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Drag due to lift for plane swept wings, alone or in combination with a body, up to high angles of attack at subsonic speeds

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DRAG DUE TO LIFT FOR PLANE SWEEP WINGS, ALONE OR IN COMBINATION WITH A BODY, UP TO HIGH ANGLES OF ATTACK AT SUBSONIC SPEEDS

CONTENTS

	Page
1. NOTATION AND UNITS	1
2. INTRODUCTION	7
2.1 Previous Issue	7
3. METHOD	7
3.1 Drag Model	7
3.1.1 Plane wing	7
3.1.2 Plane wing with body	9
3.1.3 Representation of K_{visc}	11
3.2 Calculation of Leading-edge Suction Factor	11
3.2.1 Derivation of for aerofoils	11
3.2.2 Adaptation to wings	15
3.2.3 Illustration of leading-edge suction variation	19
3.2.4 Improved method	20
3.3 Wing Lift	25
3.3.1 Wing alone	25
3.3.2 Wing-body combinations	28
4. ACCURACY AND APPLICABILITY	29
4.1 Assessment of Model	29
4.2 Accuracy	31
5. DERIVATION AND REFERENCES	44
5.1 Derivation	44
5.2 References	46
6. EXAMPLE	47
FIGURES 1 to 3	66 to 68

DRAG DUE TO LIFT FOR PLANE SWEEP WINGS, ALONE OR IN COMBINATION WITH A BODY, UP TO HIGH ANGLES OF ATTACK AT SUBSONIC SPEEDS

1. NOTATION AND UNITS

		SI	British
A	aspect ratio of gross wing		
A_N	aspect ratio of net wing, see Sketch 1.2		
a_1	wing lift-curve slope, based on S_W , from Item No. 70011	rad^{-1}	rad^{-1}
a_{1eff}	effective wing lift-curve slope, based on S_W	rad^{-1}	rad^{-1}
a_{1N}	wing lift-curve slope of net wing, based on S_N , from Item No. 70011	rad^{-1}	rad^{-1}
C_A	axial force coefficient, positive aft, based on S_W		
C_{AB}	body axial force coefficient, based on S_W		
C_{AB0}	body axial force coefficient at zero incidence, based on S_W		
C_{AW}	wing axial force coefficient, based on S_W		
C_{AWv}	wing viscous axial force coefficient, based on S_W		
C_D	drag coefficient, based on S_W		
ΔC_D	error in predicted drag coefficient, experimental value minus predicted value		
C_{DPmin}	minimum profile drag coefficient, based on S_W		
ΔC_{Dvisc}	contribution to C_D from lift-dependent viscous drag		
C_{DW}	drag coefficient of wing in presence of body, based on S_W , see Section 3.1.2		
C_L	lift coefficient, based on S_W		
C_{Lmax}	maximum lift coefficient, based on S_W		
C_{Llev}	contribution to C_L from leading-edge vortex, see Section 3.1.1 and Section 3.3		

Issued November 1995
 With Amendments A and B, August 2007 – 68 pages
 This page Amendment B

C_{Lsev}	contribution to C_L from side-edge vortex, see Section 3.1.1 and Section 3.3		
C_{LW}	lift coefficient of wing in presence of body, based on S_W , see Section 3.3.2,		
C_N	normal force coefficient, based on S_W		
C_{Naug}	augmentation in wing normal force coefficient, based on S_W , see Section 3.3.1		
C_{NB}	normal force coefficient of body, based on S_W		
ΔC_{NB}	correction to forebody-cylinder normal force coefficient to allow for boat-tail, based on S_W		
C_{NBC}	cross-flow normal force coefficient of forebody-cylinder, based on S_p		
$C_{NB\alpha}$	normal-force-curve slope of forebody-cylinder, based on S_B	rad ⁻¹	rad ⁻¹
C_{Nlev}	contribution to C_N from leading-edge vortices, see Section 3.3.1		
C_{Nsev}	contribution to C_N from side-edge vortices, see Section 3.3.1		
C_{NW}	normal force coefficient of gross wing, based on S_W		
C_p	pressure coefficient		
C_{plim}	limit on peak pressure coefficient used in prediction of σ , see Section 3.2.1		
C_s	theoretical thin-wing leading-edge suction force coefficient acting normal to leading edge in plane of wing, based on S_W		
C_{st}	theoretical thin-wing side-edge suction force coefficient acting normal to tip chord in plane of wing, based on S_W , see Section 3.3		
c	chord	m	ft
c_n	normal-section chord, see Sections 3.2.2 and 3.2.4	m	ft
c_s	local theoretical thin-wing suction force coefficient normal to leading edge, based on c		
c_{sn}	theoretical thin-wing normal-section suction force coefficient, based on c_n , see Sections 3.2.2 and 3.2.4		
$\bar{c}$	geometric mean chord of gross wing	m	ft
D	body maximum diameter	m	ft

