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Drag due to lift for non-planar swept wings up to high angles of attack at subsonic speeds

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DRAG DUE TO LIFT FOR NON-PLANAR WINGS UP TO HIGH ANGLES OF ATTACK AT SUBSONIC SPEEDS

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1. NOTATION AND UNITS

		SI	British
A	aspect ratio of gross wing		
A_N	aspect ratio of net wing		
a_1	theoretical gross wing lift-curve slope, based on S_W	rad^{-1}	rad^{-1}
$a_{1\text{eff}}$	effective wing lift-curve slope, based on S_W	rad^{-1}	rad^{-1}
a_{1N}	theoretical lift-curve slope of net wing, based on S_N	rad^{-1}	rad^{-1}
C_{AB}	body axial force coefficient, based on S_W		
C_{AB0}	body axial force coefficient when body longitudinal axis is at zero angle of attack, based on S_W		
C_{AWv}	wing viscous axial force coefficient, based on S_W		
C_D	drag coefficient, based on S_W		
C_{Daug}	contribution to C_D from lift augmentation		
C_{DB}	body drag coefficient, based on S_W		
C_{DC}	contribution to C_D from pressure distribution over cambered wing		
C_{Dlev}	contribution to C_D from leading-edge vortex		
C_{DPmin}	minimum profile drag coefficient		
C_{Dsev}	contribution to C_D from side-edge vortex		
ΔC_{Dtv}	minimum value of C_D for trailing-vortex drag polar, see Sections 3.3.1 and 7.3.9		
ΔC_{Dvisc}	contribution to C_D from lift-dependent viscous drag		
C_{DW}	wing drag coefficient, based on S_W		
C_L	lift coefficient, based on S_W		
C_{Laug}	contribution to C_L from lift augmentation		
C_{LB}	body lift coefficient, based on S_W		

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C_{LC}	contribution to C_L due to camber neglecting viscous and suction forces		
C_{Llev}	contribution to C_L due to leading-edge vortices		
C_{LT}	contribution to C_L due to twist neglecting viscous and suction forces		
ΔC_{Ltv}	value of C_L for minimum value of C_D for trailing-vortex drag polar, see Sections 3.3.1 and 7.3.9		
C_{LW}	wing lift coefficient, based on S_W		
C_{Lmin}	wing lift coefficient for minimum viscous drag coefficient, see Section 3.3 and Appendix B		
C_{NB}	normal force coefficient of body, based on S_W		
ΔC_{NB}	contribution to C_{NB} to allow for boat-tail, based on S_B		
C_{NBC}	cross-flow normal force coefficient of forebody-cylinder, based on S_p		
$C_{NB\alpha}$	normal-force-curve slope of forebody-cylinder, based on S_B	rad^{-1}	rad^{-1}
C_{plim}	limiting pressure coefficient used in analysis of σ as described in Section 3.2 of Item No. 95025 (Derivation 10)		
ΔC_p	local static pressure coefficient increment across thin cambered surface		
ΔC_{p1}	part of ΔC_p due to flat plate angle of attack loading		
$(\Delta C_{p1})_{VL}$	value of ΔC_{p1} over vortex lattice cell		
ΔC_{p2}	part of ΔC_p due to camber only		
$(\Delta C_{p2})_{VL}$	value of ΔC_{p2} over vortex lattice cell		
C_s	theoretical leading-edge suction coefficient for thin planar wing acting normal to leading edge in plane of wing, based on S_W		
C_{st}	theoretical side-edge suction force coefficient for thin planar wing acting normal to tip chord in plane of wing, based on S_W		
c	chord	m	ft
c_{aC}	local axial force coefficient due to pressure distribution on cambered section		
c_{alev}	local axial force coefficient due to leading-edge vortex, based on c		
c_l	theoretical local lift coefficient, based on c		
c_{nlev}	local normal force coefficient due to leading-edge vortex, based on c		

