

EXAMPLE OF ANALYSIS AND REPRESENTATION OF STATIC PRESSURE CORRECTION DATA IN STEADY FLIGHT

1. NOTATION AND UNITS*

		British
a	speed of sound	ft/s
C_{p_s}	pressure coefficient defined with respect to kinetic pressure, q	
G_1, G_2	two-dimensional transonic similarity parameters defined in Section 6.3	
g	acceleration due to gravity	ft/s ²
l	coefficient in Equations (6.6) and (6.7)	
M	Mach number in freestream, V/a	
M_{local}	Mach number of local flow at static pressure probe sensing ports	
m	exponent in Equations (6.6) and (6.7)	
p_a	ambient static pressure [†]	lbf/ft ²
p_s	sensed static pressure [†] in static system in steady-level-flight conditions	lbf/ft ²
p_{s_i}	registered (indicated) value of static pressure [†] (note, for steady level flight, $p_{s_i} = p_s$)	lbf/ft ²
p_t	total pressure [†] (pressure of air brought to rest isentropically relative to aircraft)	lbf/ft ²
q	kinetic pressure of freestream, $\frac{1}{2}\gamma p_a M^2$, $\frac{1}{2}\rho V^2$	lbf/ft ²
q_{local}	kinetic pressure of local flow at static pressure probe sensing ports	lbf/ft ²
R	gas constant for air	lbf ft/slug K
T_a	ambient temperature	K
$t_{95,v}$	Student's "t" statistic at 95% level of confidence for v degrees of freedom, see Reference 6	
V	true airspeed (resultant aircraft velocity relative to air)	ft/s

For footnote see end of Notation.

* Since this Data Item takes the form of an Example, only one set of units is given – corresponding to those used in the tests that form the basis of the Example.

$var[]$	variance of enclosed parameter, for example $var[p_a], var[q]$	
Z	geometric height	ft
α	angle of attack between fuselage longitudinal datum and projection of aircraft velocity vector into plane of symmetry	deg
α_{local}	angle between probe (and fuselage) longitudinal datum and projection into plane of symmetry of local velocity vector	deg
γ	ratio of specific heat capacities of air, taken as 1.4	
Δp_s	static pressure correction, $(p_a - p_s)$, see Section 4.1	lbf/ft ²
$\varepsilon[]$	residual value (scatter) of enclosed parameter, taken as difference between measured and calculated values, for example $\varepsilon[C_p], \varepsilon[p_a]$	
v	number of degrees of freedom	
$\tilde{\omega}[]$	precision of enclosed parameter, defined using 95% confidence interval, for example $\tilde{\omega}[p_a], \tilde{\omega}[p_s]$	
ρ	density of air	slug/ft ³
$\phi[]$	arbitrary function of enclosed parameter, for example, $\phi[Z], \phi[\alpha_{local}]$	

Subscripts

<i>CALC</i>	denotes value calculated using derived calibrations in Equations (5.1), (6.5), (6.8) and (6.10)
<i>FLOW</i>	denotes contribution due to presence of airframe
<i>REF</i>	denotes parameter value obtained from reference source, i.e. tower or calibrated aircraft
<i>PROBE</i>	denotes contribution due to presence of probe for incompressible flow
<i>PROBE, M</i>	denotes contribution due to presence of probe for compressible flow (evaluated, here, at $\alpha = 0$ only)
$\alpha = 0$	denotes value at zero angle of attack (note, $\alpha = 0$ when $\alpha_{local} = 0$)

† Note that these pressures are absolute.

2. INTRODUCTION

This Data Item illustrates, by example, methods of collecting information and correlating it within a theoretical framework to provide static pressure corrections* as a functional relationship covering the flight envelope of an aircraft.

The Example takes the specific case of a small subsonic military aircraft with a compensated pressure sensing head fitted to a nose boom and utilises low- and high-speed flight-test data for the aircraft in the “clean” configuration. Because the precision of the static pressure transducer was no better than $\hat{\omega}[p_s] = \pm 1 \text{ lbf/ft}^2$ at the 95% level of confidence it is neither feasible nor desirable to attempt to identify the effects of other than the more significant parameters on the static pressure corrections.