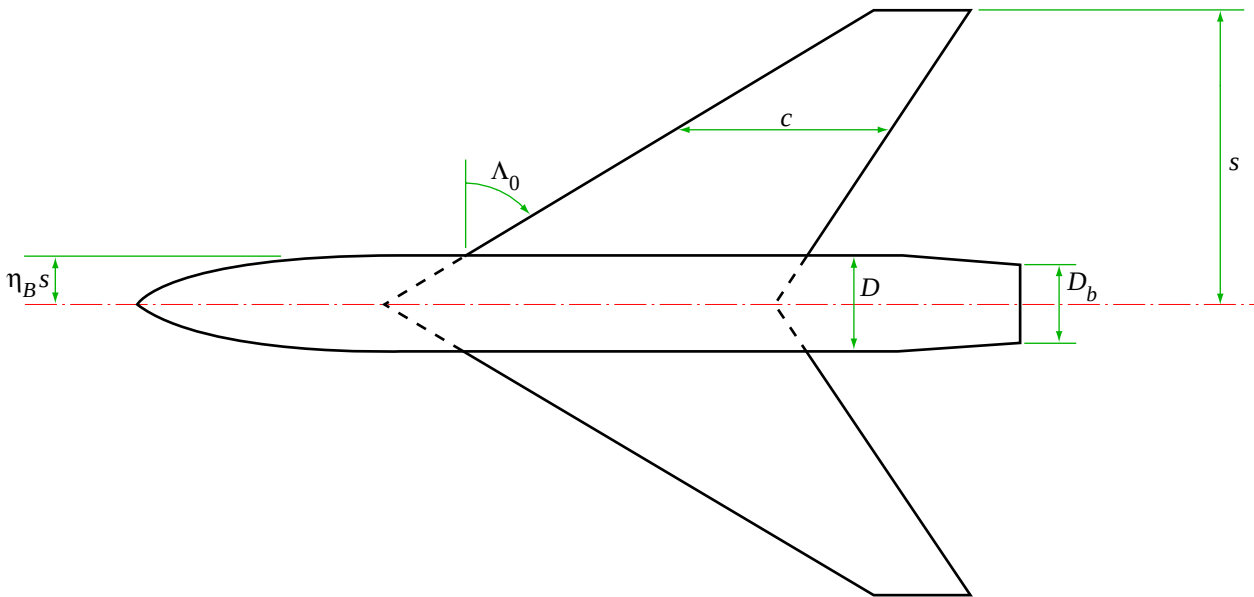
This page Amendment B

D_b	body base diameter	m	ft
e_1, e_2 e_3, e_4	empirical correlating powers, see Section 3.2.4, Equations (3.56) and (3.59) to (3.61)		
F	$-\gamma C_{plim}$, see Sections 3.2.2 and 3.2.4 and Figure 1		
F_t	factor used in evaluation of C_{st} , see Section 3.3.1 and Figure 2		
F_{VB}	factor limiting amount of leading-edge vortex lift that can be realised, see Section 3.3.1, Figure 3 and Appendix C of Item No. 96025 (Reference 47)		
K	empirical correlating factor, see Section 3.2.4		
K_{visc}	lift-dependent viscous drag factor, see Section 3.1.3		
K_{VI}	constant in Equation (3.69), see Equation (3.70)		
K_{Vt}	constant in Equation (3.73), see Equation (3.74)		
l_a	length of boat-tail	m	ft
l_c	characteristic length in Equation (3.71), streamwise distance from leading-edge tip to trailing-edge apex, see Sketch 1.3	m	ft
M	free-stream Mach number		
M_e	effective local Mach number in calculation of σ , see Equations (3.30) and (3.40)		
M_n	normal-section Mach number, $M \cos \Lambda_0$		
p	parameter in Equation (3.23)		
Q	empirical correlating factor, see Section 3.2.4, Equation (3.58)		
R	free-stream Reynolds number, based on $\bar{c}$		
R_n	normal-section Reynolds number, based on c_n , see Section 3.2.2 and 3.2.4		
r_i	leading-edge radius index, see Section 3.2.4		
r_{in}	value of r_i for normal section, see Section 3.2.4, Equation (3.57)		
S_B	body maximum cross-sectional area	m ²	ft ²
S_p	body planform area (neglecting reduction in area due to boat-tail)	m ²	ft ²

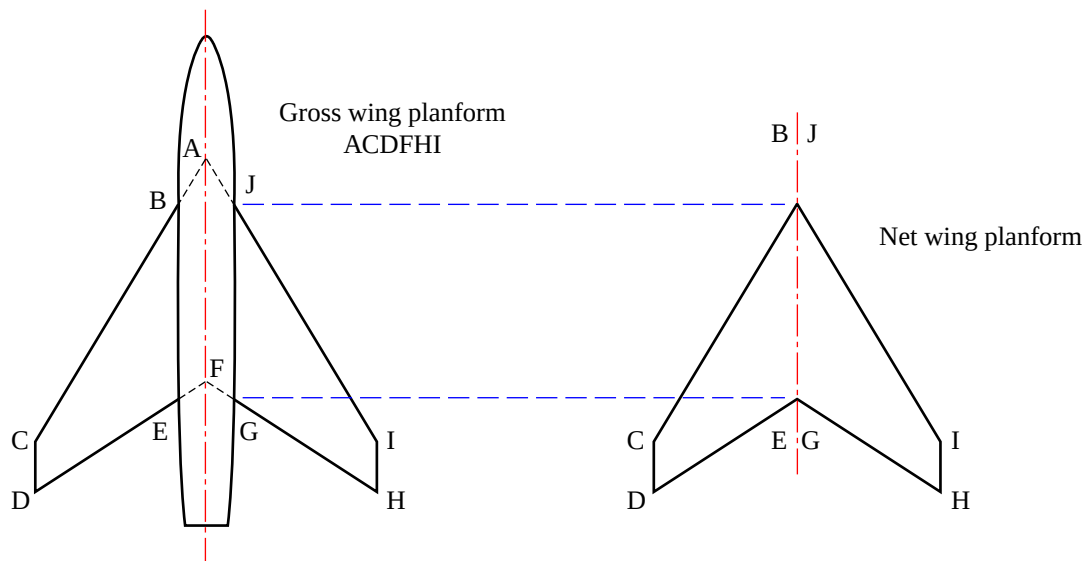
S_N	planform area of net wing, see Sketch 1.2	m ²	ft ²
S_W	planform area of gross wing	m ²	ft ²
s	semi-span of gross wing	m	ft
t	streamwise-section maximum thickness	m	ft
t_n	normal-section maximum thickness, see Sections 3.2.2 and 3.2.4	m	ft
x	streamwise distance along chord line aft from leading edge	m	ft
z	distance normal to chord line	m	ft
α	angle of attack of wing centre-line and body longitudinal axis	deg	deg
α_s	initial angle of attack at which leading-edge suction is lost, see Section 3.3.1 and Sketch 3.12	deg	deg
β	compressibility parameter, $(1 - M^2)^{1/2}$		
β_e	$(1 - M_e^2)^{1/2}$		
β_n	$(1 - M_n^2)^{1/2}$		
γ	ratio of specific heat capacities of air		
δ	$(1 + \delta)$ is lift-dependent drag factor due to trailing-vortex wake		
η	spanwise distance from centre-line as fraction of semi-span		
η_B	value of η at body side		
Λ_0	leading-edge sweep angle	deg	deg
Λ'_0	transformed leading-edge sweep, $\tan^{-1}(\beta^{-1}\tan\Lambda_0)$	deg	deg
$\Lambda_{1/4}$	quarter-chord sweep	deg	deg
$\Lambda_{1/2}$	half-chord sweep	deg	deg
Λ_1	trailing-edge sweep	deg	deg
λ	taper ratio of gross wing, ratio of tip chord to centre-line chord		
λ_N	taper ratio of net wing, see Sketch 1.2		
ξ	chordwise position of maximum thickness of streamwise section as fraction of chord		
ρ	streamwise-section leading-edge radius	m	ft

This page Amendment B

ρ_n	normal-section leading-edge radius, see Sections 3.2.2 and 3.2.4	m	ft
σ	local leading-edge suction factor, Equation (3.39) and Sketch 3.7		
$\bar{\sigma}$	mean leading-edge suction factor, Equation (3.3)		
τ, τ_a	section trailing-edge angles, defined in Item No. Aero W.01.01.05 (Reference 41)		



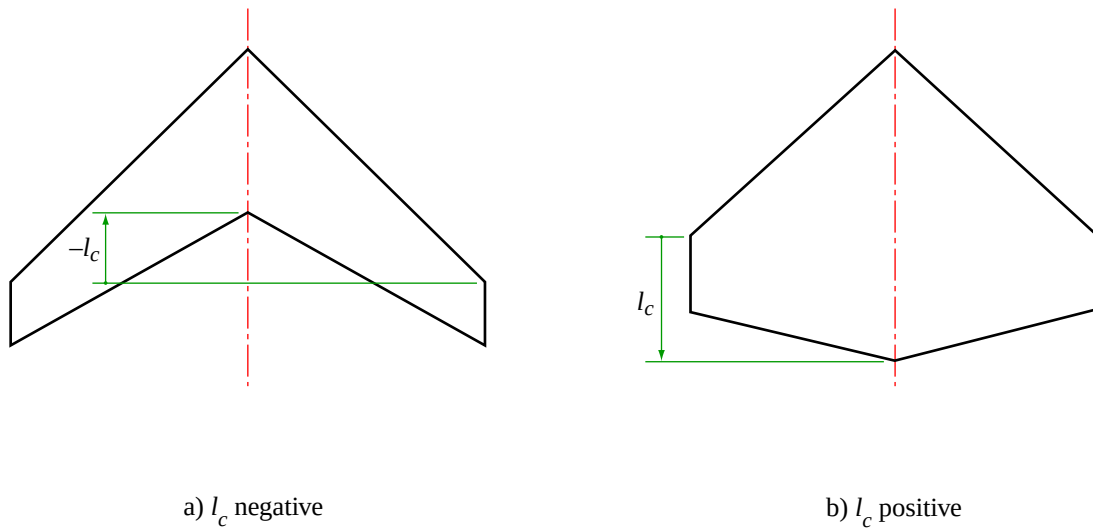
Sketch 1.1 Wing-body geometry



$$A_N = \frac{(1 + \lambda)(1 - \eta_B)A}{1 - (1 - \lambda)\eta_B + \lambda}, \quad \lambda_N = \frac{\lambda}{1 - (1 - \lambda)\eta_B},$$

$$\frac{S_N}{S_W} = \frac{(1 - \eta_B)[1 - (1 - \lambda)\eta_B + \lambda]}{1 + \lambda}.$$

Sketch 1.2 Definition of net wing



Sketch 1.3 Illustration of length l_c

2. INTRODUCTION

This Data Item gives a method for the prediction of drag due to lift for plane swept wings of straight taper, alone or in combination with a body. The method applies at subsonic speeds and when boundary-layer transition occurs near the wing leading edge. It uses an estimate of attainable wing leading-edge suction to establish a value for the wing lift-dependent drag between the limits appropriate full and zero leading-edge suction. The resolved components of the wing viscous axial force and of body axial force and normal force provide additional contributions to the total drag.