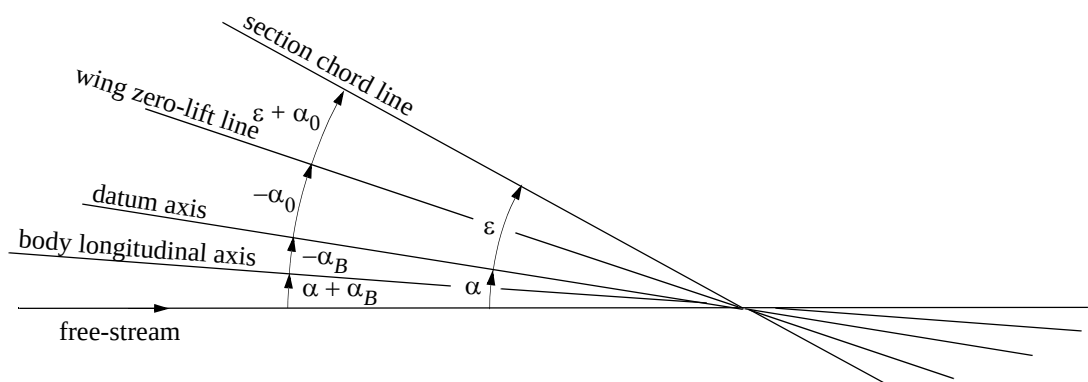
c_s	local theoretical thin-wing suction force coefficient normal to leading edge, based on c		
$\bar{c}$	geometric mean chord of wing	m	ft
D	body maximum diameter	m	ft
D_b	body base diameter	m	ft
F	limiting pressure factor, $-\gamma C_{plim}$, used in the evaluation of σ , see Figure 4 and Appendix A		
F_t	factor used in evaluation of C_{st} , see Figure 2		
F_{VB}	factor limiting amount of leading-edge vortex lift that can be realised, see Section 3.4, Figure 1 and Appendix C		
K_{visc}	lift-dependent viscous drag factor, see Section 3.3 and Appendix B		
K_σ	factor applied to limiting pressure coefficient to reduce maximum possible attainable suction, see Sections 3.1.1, 5.2 and 7.3.6		
K_{lcs}	factor applied to a_1 and a_{1eff} to allow empirical modification, see Sections 3.3.2.1 and 7.3.7		
K_{lev}	factor applied to weaken or remove leading-edge vortex lift and drag contributions, see Sections 3.4.1 and 7.3.8		
K_{sev}	factor applied to weaken or remove side-edge vortex lift and drag contributions, see Sections 3.4.1 and 7.3.8		
K_1, K_2	constants in modelling of ΔC_{p1} , ΔC_{p2}		
l_a	boat-tail length	m	ft
l_c	characteristic length in evaluation of lift augmentation, streamwise distance from leading-edge tip to trailing-edge apex, Equation (3.18)	m	ft
M	free-stream Mach number		
p	parameter in Equation (4.3)		
R	Reynolds number based on $\bar{c}$		
r	local chordwise distance of centroid of leading-edge vortex force aft of leading-edge, see Sketch 3.4 and Appendix A	m	ft
r_i	leading-edge radius index, Equation (A3.16)		
S_B	body maximum cross-section area	m ²	ft ²
S_p	body planform area (neglecting reduction in area due to boat-tail)	m ²	ft ²
S_N	planform area of net wing, see Sketch 1.1b	m ²	ft ²

