

DELAMINATION AND FREE EDGE STRESSES IN COMPOSITE LAMINATES SUBJECTED TO UNIFORM PRESCRIBED AXIAL STRAIN AND TEMPERATURE CHANGE

1. ASSUMPTIONS, SOLUTION OUTLINE AND SOME NUMERICAL RESULTS

1.1 Introduction

This Data Item, together with ESDUpac A9528, provides a Fortran computer program for calculating the stresses in the free edge regions of symmetric laminated plates subjected to a uniform prescribed axial strain and temperature change. It also predicts the onset of delamination.

The Item consists of three parts and an appendix. The first part describes the theoretical solution to the title problem and discusses some numerical results. The second part describes the input and output and gives example calculations. The third part gives notes on the method of solution used, the resulting accuracy and program testing. Appendix A follows Section 3 to describe the application of the principle of minimum complementary energy in the analysis.

The program is available in two formats: (a) a Fortran source code with example input and output files, and (b) an executable program for PCs. Both versions can be found on the Compact Disc and on disk in the Sub-series Software Volume.

Classical laminated plate theory is widely used to calculate the elastic stresses in flat laminated plates built up from thin orthotropic layers of continuous fibre reinforced composite as shown in Sketch 1.1. Data Item No. 94004* can be used to calculate these stresses for any general loading case. However, at free edge regions, classical plate theory does not account for the more general three dimensional nature of the stress field which allows the free edge surface to have zero values of direct stress and shear stresses. Edge stresses arise when the forces in a layer, predominantly in the fibre direction within that layer, have to be transferred to another layer as the fibres approach the free edge (for example in laminates containing angle plies). This implies stresses in the laminate in the through-the-thickness direction which can be large enough to cause local delamination. The magnitude of the edge stresses is increased when, for given laminate thickness, a smaller number of layers is used and each contains a larger number of plies.

The accompanying ESDUpac A9528 provides a Fortran computer program for the approximate calculation of the edge stresses for symmetric laminated plates that have free edges, and that are subjected to a prescribed axial strain and uniform temperature change. Stresses for the applied mechanical strain and for the temperature change are calculated separately, and then combined to give the total stresses arising from both effects. Also a failure criterion is used to predict edge delamination and the interface at which it occurs. The plates are considered to be built up from thin orthotropic homogeneous layers with perfect bonding between them. The layers may have different material elastic properties, thicknesses and orientations of their principal axes of orthotropy but must form a symmetric lay-up about the mid-plane of the plate, that is, the sub-matrix that couples the in-plane and flexural stiffness sub-matrices must be zero. However, the in-plane sub-matrix may be fully populated, and in such cases additional shear strains will be induced. This shear deformation is assumed to take place without any restraint from the surrounding structure.

The method is capable of analysing plates with several hundred different layers without difficulty. This gives it a major advantage over finite element methods which would have difficulty in coping with the large size of the problem.

* Data Item No. 94004 "Stress analysis of laminated flat plates".

1.2 Notation

Both SI and pound force, inch units are quoted but any coherent system of units may be used.

1.2.1 Layer and plate properties and reference axes

A_{11} to A_{33}	values of the in-plane stiffness sub-matrix (see Section 2.4.3)	N/m	lbf/in
B_{2k} , B_{4k}	layer constants in expressions for stresses which ensure stress continuity across layers	m	in
B_{5k}	layer constant in expressions for stresses which ensures stress continuity across layers	m ²	in ²
C	reference number for layer type		
C_{max}	number of layer types		
d	distance from free edge over which stresses are averaged	m	in
E_{α} , E_{β} , E_z	moduli of elasticity of layer in α -, β - and z -directions respectively	N/m ²	lbf/in ²
$e_{\theta x}$, $e_{\theta y}$, $e_{\theta xy}$	free thermal expansions defined by Equations (A7.5) and (A7.6)		
$G_{\alpha\beta}$, $G_{\alpha z}$, $G_{\beta z}$	shear moduli of layer in (α, β) , (α, z) and (β, z) directions respectively	N/m ²	lbf/in ²
h	total plate thickness	m	in
M	total number of layers		
m	layer identification number		
N	number of interfaces for which output values are required (see Table 2.1)		
S_{11} , S_{12} , S_{13} , S_{16}	elements in layer compliance matrix	m ² /N	in ² /lbf
t_{nom}	nominal ply thickness	m	in
t_k	thickness of k th layer	m	in
X	parameter used in layer material check (see Equation (2.1))		
z_k	value of z at mid-plane of k th layer	m	in
z'_k	$z_k + t_k/2$	m	in
$0(\alpha, \beta, z)$	principal axes of orthotropy of a layer (see Sketch 1.1)		

$0(x,y,z)$	plate main reference axes (see Sketch 1.1) with $0x$ measured along free edge		
$\alpha_{1k}, \alpha_{2k}, \dots, \alpha_{5k}$	layer constants in expression for stresses	N/m ²	lbf/in ²
$\mu_\alpha, \mu_\beta, \mu_z$	coefficients of thermal expansion	°C ⁻¹	°C ⁻¹
ψ_k	angle of principal axis of orthotropy, α_k , of layer measured from plate reference axis, $0x$, as shown in Sketch 1.1	degree	degree