The overall model is detailed in Section 3.1. Section 3.2 explains how the leading-edge suction is evaluated for aerofoils and wings. The method incorporates a prediction of the variation of the wing lift coefficient with angle of attack which is described in Section 3.3. The lift prediction includes the non-linear contribution from leading-edge vortex flow that is present for highly-swept wings, together with a side-edge vortex contribution, but the method is limited to angles of attack where the full realisation of the predicted potential flow contribution is not degraded by flow separation over the main part of the wing. The accuracy and applicability are addressed in Section 4. Section 5 lists the Derivation and References. An extensive worked example is provided in Section 6.

2.1 Previous Issue

The original issue of this Data Item incorporated only the correlation of aerofoil attainable leading-edge suction presented by Carlson *et al.* in Derivations 28 and 29, with some modification in the dependence on Reynolds number and additional constraints on the magnitude of the limiting pressure coefficient. The extension of the method to wings by means of a pseudo-symmetric normal section was based on the recommendation of those reports. Later work by Carlson *et al.* incorporated extensions of the original theoretical and experimental databases, which improved the suction correlation and gave a simpler extension to wings, see Derivation 36. The later work, still with some modification, provides the basis of the method now recommended in this Data Item. Reference is still made to the earlier original work because it introduces the ideas behind the correlation of attainable leading-edge suction.

A companion work, Item No. 96025 (Reference 47), extends the method of this Item to wings with camber and twist. A computer program, ESDUpac A9625, accompanies that Item as the extended method is unsuited to hand calculation.

3. METHOD

3.1 Drag Model

3.1.1 Plane wing

For a plane wing the axial force is the difference between the viscous axial force and the leading-edge suction force acting in the axial direction. Therefore, with full leading-edge suction

$$C_A = C_{AWV} - C_s \cos \Lambda_0. \quad (3.1)$$

If, as incidence increases, the flow over the wing begins to separate from the leading edge some suction force is lost. A mean leading-edge suction factor $\bar{\sigma}$ is introduced to represent the ratio of the suction force that is actually achieved to the maximum possible value, and the axial force is written

$$C_A = C_{AWV} - \bar{\sigma} C_s \cos \Lambda_0. \quad (3.2)$$

The value of $\bar{\sigma}$ for a wing is obtained by spanwise integration of the local normalised theoretical

leading-edge suction distribution $c_{sc}/C_s \bar{c}$ for a thin wing factored by the local correction factor σ that is the ratio of the suction attainable on an actual wing section to the thin-wing value, so that

$$\bar{\sigma} = \int_0^1 \frac{\sigma c_s c}{C_s \bar{c}} d\eta. \quad (3.3)$$

Item No. 94037 (Derivation 34) or the corresponding program in Item No. 94038 (Derivation 35) provides a method for obtaining $c_{sc}/C_s \bar{c}$. The factor σ depends on the local flow conditions, incidence and section geometry and is determined by a semi-empirical method. Its prediction is described in Section 3.2.

With the axial force expressed in terms of resolved components of lift and drag,

$$C_A = C_D \cos \alpha - C_L \sin \alpha, \quad (3.4)$$

it follows that

$$\begin{aligned} C_D &= C_A \sec \alpha + C_L \tan \alpha \\ &= C_{AWv} \sec \alpha - \bar{\sigma} C_s \cos \Lambda_0 \sec \alpha + C_L \tan \alpha. \end{aligned} \quad (3.5)$$

The lift coefficient can be expressed as

$$C_L = a_1 \sin \alpha \cos^2 \alpha - (C_{AWv} - \bar{\sigma} C_s \cos \Lambda_0) \sin \alpha + C_{Llev} + C_{Lsev}, \quad (3.6)$$

where the first term follows the representation by Polhamus (Derivation 12) of the resolved potential normal force coefficient in the absence of leading-edge suction, $C_N = a_1 \sin \alpha \cos \alpha$, with a_1 the magnitude of the wing lift-curve slope in radians, as may be obtained from Item No. 70011 (Derivation 15). The second term is the resolved axial force coefficient and C_{Llev} and C_{Lsev} are the contributions from any leading-edge or side-edge vortices that form (see Section 3.3.1).

It is possible to rewrite Equation (3.5) in the form

$$C_D = C_{AWv} \cos \alpha - \bar{\sigma} C_s \cos \Lambda_0 \sec \alpha + (C_L + C_{AWv} \sin \alpha) \tan \alpha, \quad (3.7)$$

where the first term is the sum of the terms $C_{AWv} \sec \alpha$ and $(-C_{AWv} \sin \alpha) \tan \alpha$, algebraically simplified.

From Equation (3.6)

$$(C_L + C_{AWv} \sin \alpha) = a_1 \sin \alpha \cos^2 \alpha + \bar{\sigma} C_s \cos \Lambda_0 \sin \alpha + C_{Llev} + C_{Lsev}. \quad (3.8)$$

However, since $C_{AWv} \sin \alpha$ is small compared to C_L , $C_L \approx (C_L + C_{AWv} \sin \alpha)$ so that

$$C_L \approx a_1 \sin \alpha \cos^2 \alpha + \bar{\sigma} C_s \cos \Lambda_0 \sin \alpha + C_{Llev} + C_{Lsev}, \quad (3.9)$$

see also Section 3.3.1, later.

With this simplification, Equation (3.7) becomes

$$C_D = C_{AWv} \cos \alpha - \bar{\sigma} C_s \cos \Lambda_0 \sec \alpha + C_L \tan \alpha. \quad (3.10)$$

This page Amendment B

The first term in Equation (3.7) is the resolved viscous axial force coefficient which can be written as the sum of a component independent of lift, C_{DPmin} , and a lift-dependent component, ΔC_{Dvisc} , i.e.

$$C_{AWv} \cos \alpha = C_{DPmin} + \Delta C_{Dvisc}, \quad (3.11)$$

so that Equation (3.7) becomes

$$C_D = C_{DPmin} + \Delta C_{Dvisc} - \bar{\sigma} C_s \cos \Lambda_0 \sec \alpha + C_L \tan \alpha. \quad (3.12)$$

The estimation of C_{DPmin} may be made through the application of a form factor to a flat-plate skin friction coefficient. For example, Item Nos Aero W.02.04.01 to 03 (Derivations 1 to 3) contain data for aerofoils. For wings with sweep $\Lambda_{1/4} \leq 45^\circ$ and section thickness to chord ratio in the range $0.1 \leq t/c \leq 0.2$, C_{DPmin} may be estimated by using the more advanced method of ESDU 07002, Reference 48. Alternatively, an empirical value may be used. Section 3.1.3 discusses the evaluation and form of ΔC_{Dvisc} .