The statistical and physical bases of the process of calibrating the static pressure sensing system are explored and the techniques necessary to the procedure are demonstrated. From a relatively modest set of flight data that were collected under non-ideal conditions, a valid calibration is derived: the calibration, when used as such, is demonstrated to be precise to the approximate order of $\pm 0.2 \text{ lbf/ft}^2$. Furthermore, the correlation is effected with a confidence interval of $\pm 1.1 \text{ lbf/ft}^2$. This confidence interval is within the range of estimates made from first principles for the relatively simple system of instrumentation fitted to the test aircraft.

An overall introduction to air data system parameters, errors and calibration laws is given in Data Item No. 86031 (Reference 5). Data Item No. 83029 (Reference 3) considers, generally, the data required for the correction of flight-test anemometric data and describes the correction processes on the assumption that the necessary data are available. Data Item No. 86006 (Reference 4) considers all likely sources of errors for pitot and static pressures in steady flight and describes, in principle, how corrections are obtained.

The test procedures outlined in Section 3.1.3 of Reference 3 and Section 5.2 of Reference 4 are built upon in the present Item – albeit for a case where a relatively unsophisticated installation is used in a flight-testing environment. Note that References 3 and 4 consider both subsonic and supersonic testing and include examples of pressure correction data for a supersonic nose-boom mounted pressure head.

3. FLIGHT TEST OBJECTIVES

The pressure correction data that are considered in this Item were collected to facilitate the analysis of data from other flight testing. Two parts of the flight envelope were of particular interest.

- (i) Low speed stalling performance of the aircraft was a matter of guarantee.
- (ii) Demonstration of the level flight envelope and some cruise performance was a requirement.

To investigate the stalling performance it was necessary to identify the value of the maximum lift coefficient within tight limits; it follows that the static-pressure correction at low speed and high angle of attack was required to be known within correspondingly tight limits. Flight tests for identifying pressure errors at such conditions were precluded both for safety and because of the difficulty in establishing a firm reference pressure with which to compare. Thus, the data that were collected under non-hazardous angle-of-attack conditions needed to be extrapolated to the stall condition.

At the higher Mach numbers, near to the limits of the aircraft's level-flight envelope, there was no specific need to extrapolate far beyond the range of the measurements. However, the collection of data needed to be correlated so that interpolations could be made. Thus, a complete calibration of the system was sought at low cost using a convincing and convenient model.

* The so named *Pressure Error Corrections*.

4. DEFINITIONS AND MODELLING ASSUMPTIONS

4.1 Definitions

The convention is adopted that the ambient static pressure is obtained from the pressure sensed in the measuring system by adding a correction, i.e.

$$\text{ambient pressure} = (\text{pressure sensed at probe}) + (\text{pressure correction}) \quad (4.1)$$

$$p_a = p_s + \Delta p_s. \quad (4.2)$$

A **Pressure Correction Coefficient** is defined as the quotient of the pressure correction and the kinetic pressure, q , in the freestream,

$$C_{p_s} = \frac{\Delta p_s}{q} = \frac{p_a - p_s}{q}. \quad (4.3)$$

In the modelling assumptions of this Data Item, the concept of the correction is central – so that steps moving from the sensed pressure at the probe towards the goal of the correct value of the ambient pressure are additive*.

4.2 Modelling Assumptions

4.2.1 General

In order to derive values of ambient static pressure from the static pressure sensed by a pressure head situated in front of a moving body, it is necessary to consider two sources of systematic error. Firstly, the presence of the body in the flowfield modifies the pressure at any given location; secondly, the design of pressure heads is such that the pressure registered by the head is different from the pressure to be sensed. This may be stated as,

$$\text{overall difference} = (\text{airframe contribution}) + (\text{probe contribution}) \quad (4.4)$$

and expressed in terms of pressures as,

$$\Delta p_s = (p_a - p_s) = (p_a - p_{s_{FLOW}}) + (p_{s_{FLOW}} - p_s) \quad (4.5)$$

or
$$p_a = p_s + (p_a - p_{s_{FLOW}}) + (p_{s_{FLOW}} - p_a). \quad (4.6)$$

The increments on the right hand side of Equation (4.6) may be regarded as the numerators of pressure coefficients. The first is the pressure change that arises when the body in question is placed in a moving stream. Thus, the coefficient is appropriately defined with respect to freestream kinetic pressure,

$$C_{p_{s_{FLOW}}} = \left(\frac{p_a - p_{s_{FLOW}}}{q} \right). \quad (4.7)$$

The second increment in Equation (4.6) is a slightly different case; if the pressure sensing head were to be isolated from the body – in a wind tunnel for example – then it would be entirely appropriate to use the freestream conditions to describe the onset flow to the probe. But, in this Example, the probe is isolated