1.2.2 Stresses, Strains and Failure Criterion

CI, MI, TI	failure criterion identification letters related to the choice of CL, ML , or TL respectively (see Table 2.1)		
CL	factor by which both mechanical and thermal stresses have to be multiplied to cause failure (see Section 1.5, Equation (1.10))		
ML	factor by which mechanical stresses have to be multiplied in order to cause failure with thermal stresses constant (see Section 1.5, Equation (1.8))		
TL	factor by which thermal stresses have to be multiplied in order to cause failure, with mechanical stresses constant (see Section 1.5, Equation (1.9))		
ΔT	uniform temperature change (final temperature minus initial stress free temperature)	°C	°C
$\bar{\epsilon}_x$	applied uniform axial strain in $0x$ direction		
$\bar{\epsilon}_{xT}$	free thermal strain in $0x$ direction, obtained from laminate theory		
ϕ	exponential factor used in expressions for stresses	m ⁻¹	in ⁻¹
λ	non-dimensional exponential factor used in expressions for stresses		
$\sigma_{xk}, \sigma_{yk}, \sigma_{zk}$	layer direct stresses	N/m ²	lbf/in ²
$\bar{\sigma}_{yk}, \bar{\tau}_{xyk}$	layer direct and shear stresses obtained from classical laminated plate theory	N/m ²	lbf/in ²
σ_{xd}	applied stress to cause edge delamination	N/m ²	lbf/in ²

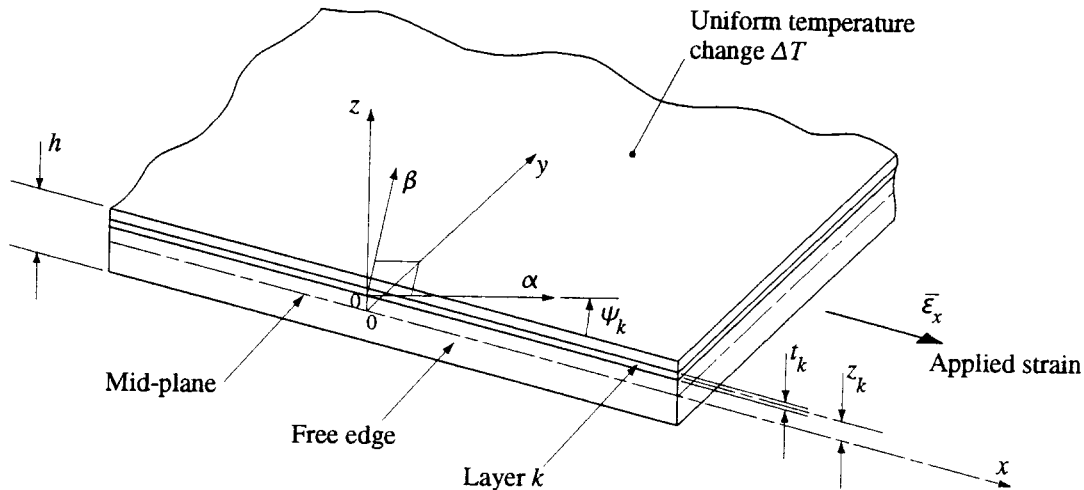
$\tau_{xyk}, \tau_{xzk}, \tau_{yzk}$	layer shear stresses	N/m ²	lbf/in ²
$\nu_{\alpha\beta}$	Poisson's ratio for layer resulting from load applied in α -direction with induced strain in β -direction		
$\nu_{\alpha z}, \nu_{\beta z}$	Poisson's ratios for layer resulting from load applied in α - and β -directions, respectively, with induced strains in z-direction		

1.2.3 Suffixes

A	used to denote average applied interlaminar stresses
F	used to denote interlaminar failure stresses
j, k	layer identification numbers starting with 1 at top (positive z)
n	number of adjacent plies in same direction having same nominal thickness and properties (see Section 1.6.2)
t, c	used to denote tension and compression stresses respectively in Equations (1.8) to (1.10)
T	used to denote thermal stresses, strains and complementary energy
x, y, z	relate to x-, y- and z-directions respectively
α, β	relate to α - and β -directions respectively
θ	used in free thermal expansion terms (see Equations (A7.4) and (A7.5))

1.2.4 Sign Convention

Tensile stresses and strains positive. Compressive stresses and strains negative. Allowable stresses and strains positive (tensile or compressive).



Sketch 1.1 Laminated plate geometry

1.3 Statement of Basic Assumptions and Solution Outline for Axial Strain

The analysis for the free edge stresses is based on linear elastic theory and follows the method presented in Derivations 1 and 2 which give an approximate analytical solution to the problem. The laminated plate shown in Sketch 1.1 is subjected to a constant uniaxial strain $\bar{\epsilon}_x$. Under this condition the stresses will not vary with x and will be a function of y and z only. It is assumed that the dependence of the stress components on y and z can be separated, and because the stresses drop away rapidly from the free edge exponential functions are used to represent them. Expressions for the stress components are given in the main text that follows and further details of the solution are given in Appendix A.

The prediction of ply delamination is based on Derivation 3. The stresses are averaged over a chosen distance from the free edge and then used in a quadratic failure stress criterion to predict when delamination is likely to occur.