For no loss of leading-edge suction ($\bar{\sigma} = 1$) and no vortex lift contributions, a resolution of the wing lift and trailing-vortex drag in inviscid flow gives

$$C_s \cos \Lambda_0 = C_L \sin \alpha - \left\{ (1 + \delta) C_L^2 / (\pi A) \right\} \cos \alpha, \quad (3.13)$$

and, with substitution for $C_s \cos \Lambda_0$, Equation (3.12), becomes

$$C_D = C_{DPmin} + \Delta C_{Dvisc} + (1 + \delta) C_L^2 / (\pi A). \quad (3.14)$$

If all leading-edge suction is lost ($\bar{\sigma} = 0$), Equation (3.12) gives

$$C_D = C_{DPmin} + \Delta C_{Dvisc} + C_L \tan \alpha. \quad (3.15)$$

Plotted against C_L , Equations (3.14) and (3.15) form minimum and maximum boundaries, respectively, for C_D . The predicted C_D will lie between those boundaries if there is a partial loss of leading-edge suction ($0 < \bar{\sigma} < 1$).

3.1.2 Plane wing with body

The expression for the drag coefficient of a wing-body combination requires the introduction of the resolved components of the body axial and normal force coefficients, C_{AB} and C_{NB} , see below. Comparisons with experimental data have shown that these can be simply added to the gross wing estimate, subject to some modification of the wing viscous drag terms and the employment of an effective lift-curve slope for the wing, a_{1eff} , see Section 3.3.2.

The drag coefficient for the wing-body combination is then

$$C_D = C_{DW} + C_{NB} \sin \alpha + C_{AB} \cos \alpha, \quad (3.16)$$

where, via Equation (3.10),

$$C_{DW} = C_{AWv} \cos \alpha - \bar{\sigma} C_s \cos \Lambda_0 \sec \alpha + C_{LW} \tan \alpha, \quad (3.17)$$

This page Amendment B

and, via Equation (3.9),

$$C_{LW} = a_{1eff} \sin \alpha \cos^2 \alpha + \bar{\sigma} C_s \cos \Lambda_0 \sin \alpha + C_{Llev} + C_{Lsev}, \quad (3.18)$$

as given in Section 3.3.2, later.

Reference to Equation (3.11) provides the term $C_{AWv} \cos \alpha$ in the form

$$C_{AWv} \cos \alpha = C_{DPmin} + \Delta C_{Dvisc} (S_N / S_W), \quad (3.19)$$

so that
$$C_{DW} = C_{DPmin} + \Delta C_{Dvisc} (S_N / S_W) - \bar{\sigma} C_s \cos \Lambda_0 \sec \alpha + C_{LW} \tan \alpha, \quad (3.20)$$

where C_{DPmin} is here to be estimated for the exposed wing, but based on the gross wing area S_W . A net wing to gross wing area ratio correction, see Sketch 1.2, is applied as shown to the gross wing value of ΔC_{Dvisc} obtained from Equation (3.26).

Methods to estimate C_{NB} for simple body shapes are available in the Aerodynamics Series. The normal force coefficient for forebody–cylinder combinations can be obtained from Item No. 89014 (Derivation 31). The reduction in normal force due to rear-body taper can be estimated for conical boat-tails of up to 2 calibres in length and boat-tail angles up to 10° by the method in Item No. 87033 (Derivation 30), and those results may be used as an approximation for other tapering afterbodies. A program based on Item Nos 89014 and 87033 is available in Item No. 90034 (Derivation 32).

The body normal force coefficient is then

$$C_{NB} = C_{NB\alpha} (S_B / S_W) \sin \alpha \cos \alpha + C_{NBC} (S_p / S_W) + \Delta C_{NB} (S_B / S_W), \quad (3.21)$$

where the first two terms come from Item No. 89014 and the last from Item No. 87033.

For present purposes, for angles of attack up to 30° it is sufficient to take

$$C_{NB\alpha} = 2 + 0.001386 e^{5.376 M} \text{rad}^{-1}, \quad (3.22)$$

and, for $M \geq 0.5$,

$$C_{NBC} = 0.58 \sin^2 \alpha + p [(\alpha - 17.5)/12.5] [0.02 + 0.008265 e^{5.2(M - 0.5)}] \quad (3.23)$$

with C_{NBC} taking the sign of α , and where $p = 1$ if $\alpha > 17.5^\circ$ and $p = 0$ otherwise.

The values of C_{NBC} at $M = 0.5$ hold for $0 \leq M < 0.5$.

The boat-tail contribution is

$$\Delta C_{NB} = -2 \left(1 - \left(\frac{D_b}{D} \right)^2 \right) [1 - \sin^{0.6}(\alpha)] \sin \alpha \cos \alpha - \frac{l_a}{D} \left(1 - \frac{D_b}{D} \right) \sin \alpha |\sin \alpha| \quad (3.24)$$

The body axial force coefficient C_{AB} will typically take the form $C_{AB0} \cos^2 \alpha$, where C_{AB0} is the value at zero incidence. The value of C_{AB0} may be estimated using Item Nos 68020, 76033, 77020 and 78019 (Derivations 14, 23, 25 and 27), for example, or an experimental value may be employed. No allowance is made for the effect of wing-body interference but Reference 45 contains some data on the subject. Any axial force arising on rear-facing body areas at high incidence as a result of suction due to trailing forebody vortices is also neglected.

This page Amendment B

3.1.3 Representation of K_{visc}

For a planar wing with full leading-edge suction the lift-dependent viscous drag coefficient can be represented, to a good approximation, as

$$\Delta C_{Dvisc} = K_{visc} C_L^2, \quad (3.25)$$

where K_{visc} is independent of C_L , see Item No. 66031 (Reference 44). Comparisons with experimental drag polars showed that at high C_L , when leading-edge suction was being lost, the contribution from Equation (3.25) became too large. As a modification to counter this, $\bar{\sigma}^2$ was introduced to provide a necessary and sufficient reduction factor and

$$\Delta C_{Dvisc} = \bar{\sigma}^2 K_{visc} (a_1 \alpha \pi / 180)^2 \quad (3.26)$$

is the representation adopted in the method, the linear representation of C_L being consistent with the empirical values of K_{visc} to be employed.

For wings with sweep $\Lambda_{1/4} \leq 45^\circ$ and thickness to chord ratio $0.1 \leq t/c \leq 0.2$, the method of ESDU 07002 (Reference 48) provides K_{visc} . Otherwise an estimate may be obtained by the method described in Appendix B of Item No. 96025, as illustrated in the worked example in Section 6.*

