c(\xi, \eta) = c(\eta) \left[\xi \int_0^1 \theta(\xi, \eta) d\xi - \int_0^\xi \theta(\xi', \eta) d\xi' \right]. \quad (9.14)$$

In Equations (9.8) and (9.12) to (9.14), to obtain the effective additional incidence, twist and camber for any particular case, the expression for $\delta(\xi, \eta)$ in Equation (9.4) is used with the relevant values of the $e_{k,l}$ coefficients, together with a value for C_L . Since the effects of upwash are proportional to C_L and also, for a given case of transformed planform and relative size of wind-tunnel cross-section, proportional to β , it is convenient to obtain $\Delta \bar{\alpha}/\beta C_L$, $\alpha_t(\eta)/\beta C_L$, and $z_c(\xi, \eta)/\beta C_L c(\eta)$, which are the effective additional incidence, twist and (dimensionless) camber *per unit* βC_L . Thus

$$\frac{\Delta \bar{\alpha}}{\beta C_L} = \frac{(s/b')^2}{A_T(h'/b')} \int_0^1 \delta(\xi, 0) d\xi, \quad (9.15)$$

$$\frac{\alpha_t(\eta)}{\beta C_L} = \frac{(s/b')^2}{A_T(h'/b')} \left[\int_0^1 \delta(\xi, \eta) d\xi - \int_0^1 \delta(\xi, 0) d\xi \right], \quad (9.16)$$

$$\frac{z_c(\xi, \eta)}{\beta C_L c(\eta)} = \frac{(s/b')^2}{A_T(h'/b')} \left[\xi \int_0^1 \delta(\xi, \eta) d\xi - \int_0^\xi \delta(\xi', \eta) d\xi' \right]. \quad (9.17)$$

Results for these parameters are provided for the cases considered in Section 4 by use of ESDUpac A9514, see Appendix A.