* See ESDU Data Item No. 86006 (Reference 4) for a discussion of the relation between *Corrections* and *Errors*.

from the freestream by the pressure field of the body: the probe senses (and is influenced by) local conditions. It is thus operating at a Mach number and a kinetic pressure different from those in the freestream. Thus the pressure differential due to the probe is written:

$$(p_{s_{FLOW}} - p_s) = q_{local} \times C_{p_s PROBE} \quad (4.8)$$

In terms of coefficients Equation (4.5) is written

$$C_{p_s} = C_{p_s FLOW} + \left(\frac{q_{local}}{q} \right) C_{p_s PROBE} \quad (4.9)$$

It may be expected that the static pressure coefficient C_{p_s} depends on both Mach number and angle of attack*.

4.2.2 Mathematical model

In order to establish a calibration for such a sensing system, it might be thought necessary to fly the subject aircraft over the whole of its envelope against a known reference standard for freestream (*i.e.* ambient) pressure. Such a course is impractical in terms of both cost and time: some means must be sought to avoid such an expensive activity.

Consider the airframe contribution: subsonic linearised theory can be used to show[†] that the static pressure coefficient ahead of a body (of revolution) is independent of the freestream Mach number. The pressure coefficient is, however, influenced by the angle of attack independently of Mach number. This provides an indication for a method for both testing and correlation because, as a consequence, the airframe contribution can be identified in incompressible flow and that effect can be attributed to the airframe *throughout the subsonic speed envelope*. So the sensing head can be calibrated over an appropriate range of angle of attack in a low-speed wind tunnel; flight tests can be carried out at low speed and low altitude and a means sought to extrapolate the low-speed characteristics of the probe over the speed envelope.

To do this, the pressure head could be calibrated in a high-speed wind tunnel. However, prudence would mandate that flight testing should be carried out to ensure that there were no unexpected installation effects. Since flying has to be arranged it is therefore defensible to eliminate the cost of the high-speed wind-tunnel testing from the exercise. Any variation in pressure coefficient with Mach number identified from flight can be ascribed to the probe. The modelling for the calibration is therefore in accordance with the relationship

$$C_{p_s}[\alpha, M] \sim C_{p_s FLOW}[\alpha] + \left(\frac{q_{local}}{q} \right) C_{p_s PROBE}[\alpha_{local}, M_{local}] \quad (4.10)$$

and the test schedule can be designed to enable the deduction of the component variations.

* In this exercise the effects of sideslip are not considered, see Section 7.3

† See Reference 7.

4.2.3 Assumptions in mathematical model

The validity of the mathematical model as a representation of physical reality rests upon the assumptions that are implicit in the establishment of the procedure. There are seven assumptions underlying the process, as follows.

- A** Equation (4.10) is valid: the pressure coefficient is dependent only on freestream Mach number and fuselage angle of attack.
- B** The probe^{*} effect varies with local angle of attack and local Mach number but the variation with α is independent of the variation with M .
- C** There is no coupling between the probe effect and the airframe effect: that is, the principle of superposition holds.
- D** The airframe effect varies only with angle of attack – it is independent of Mach number.
- E** For the purposes of the sought-for correlation, the local and freestream kinetic pressures are equal.
- F** For the purposes of correlation, local Mach number is identical to freestream Mach number.
- G** The form of Transonic Similarity identified is valid for Mach number approaching unity.

5. TEST PROCEDURES

5.1 Wind-Tunnel Testing

In order to minimise the cost of the test programme, no use was made of a high-speed wind tunnel. The probe was calibrated only in a low-speed wind tunnel. Care was taken when designing the support system to ensure that the probe tip was located at the same longitudinal, lateral and vertical position in the working section for all angles of attack. The data collected are summarised in Table 10.1 and are plotted in Figure 1. The following empirical fit to the data was chosen in order to obviate the use of look-up tables in the flight data reduction and handling system.

$$C_{p_s \text{ PROBE, CALC}} = -0.04 \cos \alpha_{local} + 0.0833 \cos 2\alpha_{local} + \phi[\alpha_{local}] \quad (5.1)$$

$$\text{where } \phi[\alpha_{local}] = 0.265(0.0419 - \sin \alpha_{local})^2 \quad \alpha_{local} \leq 18 \text{ deg}$$

$$\text{and } \phi[\alpha_{local}] = 0.019 + 0.84(0.0419 - \sin \alpha_{local})^4 \quad \alpha_{local} > 18 \text{ deg}.$$