3.2 Calculation of Leading-edge Suction Factor σ

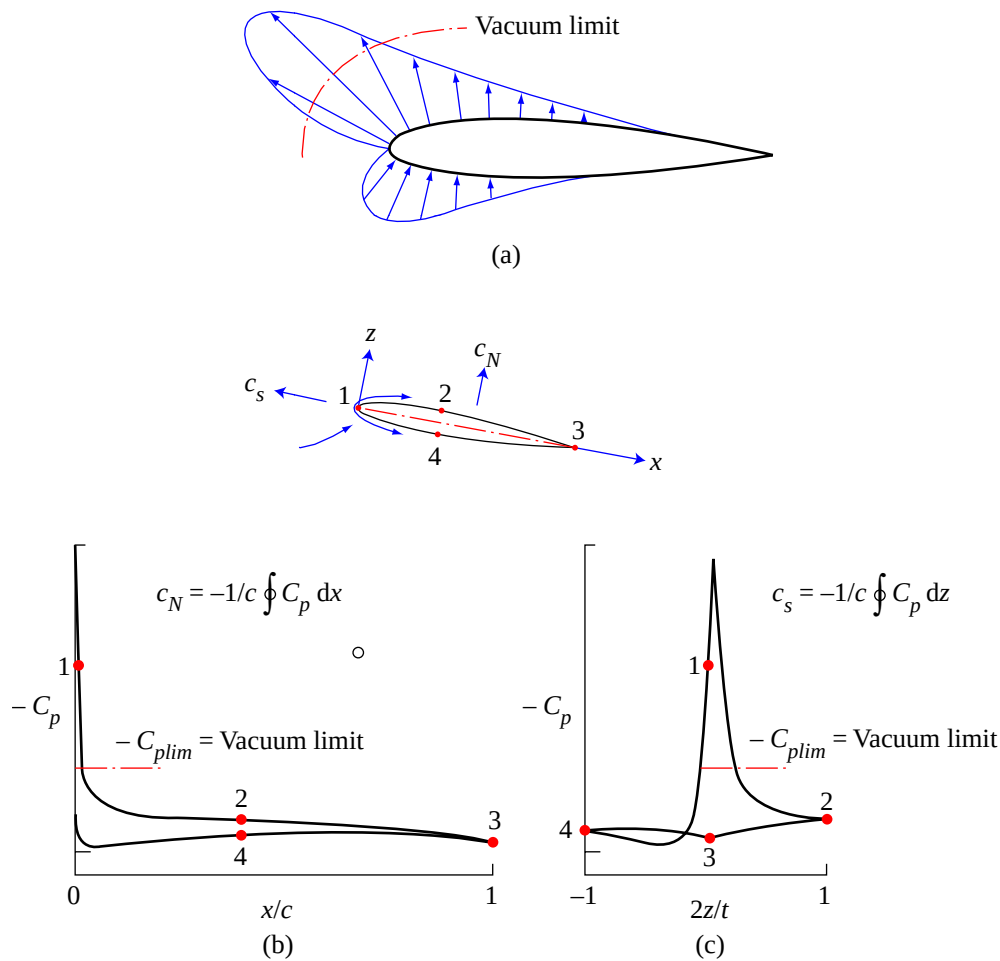
The method of Derivations 28 and 29 for the calculation of σ is described below. Section 3.2.1 covers the original development for aerofoils and Section 3.2.2 the extension to wings. The subsequent improvements in these methods, see Derivation 36, are given in Section 3.2.4.

3.2.1 Derivation of σ for aerofoils

In Derivations 28 and 29, the factor σ was constructed by first running parametrically a theoretical two-dimensional aerofoil program (Reference 45) in order to define the dependence of leading-edge suction on aerofoil geometry and the theoretical lower limit on pressure coefficient. Comparisons of predicted axial force with experimental data were then used to define an empirical lower limit on pressure coefficient in terms of local Mach number and Reynolds number.

Following the arguments and results of Derivations 28 and 29, a typical pressure distribution for a symmetrical aerofoil at a moderate angle of attack is illustrated in Sketch 3.1(a). The pressure distribution can be plotted as a function of chordwise distance and also as a function of distance normal to the chord line, Sketches 3.1(b) and 3.1(c). Clockwise contour integration of the former (with respect to x) gives the section normal force and similar integration of the latter (with respect to z) gives the leading-edge suction force. The salient feature of the theoretical pressure distribution is the large negative pressure coefficient near the aerofoil nose. As indicated in Sketch 3.1, this value can greatly exceed the vacuum limit, $-2/\gamma M^2$. Imposition of that theoretical limit results in a significant decrease in leading-edge suction force with little change in normal force.

* In its earliest form, Item No. 95025 took a constant value of 0.018 for K_{visc} , in the absence of reliable prediction methods.



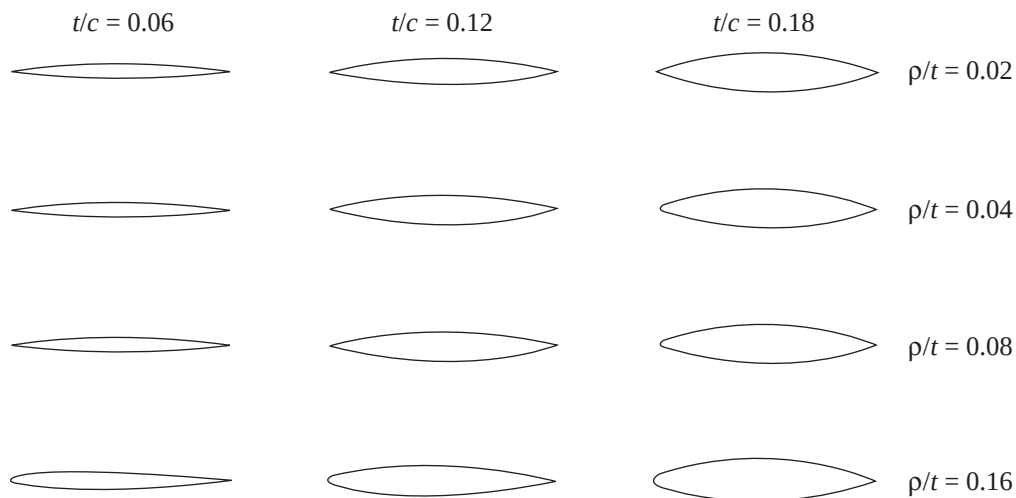
Sketch 3.1 Aerofoil pressures

An initial estimate of the suction factor σ was established as the ratio of integrals of computer-generated pressure distributions of the type illustrated in Sketch 3.1(c), with and without a limiting pressure coefficient set equal to the vacuum limit. Sketch 3.2 illustrates the family of symmetrical aerofoils studied, each having maximum thickness at mid-chord.

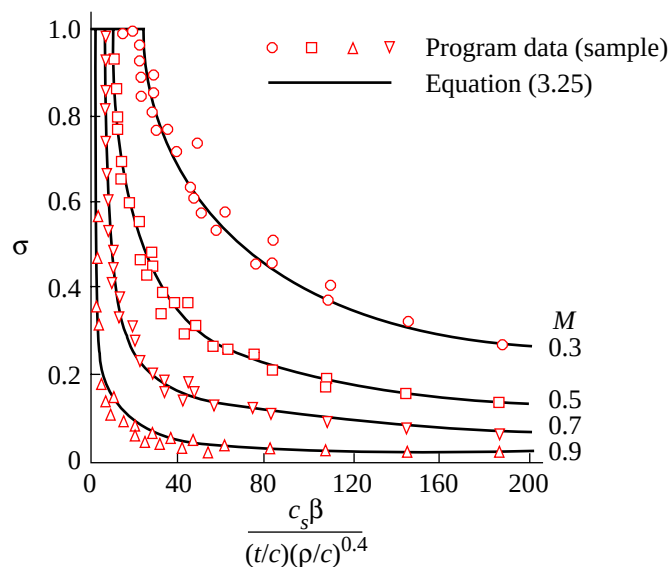
For a given Mach number σ was found to correlate well with the parameter $c_s \beta / [(t/c)(\rho/c)^{0.4}]$, see Sketch 3.3, to which there is a curve fit

$$\sigma = 2 \frac{\beta^2}{M} \left[\frac{(t/c)(\rho/c)^{0.4}}{c_s \beta} \right]^{0.6}, \quad (3.27)$$

but values of σ greater than unity are replaced by $\sigma = 1$.