1.3.1 Stress distribution for axial strain - general case

From previously published work, an expression is chosen for the 'y' dependence of the σ_{yk} stress in the form

$$\sigma_{yk} = \alpha_{1k} e^{-\phi y} + \alpha_{2k} e^{-\lambda \phi y} + \alpha_{3k} \quad (1.1)$$

whilst the y dependence for the τ_{xyk} shear stress is chosen in the form

$$\tau_{xyk} = \alpha_{4k} + \alpha_{5k} e^{-\phi y} \quad (1.2)$$

In the above expressions $\alpha_{1k} \dots \alpha_{5k}$ are layer constants which are determined from stress boundary conditions, and the exponential factors λ and ϕ are determined from the principle of minimum

complementary energy (see Appendix A). The y dependence for the remaining stress components is determined from the stress equilibrium equations in each layer. The dependency on z is also found from the stress boundary conditions and from the stress equilibrium equations. In this case, the functions are found to be simple polynomial expressions in z , up to the second degree. In this way, it is ensured that the top and bottom surfaces of the plate are stress free, as are the longitudinal surfaces at the free edges. Also, the classical laminated plate stresses are recovered at large distances from the free edges and stress continuity is satisfied at each layer interface. However, interface displacement continuity is not ensured, but this does not appear to affect significantly the stresses obtained for the cases considered. The final expressions for the stresses are as follows.

$$\left. \begin{aligned} \sigma_{yk} &= \bar{\sigma}_{yk} \left[1 + \left(\frac{\lambda}{1-\lambda} \right) e^{-\phi y} + \left(\frac{1}{\lambda-1} \right) e^{-\lambda \phi y} \right], \\ \sigma_{zk} &= \bar{\sigma}_{yk} \left[\left(\frac{\lambda}{1-\lambda} \right) \phi^2 e^{-\phi y} + \left(\frac{1}{\lambda-1} \right) \lambda^2 \phi^2 e^{-\lambda \phi y} \right] \times \left[\frac{z^2}{2} + z B_{4k} + B_{5k} \right], \\ \tau_{xyk} &= \bar{\tau}_{xyk} [1 - e^{-\phi y}], \\ \tau_{xzk} &= -\bar{\tau}_{xyk} \phi e^{-\phi y} [z + B_{2k}], \\ \tau_{yzk} &= \bar{\sigma}_{yk} \left[\left(\frac{\lambda}{1-\lambda} \right) \phi e^{-\phi y} + \left(\frac{1}{\lambda-1} \right) \lambda \phi e^{-\lambda \phi y} \right] \times [z + B_{4k}], \\ \sigma_{xk} &= \frac{1}{S_{11k}} \left[\bar{\epsilon}_x - S_{12k} \sigma_{yk} - S_{13k} \sigma_{zk} - S_{16k} \tau_{xyk} \right] \end{aligned} \right\} \quad (1.3)$$

where

$$\left. \begin{aligned} B_{2k} &= -\frac{1}{\bar{\tau}_{xyk}} \sum_{j=1}^{k-1} (\bar{\tau}_{xyj} t_j) - z'_k, \\ B_{4k} &= -\frac{1}{\bar{\sigma}_{yk}} \sum_{j=1}^{k-1} (\bar{\sigma}_{yj} t_j) - z'_k, \\ B_{5k} &= \frac{1}{\bar{\sigma}_{yk}} \sum_{j=1}^{k-1} (\bar{\sigma}_{yj} t_j z_j) + z_k'^2/2. \end{aligned} \right\} \quad (1.4)$$

and

In Section 1.6, numerical results show that the resulting stress fields agree quite well with results obtained from a finite element method. However, in some cases, other theories have predicted stress singularities at the free edge, and these are not captured by the present method, nor by the finite element method. Such singularities cannot occur in real laminates owing to the redistribution of the stresses local to the edge. Furthermore, they are often so local to the free edge region that they take place over a distance of a few fibre diameters and this calls into question the validity of the basic assumption that the material can be treated as a homogeneous material in this region. From the point of view of predicting failure, exact values of the stresses at the free edge are not too important since there is evidence to show that delamination is controlled by the stresses averaged over some distance from the edge (see Derivation 3).

1.3.2 Stress distribution for axial strain - special cases

In certain types of laminated plates, such as the $[0/90]_s$ cross-ply and $[+\psi/-\psi]_s$ angle-ply lay-ups, it is found that some of the stress components are everywhere zero. In these cases, the general formulation given in Section 1.3.1 breaks down, and simpler expressions have been obtained for the stress components. The program automatically chooses the appropriate formulation.

1.3.2.1 $[0/90]_s$ Cross-ply plates

Here, the two shear stress components, τ_{xyk} and τ_{xzk} are everywhere zero, and the remaining stresses are given by

$$\left. \begin{aligned} \sigma_{yk} &= \bar{\sigma}_{yk} [1 - e^{-\phi y} - \phi y e^{-\phi y}], \\ \sigma_{zk} &= \bar{\sigma}_{yk} \phi^2 e^{-\phi y} [1 - \phi y] \left[\frac{z^2}{2} + z B_{4k} + B_{5k} \right], \\ \tau_{yzk} &= -\bar{\sigma}_{yk} \phi^2 y e^{-\phi y} [z + B_{4k}] \end{aligned} \right\} \quad (1.5)$$

and

$$\sigma_{xk} = \frac{1}{S_{11k}} \left[\bar{\epsilon}_x - S_{12k} \sigma_{yk} - S_{13k} \sigma_{zk} \right]$$

where B_{4k} and B_{5k} are given by Equations (1.4).

1.3.2.2 $[+\psi/-\psi]_s$ Angle-ply plates

Here, the two direct stresses σ_{yk} and σ_{zk} as well as the shear stress τ_{yzk} are everywhere zero, and the remaining stresses are given by

$$\left. \begin{aligned} \tau_{xyk} &= \bar{\tau}_{xyk} [1 - e^{-\phi y}], \\ \tau_{xzk} &= -\bar{\tau}_{xyk} \phi e^{-\phi y} [z + B_{2k}] \end{aligned} \right\} \quad (1.6)$$

and

$$\sigma_{xk} = \frac{1}{S_{11k}} \left[\bar{\epsilon}_x - S_{16k} \tau_{xyk} \right]$$

where B_{2k} is given by Equations (1.4).