S_W	planform area of gross wing, see Sketch 1.1b	m ²	ft ²
s	semi-span of gross wing	m	ft
t	streamwise-section maximum thickness	m	ft
α	angle of attack of datum axis, see Sketch 1.1a	deg	deg
α_B	angle between longitudinal body axis and datum axis, shown negative in Sketch 1.1a	deg	deg
α_{corr}	local correction to α in evaluation of leading-edge suction for non-planar wing, see Section 3.1 and Sketch 3.6a	deg	deg
α_{FVB}	effective angle of attack for evaluation of F_{VB}	deg	deg
α_l	local angle of attack, $\alpha + \varepsilon$ on wing and $\alpha + \alpha_B$ on body	deg	deg
α_{s1}, α_{s2}	angles of attack that define range of α for no loss of leading-edge suction, see Section 3.1 and Sketch 3.3, otherwise both take value of α for maximum $\bar{\sigma}$	deg	deg
α_{min}	angle of attack for minimum viscous drag coefficient	deg	deg
α_0	angle of attack of datum axis for zero wing lift when suction and vortex terms are neglected, shown negative in Sketch 1.1a, Equation (3.11)	deg	deg
β	compressibility parameter, $(1 - M^2)^{1/2}$		
ε	section twist angle with respect to datum axis, positive leading edge up, see Sketch 1.1a	deg	deg
ε_t	value of ε at wing tip	deg	deg
ε_0	angle between chord line and tangent to camber line at section leading edge, positive leading-edge down, see Sketch 3.2	deg	deg
ζ_C	camber ordinate as fraction of chord		
θ	angle between normal to section chord line and normal to section surface at chordwise location r , positive for forward rotation, see Sketch 3.5; defined for upper surface if $\alpha \geq (\alpha_{s1} + \alpha_{s2})/2$ or lower surface if $\alpha \leq (\alpha_{s1} + \alpha_{s2})/2$	deg	deg
η	spanwise distance from centre-line as fraction of semi-span		
η_B	spanwise distance from centre-line to body side as fraction of semi-span, $D/2s$		
Λ_0	leading-edge sweep	deg	deg
Λ'_0	transformed leading-edge sweep, $\tan^{-1}(\beta^{-1} \tan \Lambda_0)$	deg	deg
λ	taper ratio of gross wing, ratio of tip chord to centre-line chord		
λ_N	taper ratio of net wing, ratio of tip chord to root chord		

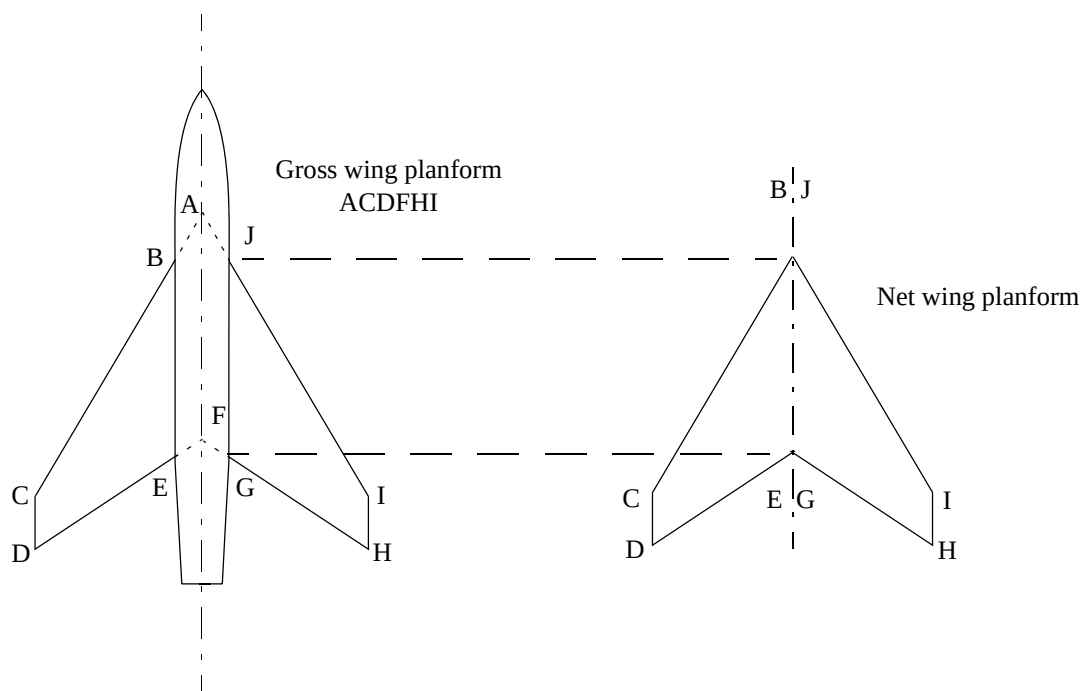
ξ	chordwise distance aft from leading edge as fraction of chord		
$\Delta \xi$	chord of vortex lattice cell		
ξ_m	chordwise position of maximum thickness of streamwise-section as fraction of chord		
ρ	streamwise-section leading-edge radius	m	ft
σ	local leading-edge suction factor		
$\bar{\sigma}$	mean leading-edge suction factor, see Equation (3.1)		