Sketch 3.2 Aerofoil geometries of parametric study in original method



Sketch 3.3 Correlation of σ as calculated with $C_{plim} = -2/\gamma M^2$

Equation (3.27) applies specifically to data where C_{plim} is equal to the theoretical limit. To enable it to be used for a general value, $0 \leq -C_{plim} \leq 2/\gamma M^2$, an equivalent Mach number M_e is introduced. It is a feature of the theoretically calculated pressure distributions that at any point on the aerofoil the pressure coefficient will vary with Mach number according to the Prandtl-Glauert rule that βC_p is constant. If it is assumed

This page Amendment B

that the limiting pressure coefficient behaves in the same way it is possible to associate an arbitrary C_{plim} at Mach number M with a vacuum limit at Mach number M_e , higher than M . Equation (3.27) can then be used in the form

$$\sigma = 2 \frac{\beta_e^2}{M_e} \left[\frac{(t/c)(\rho/c)^{0.4}}{c_s \beta} \right]^{0.6}, \quad (3.28)$$

where c_s is evaluated at Mach number M ; as before values of σ greater than unity are replaced by $\sigma = 1$.

The equivalent Mach number is obtained from the equation

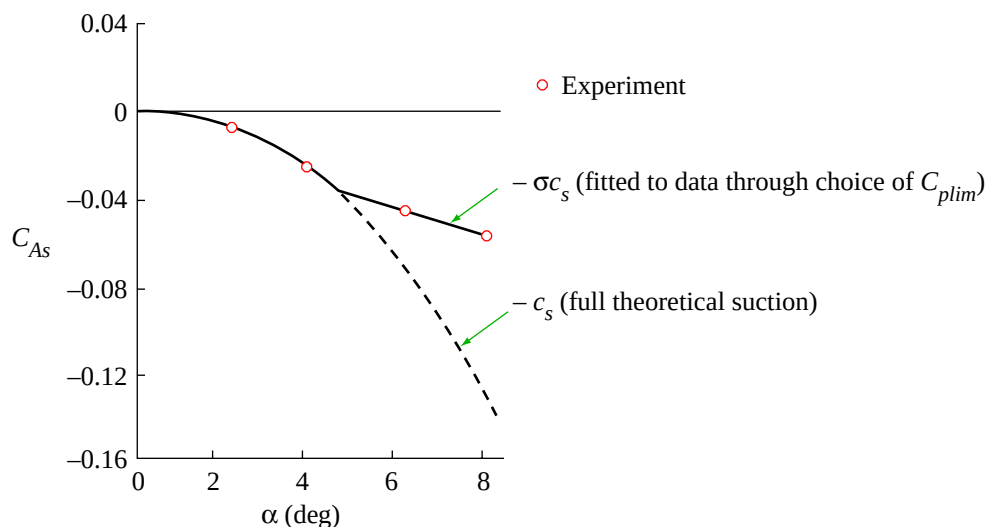
$$-\frac{2}{\gamma M_e^2} = \frac{\beta}{\beta_e} C_{plim}, \quad (3.29)$$

which is solved to give

$$M_e = 2^{1/2} \{ [1 + (\gamma C_{plim} \beta)^2]^{1/2} - 1 \}^{1/2} / (-\gamma C_{plim} \beta), \quad (3.30)$$

where C_{plim} is to be determined empirically.

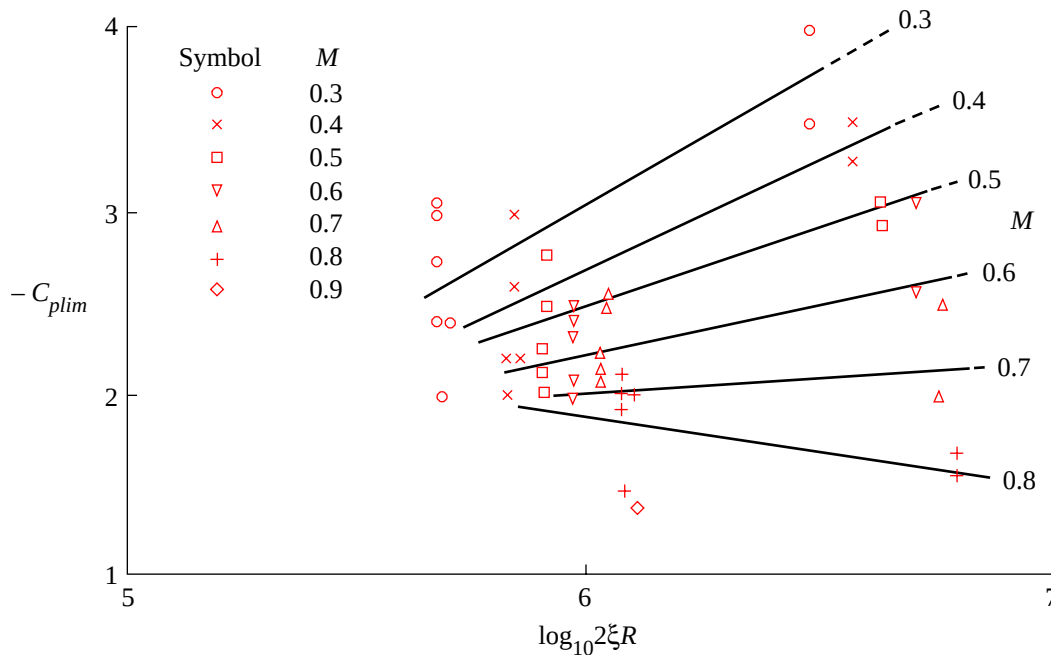
The original theoretical correlations made in Derivations 28 and 29 were confined to a set of aerofoils with maximum thickness at mid-chord in order to limit the number of geometric variables. To apply the method more generally, it was presumed that the leading-edge suction characteristics were determined almost entirely by the nose section and were little affected by geometry changes aft of the maximum thickness. An effective chord for an aerofoil with maximum thickness at ξc is therefore taken as $2\xi c$, and Equation (3.28) is evaluated with c replaced by $2\xi c$ and c_s by $c_s/2\xi$. Empirical values of C_{plim} as a function of Mach number and modified Reynolds number, $2\xi R$, can then be found by comparing the predicted axial force coefficient due to suction $-\sigma c_s$ with experimentally deduced values, see Sketch 3.4.



Sketch 3.4 Axial force due to suction

This page Amendment B

In Derivations 28 and 29 the term $\frac{1}{2}\gamma M^2 C_{plim}$ was correlated in terms of M and $2\xi R$, but it was acknowledged that the Reynolds number range was restricted, $5 \times 10^5 \leq 2\xi R \leq 6 \times 10^6$. Sketch 3.5 shows the original data, with $-C_{plim}$ plotted against $\log_{10} 2\xi R$ for fixed values of M . Extrapolation to higher Reynolds number and lower Mach numbers is discussed in Section 3.2.2. With C_{plim} established, σ can be evaluated for any plain aerofoil, given M , R , t/c , ρ/c , ξ and c_s .