1.4 Solution Outline for Uniform Temperature Change

The analysis is similar to that given for the axial strain case and follows the method presented in Derivation 4. The laminated plate shown in Sketch 1.1 is subjected to a uniform temperature change ΔT from an initial stress free condition, and is allowed to expand or contract freely. Thus there are no restraining forces at the plate boundaries. As in the case for axial strain, the edge stresses are represented by exponential expressions given by Equations (1.1) and (1.2), which are functions of y and z only.

Equations for the thermal stresses are discussed in the main text that follows, and further details of the solution are given in Appendix A.

1.4.1 Thermal stress distribution

The final equations for the thermal edge stresses are identical to those given above for the case of axial strain, except that in the three expressions for σ_{xk} (see Equations (1.3), (1.5) and (1.6)), the constant strain term ($\bar{\epsilon}_x$) is replaced by the term $(\bar{\epsilon}_x - e_{\theta xk})$. Also the bar stresses $\bar{\sigma}_{yk}$ and $\bar{\tau}_{xyk}$ are thermal stresses obtained from classical laminated plate theory when the plate is subjected to the temperature change ΔT . The exponential factors λ and ϕ are determined from the principle of minimum complementary energy, modified for the case of thermal loading (see Derivations 4 and 6, and Appendix A). It should be noted that in Derivation 4 it is pointed out that there are situations where there is no thermal solution for the exponential parameters ϕ and λ . This occurs when the edge thermal stresses are highly concentrated close to the free edge, and it then becomes impossible to represent the stress distribution by the relatively simple expressions given by Equations (1.3). Very few cases have arisen where this has happened, but when it does, the analysis is terminated with a warning message (see Section 2.4.3).

1.5 Delamination

The onset of delamination at the free edge is predicted approximately using a quadratic average stress criterion (see Derivations 3 and 5). Delamination initiation does not usually coincide with final failure and delamination often undergoes steady growth until eventually it interacts with in-plane failure modes and final fracture occurs. Delamination will be governed by the three through-the-thickness stresses σ_{zk} , τ_{xzk} and τ_{yzk} at an interface. Average values for these stresses over a chosen distance d from the free edge are defined by

$$(\sigma_{zkA}, \tau_{xzkA}, \tau_{yzkA}) = \frac{1}{d} \int_0^d (\sigma_{zk}, \tau_{xzk}, \tau_{yzk}) dy . \quad (1.7)$$

In Derivation 3, a value for d in the above expression was determined by choosing a value which best fitted the experimental failure load data. In fact, the value was fairly close to the nominal ply thickness t_{nom} , and this is supported by the work of Kim and Soni in Reference 7. For an initial estimate d may be taken as t_{nom} .

It will be found that the value of d used will have a significant effect on the prediction of ply delamination. It should therefore be chosen with care and, where possible, coupon tests should be carried out to check the theoretical predictions. In the absence of relevant experimental data, the value for d can be varied to assess the sensitivity of the solution to this parameter.

Having calculated the average stresses, the failure criterion is then used in one of the three following forms (as chosen in the program input, see Section 2.3, Table 2.1).

In the following criteria ML , TL and CL are the factors by which the appropriate stresses have to be multiplied in order to cause delamination at an interface. In the third term of each of the following three criteria the choice of the appropriate allowable stress, tensile or compressive (the denominator), is determined by the sign of the sum of the applied stresses (the numerator).

- (i) Temperature change fixed, mechanical strain is varied to obtain the value to cause delamination.

$$\left(\frac{(ML)\tau_{xzkA} + \tau_{xzkTA}}{\tau_{xzkF}} \right)^2 + \left(\frac{(ML)\tau_{yzkA} + \tau_{yzkTA}}{\tau_{yzkF}} \right)^2 + \left(\frac{(ML)\sigma_{zkA} + \sigma_{zkTA}}{\sigma_{zkFt} \text{ or } \sigma_{zkFc}} \right)^2 = 1. \quad (1.8)$$

- (ii) Mechanical strain fixed, temperature change is varied to obtain the value to cause delamination.

$$\left(\frac{\tau_{xzkA} + (TL)\tau_{xzkTA}}{\tau_{xzkF}} \right)^2 + \left(\frac{\tau_{yzkA} + (TL)\tau_{yzkTA}}{\tau_{yzkF}} \right)^2 + \left(\frac{\sigma_{zkA} + (TL)\sigma_{zkTA}}{\sigma_{zkFt} \text{ or } \sigma_{zkFc}} \right)^2 = 1. \quad (1.9)$$

Because of the quadratic failure interaction there are two solutions to the above equations and both are calculated by the program in order to find a safe range. However, because of the way that the mechanical and thermal stresses combine in certain configurations, there may not be a safe range for the laminate. One example of this is when the temperature change alone causes delamination at one interface but the mechanical loading required to prevent thermal delamination may in itself cause delamination at another interface. If such a situation arises the program outputs the stresses for the nominal condition and a warning that no safe range for the variable exists.

- (iii) Both temperature change and mechanical strain are varied by the same factor to obtain the condition to cause delamination.

$$\left(\frac{CL(\tau_{xzkA} + \tau_{xzkTA})}{\tau_{xzkF}} \right)^2 + \left(\frac{CL(\tau_{yzkA} + \tau_{yzkTA})}{\tau_{yzkF}} \right)^2 + \left(\frac{CL(\sigma_{zkA} + \sigma_{zkTA})}{\sigma_{zkFt} \text{ or } \sigma_{zkFc}} \right)^2 = 1. \quad (1.10)$$

In this case also two solutions exist. Solutions significantly removed from the applied case should be treated with caution since other mechanical/thermal cases might be more relevant.