To ensure that Equation (5.1) could be used in conjunction with aircraft data, flight testing had shown that the upwash at the station of the pressure head was such that

$$\alpha_{local} = 1.2\alpha. \quad (5.2)$$

5.2 Flight Testing

Tests were carried out at sea level (low speed) and at a height of 25 000 feet (high speed); the essentials of the techniques were similar at both heights. At sea level the test aircraft was flown past an observation

* The probe is centrally mounted, ahead of the fuselage.

tower and its vertical position with respect to an accurate barometer was recorded photographically. At the height of 25 000 ft, the test aircraft was flown in formation with another aircraft which had an attested calibration* of its air data system. The relative vertical positions of the aircraft were recorded photographically. Some details of the test techniques are included in Sections 6.2 and 6.3 (dealing, respectively, with the tests at sea level and at 25 000 ft).

At each test point, records were taken of the following parameters:

$$p_s, \quad p_{a_{REF}}, \quad p_t, \quad T_{a_{REF}}, \quad (Z - Z_{REF}), \quad \alpha_{local}.$$

The methods used to “calibrate” the atmosphere in terms of temperature and pressure are well established and are based on the premise that air is a perfect gas and that the atmosphere is in static balance – see Reference 1, for example. Thus the general equation of state for a perfect gas,

$$p_a = \rho R T_a, \quad (5.3)$$

applies and is used in the differential equation that describes the condition of static balance in the atmosphere,

$$dp_a = -\rho g dZ \quad (5.4)$$

to obtain a finite difference equation

$$p_a - p_{a_{REF}} = -\left(\frac{g p_{a_{REF}}}{R T_{a_{REF}}}\right)(Z - Z_{REF}). \quad (5.5)$$

Equation (5.5) can be used to convert the measured increments in geometric height to increments in ambient pressure. In some circumstances, the measured increments in geometric height may be too large to justify the use of the finite difference approach. In such cases Equations (5.3) and (5.4) are combined with a relation, $\phi[Z]$, between temperature and height, to describe the pressure relationship with height,

$$\frac{dp_a}{p_a} = -\frac{g}{R} \frac{dZ}{\phi[Z]}. \quad (5.6)$$

Given a formulation of the function $\phi[Z]$, Equation (5.6) can be solved.

The temperature lapse rate for the International Standard Atmosphere (Reference 1) is usually suitable for use in these and similar experimental situations. For other atmospheres, see Reference 2.

* See References 9, 10 and 11.

6. ANALYSIS OF FLIGHT DATA AND SYNTHESIS OF CALIBRATION OF STATIC PRESSURE SYSTEM

6.1 Summary of General Analysis Process

The process of analysis follows from Equation (4.10). For all practical purposes, the ratio q_{local}/q may be taken to be unity^{*}; in addition, for the limited purpose of calibrating the probe, the local Mach number, M_{local} , may be equated^{*} to the freestream value. The steps to be followed are listed below.

- (i) Values of C_{p_s} from the low-altitude, low-speed fly-pasts are used to identify the (incompressible) values of the airframe contribution, $C_{p_{s FLOW}}$, to the static pressure coefficient, *i.e.* from Equation (4.9),

$$C_{p_{s FLOW}} = C_{p_s} - C_{p_{s PROBE, CALC}}, \quad (6.1)$$

where values of $C_{p_{s PROBE, CALC}}$ are calculated using Equation (5.1). Results are presented in Table 10.2 and Figure 3. See Section 6.2 for further details and a description of the curve-fitting procedure used to identify the value of $C_{p_{s FLOW}}$ at $\alpha = 0$.

- (ii) The static pressure correction due to the probe for the effects of compressibility is denoted $C_{p_{s PROBE, M}}$ and, as indicated in “B” in Section 4.2.3, is independent of α . Values of $C_{p_{s PROBE, M}}$ are deduced using the curve-fit referred to in (i), above, (and presented as Equation (6.5) in Section 6.2) to eliminate the airframe contribution, and Equation (5.1) to adjust to $\alpha = 0$, as follows.

$$C_{p_{s PROBE, M}} = C_{p_s} - C_{p_{s FLOW, CALC}} - [C_{p_{s PROBE, CALC}} - (C_{p_{s PROBE, CALC}})_{\alpha = 0}]. \quad (6.2)$$

(By definition, all the “adjustment” terms in Equation (6.2) are independent of Mach number.) In the case of the values of C_{p_s} from the high-altitude tests (obtained at angles of attack near to zero), only the airframe adjustment, $C_{p_{s FLOW, CALC}}$, is significant and the terms in the square brackets can be ignored. Results are presented in Table 10.3 and Figures 4 and 5. See Section 6.3 for details of the correlation of values of $C_{p_{s PROBE, M}}$ with freestream Mach number.