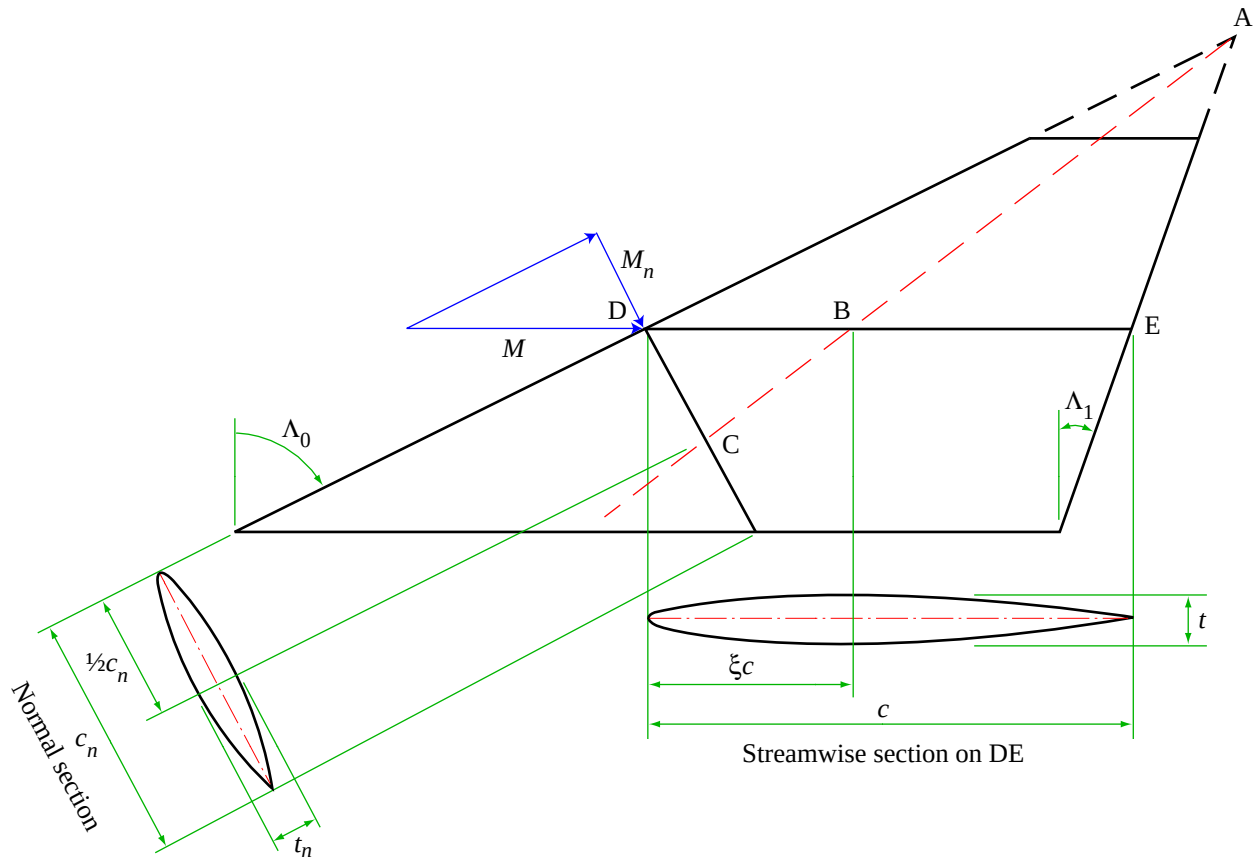


Sketch 3.5 Deduced values of C_{plim}

3.2.2 Adaptation to wings

The method is extended for use at a spanwise station on a wing of any aerofoil section by the use of simple sweep theory and the construction of a fictitious chord normal to the wing leading edge that satisfies the requirement of having maximum thickness at mid-chord. The construction of that chord is peculiar to the method of Derivations 28 and 29 and does not correspond to a true normal chord. For a given planform the length of the normal chord is determined by the position of maximum thickness of the true streamwise wing section. Sketch 3.6 demonstrates how the normal chord is constructed.

The leading and trailing edges at D and E of the streamwise section of interest are extended to a supposed wing tip at A . The line AB is constructed to connect the tip A and the point of maximum thickness B of the streamwise section, and is extended to meet at C the line drawn normal to the wing leading edge from D . The distance DC then defines $c_n/2$, the half-chord length of the normal section. This is equivalent to the assumption made in Section 3.2.1 for aerofoils with maximum thickness not at mid-chord.



Sketch 3.6 Construction for c_n

Sketch 3.6 is purely for illustration. It is not necessary for the user formally to make such constructions as equations developed in Derivations 28 and 29 are given for all necessary quantities.

From the free-stream Mach number, Reynolds number based on $\bar{c}$, and the wing geometry the following parameters are calculated.

The normal-section Mach number is

$$M_n = M \cos \Lambda_0 \quad (3.31)$$

and the Reynolds number is

$$R_n = R(c_n/\bar{c}) \cos \Lambda_0 = R(c_n/c)(c/\bar{c}) \cos \Lambda_0, \quad (3.32)$$

$$\text{with } c/\bar{c} = 2\{1 - \eta(1 - \lambda)\}/(1 + \lambda). \quad (3.33)$$

The ratio of the normal-section chord to the streamwise-section chord is

$$c_n/c = 2\xi\{\sin \Lambda_0[(1 - \xi)\tan \Lambda_0 + \xi \tan \Lambda_1] + \cos \Lambda_0\}^{-1}. \quad (3.34)$$

The thickness to chord ratio of the normal section is

$$t_n/c_n = 0.5(t/c)\xi^{-1}\sec\Lambda_0, \quad (3.35)$$

and the ratio of the leading-edge radius to chord for the normal section is

$$\rho_n/c_n = 0.5(\rho/c)\xi^{-1}\sec^2\Lambda_0. \quad (3.36)$$

The normal-section suction coefficient c_{sn} based on c_n is needed for use with the two-dimensional results of Section 3.2.1. It is related to the theoretical thin-wing value of $c_s c/\bar{c}$ calculated for the true planform by

$$c_{sn} = (c_s c/c_n)\sec\Lambda_0 = C_s(c_s c/C_s \bar{c})(\bar{c}/c)(c/c_n)\sec\Lambda_0 \quad (3.37)$$

where $c_s c/C_s \bar{c}$ is obtainable from Item No. 94037 (or the program in Item No. 94038) and, from the results of linearised theory,

$$\begin{aligned} C_s &= \beta \left(a_1 - \frac{a_1^2(1+\delta)}{\pi A} \right) \frac{\sec\Lambda'_0 \sec\Lambda_0}{[\beta^2 + \tan^2\Lambda_0]^{1/2}} \left(\frac{\pi\alpha}{180} \right)^2 \\ &= \beta A \left(\frac{a_1}{A} \right) \left(1 - \left(\frac{a_1}{A} \right) \frac{(1+\delta)}{\pi} \right) \frac{\sec\Lambda'_0 \sec\Lambda_0}{[\beta^2 + \tan^2\Lambda_0]^{1/2}} \left(\frac{\pi\alpha}{180} \right)^2, \end{aligned} \quad (3.38)$$

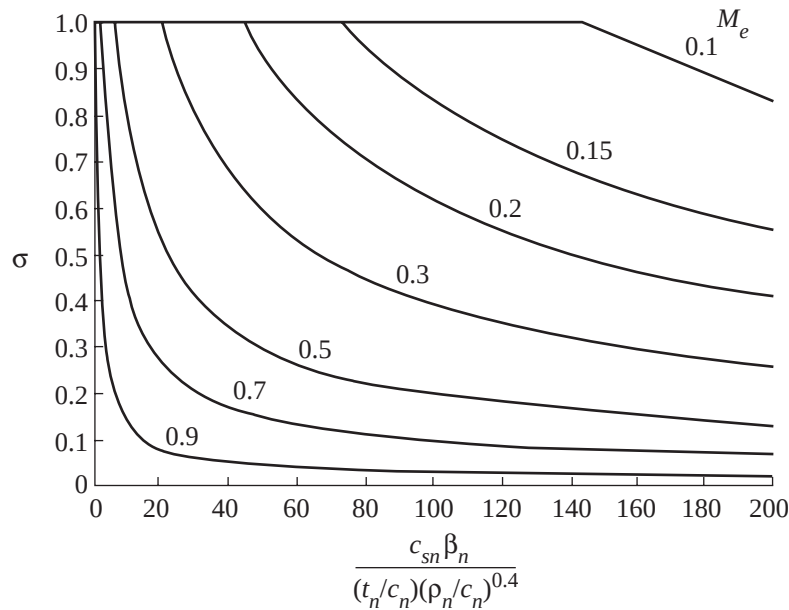
with a_1/A from Item No. 70011 (Derivation 15).