In using the above criteria it is assumed that superposition of the mechanical and thermal cases is allowed.

1.6 Some Numerical Results

1.6.1 Stresses

In Reference 8, Herakovich has produced a comprehensive set of results for the free edge stresses in cross-ply, angle-ply and quasi-isotropic laminated plates using a four-node isoparametric finite element representation. These cases provide a good bench mark for the method provided in this Data Item and correspond with the following mechanical properties for each layer.

$$\begin{aligned} E_{\alpha} &= 132 \text{ GN/m}^2, & E_{\beta} &= E_z = 10.8 \text{ GN/m}^2, \\ G_{\beta z} &= 3.38 \text{ GN/m}^2, & G_{\alpha z} &= G_{\alpha\beta} = 5.65 \text{ GN/m}^2, \\ \nu_{\beta z} &= 0.49, & \nu_{\alpha z} &= \nu_{\alpha\beta} = 0.24. \end{aligned}$$

A uniform tensile strain of 0.1 per cent is applied, with no temperature change.

In Sketches 1.2 and 1.3 the values for the two shear stresses τ_{xz} and τ_{yz} , obtained from Reference 8, have been multiplied by -1 in order to correspond with the sign convention adopted in the present analysis.

1.6.1.1 $[0/90]_s$ Cross-ply plates

Sketches 1.2(a) and (b) give some results for the normal stresses, σ_z , along the mid-plane of a $[0_2/90_2]_s$ and a $[90_2/0_2]_s$ laminate respectively. The stress shapes are very similar, but in the first case the present analysis predicts a higher stress at the free edge. The change in sign that is experienced when the order of

the lay-up is changed is an important design consideration, since the high tensile stresses at the free edge will be much more likely to cause delamination than the compressive stresses.

Sketches 1.2(c) and (d) show the distribution of σ_z and τ_{yz} along the $90^\circ/0^\circ$ interface for the two cases considered above. There is seen to be reasonable agreement with the shear stress component in Sketch 1.2(d), but the σ_z distribution from the present analysis does not show the complex behaviour predicted by the finite element method close to the free edge singularity. In the present analysis, the exponential form for σ_z is chosen deliberately to allow it to pass through zero only once close to the free edge and, from considerations of simple force equilibrium, is considered to be a realistic distribution. Once the shape has been established, it will apply at each interface in order to maintain stress continuity. This does not appear to have been achieved with the finite element method in this case, as can be seen by comparing the σ_z stress distributions in Sketches 1.2(b) and (d).

1.6.1.2 $[+\psi/-\psi]_s$ Angle-ply plates

Sketch 1.3(a) shows the σ_z and τ_{xz} stress distributions for a $[15/-15/15/-15]_s$ lay-up at the top interface. For this case the σ_z stresses, obtained from the finite element analysis, are very small compared with τ_{xz} and, in fact, are everywhere zero in the present analysis. The shear stresses τ_{xz} agree very well, showing high values at the free edge.

1.6.1.3 Quasi-isotropic plates

Sketch 1.3(b) shows the stresses for a $[90/45/0/-45]_s$ plate at the $45^\circ/0^\circ$ interface. Quite good agreement is obtained for the σ_z and τ_{yz} stresses but the τ_{xz} shear stresses are a little more difficult to follow. Sketches 1.3(c) and (d) show good correlation for the normal stresses σ_z along the mid-planes of a $[0/90/45/-45]_s$ plate and a $[45/-45/0/90]_s$ plate respectively. They thus show the effect of interchanging the position of the $0^\circ/90^\circ$ and $\pm 45^\circ$ combinations. The sign of the normal stress reverses with this change in stacking sequence which again points to a higher likelihood of ply delamination in the second case.

1.6.2 Delamination

Some experimental results for the onset of delamination are given in Derivation 3, and these provide a useful comparison with the theoretical values given by the present program. The following properties were used in the calculations, and correspond with those given in Derivation 1.

$$E_\alpha = 138 \text{ GN/m}^2, E_\beta = E_z = 14.5 \text{ GN/m}^2,$$

$$G_{\alpha\beta} = G_{\beta z} = G_{\alpha z} = 5.9 \text{ GN/m}^2,$$

$$\nu_{\alpha\beta} = \nu_{\beta z} = \nu_{\alpha z} = 0.21$$

and $t_{nom} = 0.134 \text{ mm}.$

The values for all the interface failure stresses were taken to be

$$\sigma_{zFt} = 53.9 \text{ MN/m}^2, \sigma_{zFc} = 107.8 \text{ MN/m}^2$$

and $\tau_{xzF} = \tau_{yzF} = 105 \text{ MN/m}^2.$

Table 1.1 gives values for the applied axial stress, σ_{xd} , necessary to cause delamination for various lay-up arrangements based on the 0° and $\pm 15^\circ$ ply angles. In this table, the suffix n is used to denote the number

of plies of nominal thickness (in this case 0.134 mm) that are used to make up the total layer thickness for a particular angle. Thus 0_2 indicates a single 0° layer of thickness 0.268 mm (2×0.134 mm), and $\pm 15_3$ indicates one $+15^\circ$ layer and one -15° layer each of thickness 0.402 mm (3×0.134 mm). Results are given for three different averaging distances, namely, $d = t_{nom}/2$ ($= 0.067$ mm), $d = t_{nom}$ ($= 0.134$ mm) and $d = 2t_{nom}$ ($= 0.268$ mm). In each case, this distance is maintained at a constant value as n is increased. The table shows that in each of the three cases considered, the delamination stress, σ_{xd} , decreases with increasing values for n . The maximum, mean and minimum experimental values are shown as there is some scatter in the experimental results. This scatter is not unexpected with fracture tests of this type, and in one extreme case ranges from -34 per cent to $+41$ per cent either side of the mean value. Theoretical results are shown for values of d (the distance over which the edge stresses are averaged) equal to $t_{nom}/2$, t_{nom} and $2t_{nom}$. As d is increased, so the average interlaminar stress values are decreased, and this has the effect of raising the applied delamination stress, σ_{xd} . This theoretical trend is shown very clearly in the table. Alternatively, for a constant value of d , the theory calculates increasing average edge stresses as n is increased, and this has the effect of decreasing the applied stress for edge delamination. The best correlation with experiment is obtained when $d = t_{nom}$ where agreement is within -25 per cent to 17 per cent.