- (iii) The required calibration of the static pressure sensor is synthesised (in Section 6.4) from the components identified in Sections 5.1, 6.2 and 6.3.
- (iv) The statistical implications of the whole process are considered and the random errors (precision) of the final calibration assessed in Section 7.

6.2 Identification of Airframe Contribution for Incompressible Flow

The airframe can be expected to affect the static pressure at points on its longitudinal axis close to and in front of the nose. The effect is due to both its bulk and its orientation to the flow. If the forward fuselage is approximately axisymmetric then the variation of pressure coefficient with angle of attack is symmetric about zero. As angle of attack varies from zero, a static sensor mounted on an axially placed boom moves progressively away from the stagnation streamline: that is, away from the position of maximum positive pressure at the given radial distance from the nose. Thus, as angle of attack increases, the pressure coefficient

^{*} While this assumption is acceptable for a sensor mounted on a nose probe, it may not be so for a fuselage side-mounted probe.

decreases. A theoretical method* was used to assess the magnitude of the effect; the solutions for a range of angles of attack are plotted in Figure 2. An empirical fit to the solutions is also included on the plot. It is:

$$C_{p_{s \text{ FLOW}}} = 0.073 \cos 2\alpha - 0.0088. \quad (6.3)$$

It is probable that the relation quoted is not directly applicable. However, the format is convenient and an initial basis for correlation is obtained by using the $\alpha = 0$ case as a datum and ignoring the numerical constants, to give

$$\frac{C_{p_{s \text{ FLOW}}}}{(C_{p_{s \text{ FLOW}}})_{\alpha = 0}} = \cos 2\alpha. \quad (6.4)$$

In the flight tests twelve test runs were made over a range of speed (and thus at various angles of attack); the speed ranged from the lowest considered safe up to approximately $M = 0.3$ where the angle of attack was sufficiently low to allow the identification of the effect of that parameter and to ensure the assumption of incompressible flow was not invalidated†. Pressure coefficients were calculated and the relation of Equation (5.1) was used to account for the probe effects; the appropriate data are shown in Table 10.2 and Figure 3. The latter is a plot of the incompressible pressure coefficient against angle of attack. The function used to describe the data is

$$C_{p_{s \text{ FLOW, CALC}}} = -0.0858 \times \cos 2\alpha. \quad (6.5)$$

In accord with “D” in Section 4.2.3, this function is assumed to hold over the subsonic flight envelope.

6.3 Identification of Probe Effects for Compressible Flow

To obtain test data for compressible flow conditions the formation test flights at high altitude were flown at 25 000 ft since this maintained values of angle of attack close to zero over a wide range of M . Thus the solutions of Equation (6.2)(6.2), which are shown in Table 10.3, may be studied to obtain a correlation that represents the effect of M on the probe. (Table 10.3 presents the high altitude data of test points 13 to 28; the low altitude tests are included as points 1 to 12.)

In studies of this kind it is general practice to make use of the notions of *Transonic Similarity*‡. As a consequence, Figure 4 was prepared. This is a plot of

$$\left(\frac{M^2}{1 - M^2} \right) \times C_{p_{s \text{ PROBE, } M}} \quad \text{versus} \quad \left(\frac{M^2}{(1 - M^2)^{1.5}} \right)$$

and forms the basis of the search for a simple correlation. Physically, the correlating function ought to tend to the *Prandtl-Glauert* relation as Mach number tends to zero. The characteristics of the data in Figure 4

* Since the flow is incompressible, the Laplace equation was solved for a uniform onset flow. The airframe was modelled using sources in a quadrilateral layout – on the wing, vortices were also added using the sides of the quadrilaterals as axes.

† In practice the assumption that compressibility effects are not significant is tied to the value ascribed to the function $\sqrt{(1 - M^2)}$. Values greater than 0.95 ($M \approx 0.3$) are generally accepted to imply that the flow is incompressible. For most aircraft applications a value of 0.9 ($M \approx 0.45$) can safely be taken as the transition to compressible flow for Performance work.

‡ This subject is addressed in Reference 8.