Substitution of normal-section properties for aerofoil properties in Equations (3.28), (3.30) and a revision of Sketch 3.5 gives the data necessary for evaluating the local suction factor σ for the wing.

Equation (3.28) becomes

$$\sigma = \frac{2\beta_e^2}{M_e} \left[\frac{(t_n/c_n)(\rho_n/c_n)^{0.4}}{c_{sn}\beta_n} \right]^{0.6} \quad (3.39)$$

where $\beta_e = (1 - M_e^2)^{1/2}$ and again with values of σ greater than unity replaced by $\sigma = 1$. This is illustrated in Sketch 3.7.



Sketch 3.7 Local leading-edge suction factor

Equation (3.30) becomes

$$M_e = 2^{1/2} \{ [1 + (F\beta_n)^2]^{1/2} - 1 \}^{1/2} / F\beta_n \quad (3.40)$$

where $\beta_n = (1 - M_n^2)^{1/2}$ and, for simplicity, F replaces $-\gamma C_{plim}$.

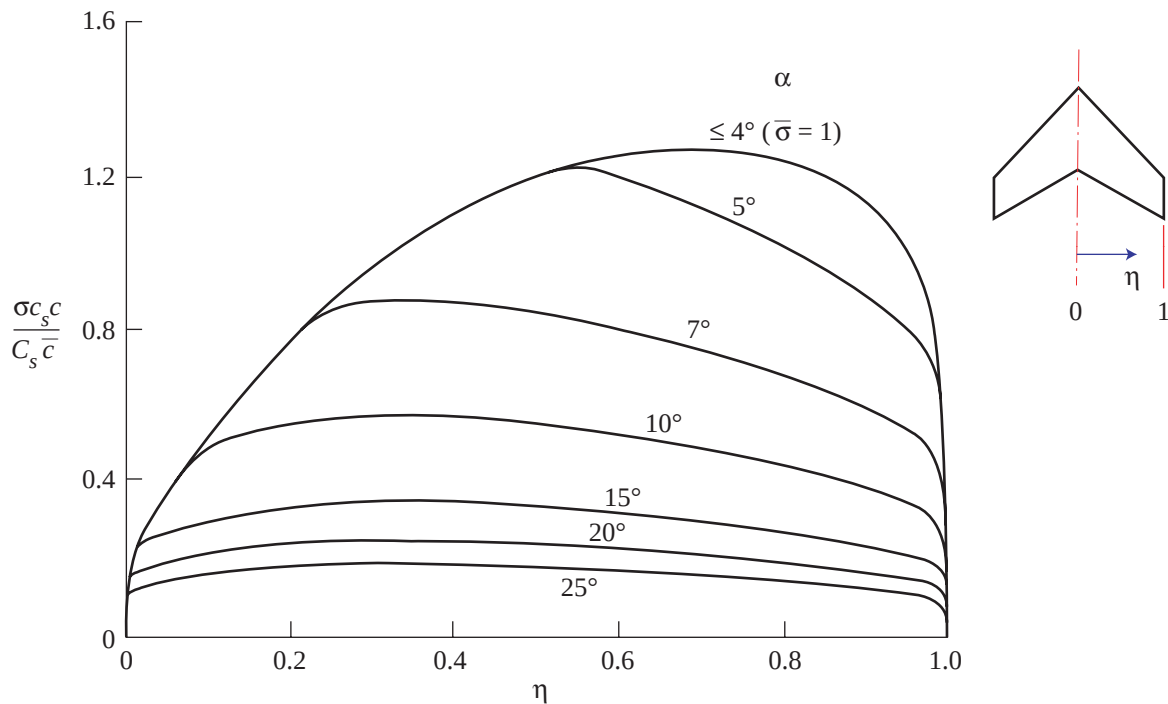
Based on the original aerofoil data shown in Sketch 3.5, which were limited to the ranges $5 \times 10^5 \leq R_n \leq 8 \times 10^6$ and $0.3 \leq M_n \leq 0.9$, Derivation 28 gave the empirical correlation

$$F = \frac{2}{M_n^2} \left[\frac{R_n \times 10^{-6}}{R_n \times 10^{-6} + 10^{(4-3M_n)}} \right]^{0.05 + 0.35(1-M_n)^2} \quad (3.41)$$

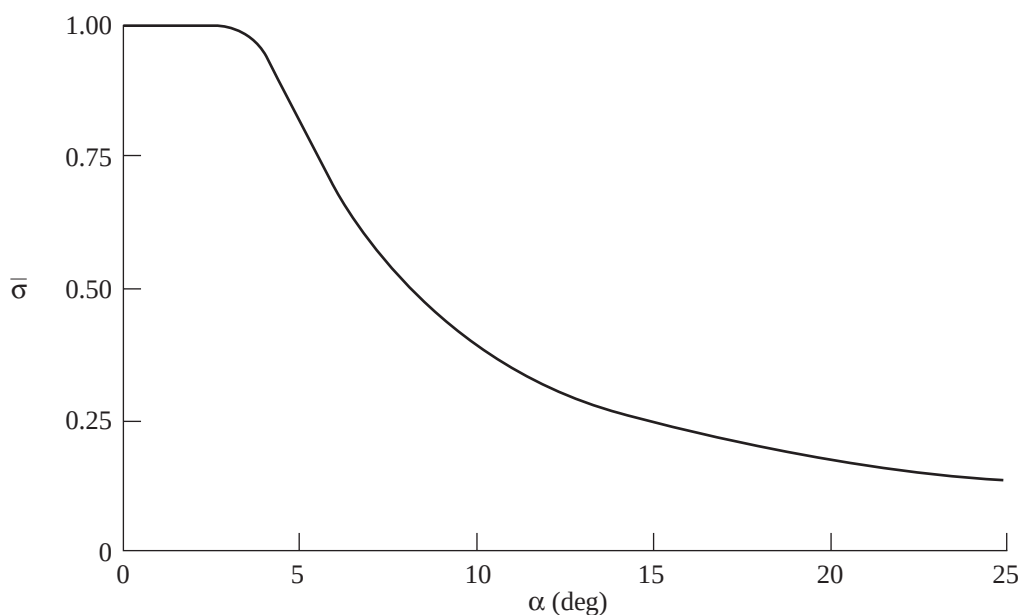
The original issue of Item No. 95025 used a modified form of that correlation, with some tentative extrapolation to higher R_n and to low M_n , with a maximum value of 14 imposed on F to correspond to an approximate practical lower bound of -10 on pressure coefficient. This is, however, now superseded by the improved correlation introduced in Section 3.2.4.

3.2.3 Illustration of leading-edge suction variation

As an illustration of patterns of behaviour, Sketch 3.8 shows the spanwise variation of $\sigma c_s c / C_s \bar{c}$ at various incidences for a straight-tapered swept wing of low aspect ratio with a constant streamwise section. The sketch indicates that the loss of leading-edge suction begins at 4° incidence and spreads progressively inboard from the tip as incidence increases. The integrated value, $\bar{\sigma}$, as a function of α is shown in Sketch 3.9.



Sketch 3.8 Normalised leading-edge suction distributions



Sketch 3.9 Variation of $\bar{\sigma}$ with α

This page Amendment B