1.6.3 Thermal effects

The various cases for delamination, given in Table 1.1, were evaluated for a temperature change of -157°C . This value was obtained from the difference between the manufacturer's cure temperature for carbon/epoxy of 177°C and a room temperature of 20°C . Values for the three thermal expansion coefficients were taken to be

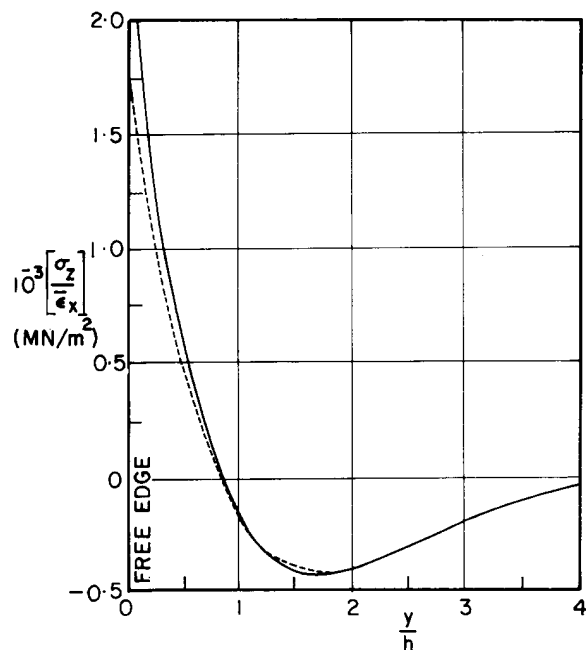
$$\mu_\alpha = 0, \mu_\beta = \mu_z = 33 \times 10^{-6}/^\circ\text{C}.$$

The numbers given in brackets () in Table 1.1 correspond with the thermal case. It can be seen that in all cases the effect of the thermal stresses is to give an increase in the applied axial stress to cause delamination. This is because of the counteractive effect of the thermal stresses on the mechanical stresses for these kinds of laminates. As in the mechanical case, in accordance with the experimental results, the delamination stress, σ_{xd} (with thermal stresses present), decreases with increasing values of n for all three lay-up arrangements. For the thermal case, the best correlation with experiment is obtained when $d = t_{nom}/2$, where agreement is within -30 per cent to $+39$ per cent either side of the mean value. This spread is worse than that obtained for the mechanical case, but a mean deviation from experiment of only $+4.2$ per cent is obtained for this particular set of results, compared with a value of -8.0 per cent for the mechanical case when $d = t_{nom}$.

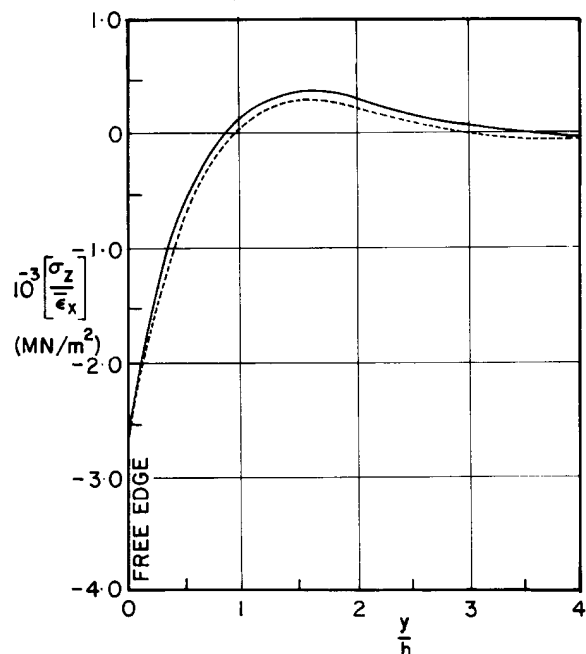
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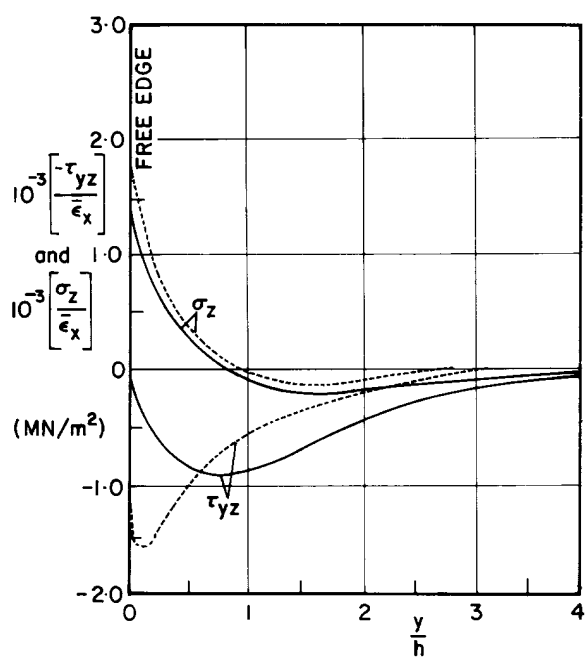
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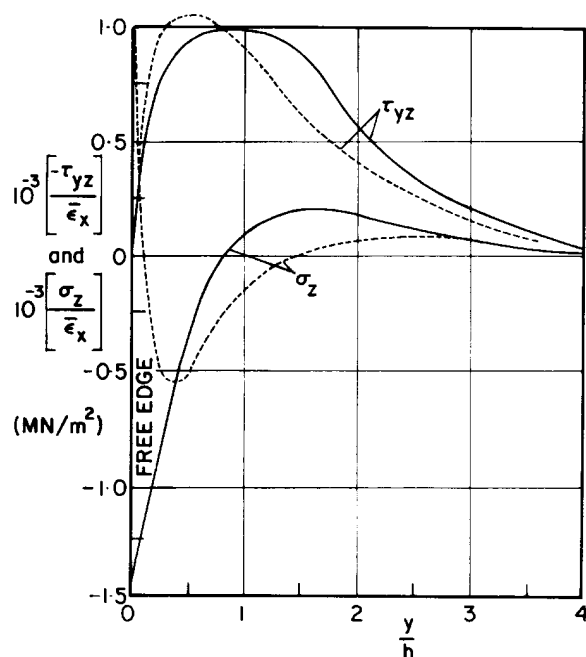
Sketch 1.2(a) $(0_2^0/90_2^0)_s$ σ_z on mid-plane



Sketch 1.2(b) $(90_2^0/0_2^0)_s$ σ_z on mid-plane



Sketch 1.2(c) $(0_2^0/90_2^0)_s$ stresses at the 90^0-0^0 interface

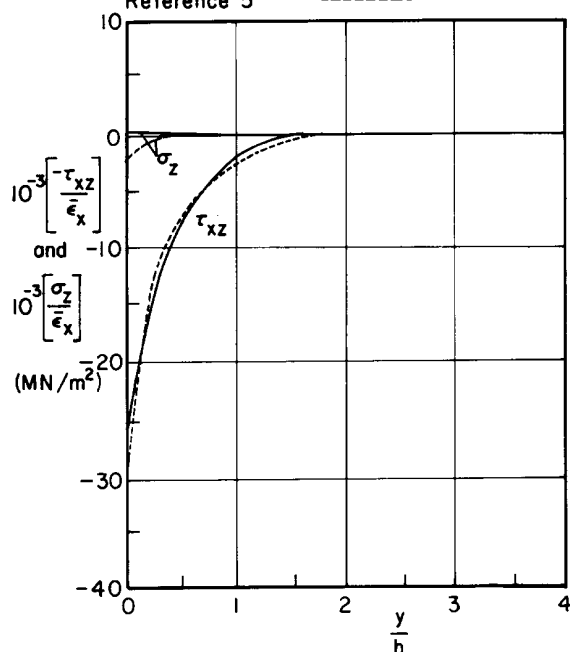


Sketch 1.2(d) $(90_2^0/0_2^0)_s$ stresses at the 90^0-0^0 interface

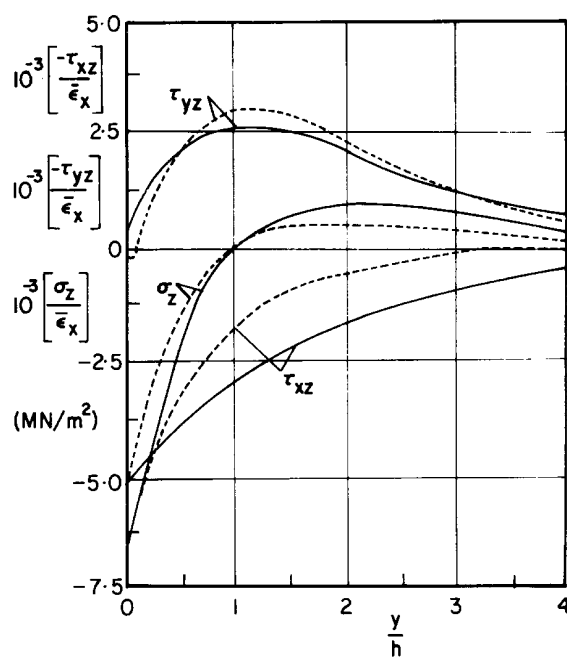
Sketch 1.2 Edge stresses in cross-ply plates

LEGEND

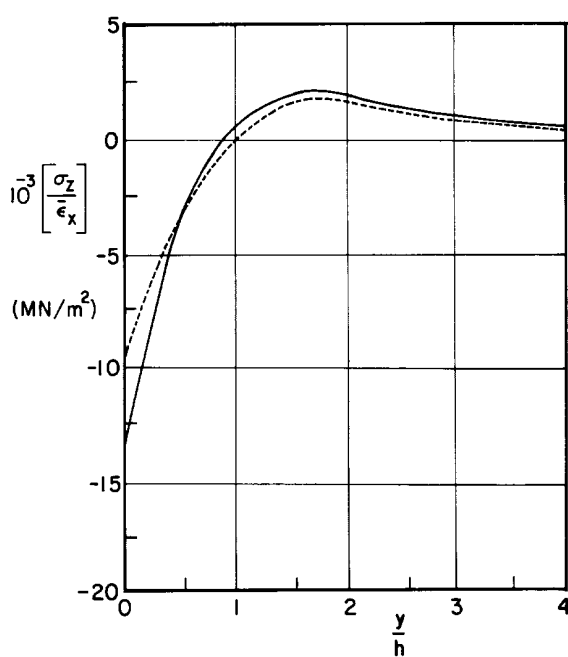
Present analysis —
Reference 5 - - -



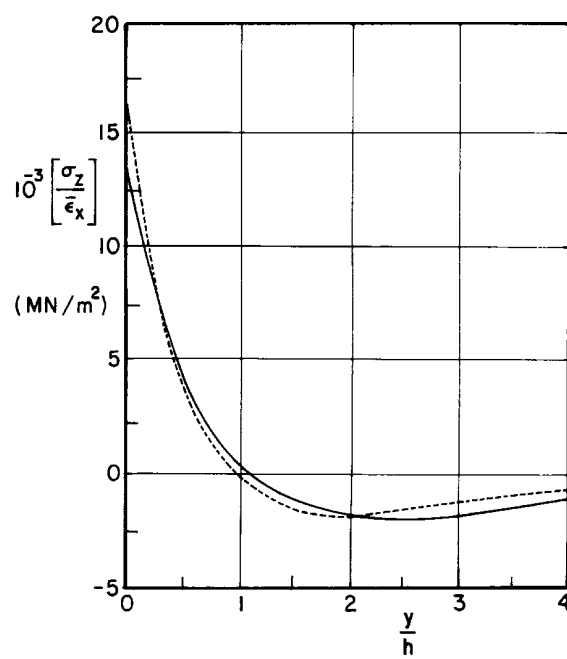
Sketch 1.3(a) $(15^\circ/-15^\circ/15^\circ/-15^\circ)_s$ stress at 15° to -15° interface



Sketch 1.3(b) $(90^\circ/45^\circ/0^\circ/-45^\circ)_s$ stresses at 45° to 0° interface



Sketch 1.3(c) $(0^\circ/90^\circ/45^\circ/-45^\circ)_s$ stress at mid-plane



Sketch 1.3(d) $(45^\circ/-45^\circ/0^\circ/90^\circ)_s$ stress at mid-plane

Sketch 1.3 Edge stresses in angle-ply plates



TABLE 1.1 COMPARISON OF EXPERIMENTAL AND THEORETICAL DELAMINATION STRESSES FOR VARIOUS LAY-UP TYPES.
(UNITS FOR STRESSES MN/m²)

Lay up arrangement [*]	σ_{xd} <i>Experimental</i>			σ_{xd} Theory											
				$d = t_{nom}/2$ (= 0.067 mm)				$d = t_{nom}$ (= 0.134 mm)				$d = 2t_{nom}$ (= 0.268 mm)			
	Min.	Mean [†]	Max.	σ_{xd}		$\frac{\text{Theory}}{\text{Expt}}^{\ddagger}$		σ_{xd}		$\frac{\text{Theory}}{\text{Expt}}^{\ddagger}$		σ_{xd}		$\frac{\text{Theory}}{\text{Expt}}^{\ddagger}$	
$[\pm 15_n/0_n]_s$															
$n = 1$	670	740	860	540	(650)	0.73	(0.88)	690	(790)	0.93	(1.07)	1040	(1140)	1.41	(1.54)
2	580	620	660	470	(590)	0.76	(0.95)	540	(650)	0.87	(1.05)	690	(790)	1.11	(1.27)
3	510	540	570	450	(570)	0.83	(1.06)	490	(610)	0.91	(1.13)	580	(700)	1.14	(1.29)
4	450	480	500	440	(560)	0.92	(1.17)	470	(590)	0.98	(1.23)	540	(650)	1.13	(1.35)
5	–	440	–	430	(560)	0.98	(1.27)	460	(580)	1.05	(1.32)	510	(630)	1.16	(1.43)
$[0_n/\pm 15_n]_s$															
$n = 1$	730	760	780	540	(650)	0.71	(0.86)	690	(790)	0.91	(1.04)	1040	(1140)	1.37	(1.50)
2	580	650	670	470	(590)	0.72	(0.91)	540	(650)	0.83	(1.00)	690	(790)	1.06	(1.22)
3	410	480	540	450	(570)	0.94	(1.19)	490	(610)	1.02	(1.27)	580	(700)	1.21	(1.45)
4	370	570	620	440	(560)	0.77	(0.98)	470	(590)	0.82	(1.04)	540	(650)	0.95	(1.14)
5	420	540	580	430	(560)	0.80	(1.04)	460	(580)	0.85	(1.07)	510	(630)	0.94	(1.17)
$[\pm 15_n]_s$															
$n = 1$	750	860	910	500	(610)	0.58	(0.71)	640	(740)	0.74	(0.86)	980	(1060)	1.14	(1.23)
2	580	610	650	440	(550)	0.72	(0.90)	500	(610)	0.82	(1.00)	630	(740)	1.03	(1.21)
3	430	510	590	420	(530)	0.82	(1.04)	460	(570)	0.90	(1.12)	550	(650)	1.08	(1.27)
4	240	380	530	410	(530)	1.08	(1.39)	440	(550)	1.16	(1.45)	500	(610)	1.32	(1.61)
5	360	400	450	410	(520)	1.03	(1.30)	430	(540)	1.08	(1.35)	480	(580)	1.20	(1.46)

^{*} For description of notation see Section 1.6.2.

[†] Numbers of specimens used for each test value are not available. However, the number of specimens used for all tests totalled 67 suggesting that the mean values are calculated from 4 or 5 tests.

[‡] Mean experimental value used.

() Figures in brackets are obtained with thermal effects included.