Subscripts

C	as in () _C denotes loading due to camber
n	denotes normal-section values
T	as in () _T denotes loading due to twist
α	as in () _α denotes incidence load grading



a) The datum axis is any convenient axis used for the definition of angle of attack



b) Definition of net wing

Sketch 1.1

2. INTRODUCTION

This Item describes a method for the prediction of the drag due to lift for swept wings of straight taper with camber and twist, alone or in combination with a body under conditions that tend to induce leading-edge flow separation and a loss of leading-edge suction at low to moderate values of lift coefficient. It is applied to wings with mild cranks through the use of an 'equivalent' straight-tapered wing, a geometric construction that is described in Addendum A of Item No. 76003 (Derivation 3). The angle of attack range includes the regime where there may be a full or partial loss of leading-edge suction and the formation of leading-edge vortex flow, but widespread flow separation over the main part of the wing is not addressed. The method applies for subsonic flow and when the boundary-layer over the whole wing is fully turbulent. The method is unsuited to hand calculations and has been programmed as ESDUpac A9625, which operates as a batch file that runs a number of other programs automatically, writing their inputs and reading their outputs to produce a final output file from one input file.

Item No. 95025 (Derivation 10) developed a method for the prediction of drag due to lift for planar wings. The method invokes the concept of attainable leading-edge suction to cater for the effects of wing section geometry (Derivations 17 and 18). For wings of low aspect ratio the lift associated with the leading-edge vortex is estimated by extended forms of the Polhamus analogy (Derivations 13, 15 and 16).

Derivations 20 to 22 and the recapitulation in Reference 43 show that the calculation of attainable suction for non-planar wings may be made by using the same equations that apply to planar wings provided that a reliable estimate can be made of the theoretical leading-edge suction distribution. They demonstrate that this can be done by using the theoretical distribution for a planar wing of identical planform in conjunction with a corrected local angle of attack. They also show that it is necessary in the calculation of vortex lift to allow for the migration of the leading-edge vortex as it travels back over the wing. It is not sufficient to assume that all the vortex lift acts at the leading edge, and the local surface slope is of consequence as it determines the direction of the force. The method of this Item incorporates both these concepts. The detailed calculation of the corrected angle of attack follows the technique in Derivations 20 to 22 and Reference 43. The prediction of the aft migration rate of the leading-edge vortex is based on the work of Derivation 19, the simplest of the published options available. As incidence increases the loss of lift due to the vortex passing over the wing trailing edge or of vortex breakdown occurring ahead of the wing trailing edge is modified through a single factor developed from a correlation presented in Reference 41.

In Derivation 24 the method for estimating attainable suction was improved over the earlier versions in Derivations 17, 18 and 21. A final modification was reported in Reference 43. The programmed method takes account of all these changes. Full details of the development of the method are not given because the basic philosophy of the technique remains as summarised in Section 3.2 of Item No. 95025. Moreover, Derivation 24 provides a comprehensive description of the development of the whole method for attainable suction and does not simply confine itself to the improvements made. It is therefore an ideal reference for users who wish to know more of the intricacies involved.

The overall model for a wing is detailed in Section 3. Section 4 indicates a simple extension to include wing-body configurations. Section 5 discusses accuracy and applicability. Section 6 lists the Derivation and References and Section 7 describes the input and output for the programmed version of the method, with examples. In the input file (see Section 7.3), after the free-stream conditions and configuration geometry have been specified, some entries allow the user to alter a number of the program default settings and so examine the influence of certain parameters or calibrate prediction against known experimental data.

Appendix A sets out the formulae for the calculation of leading-edge suction. Appendix B discusses the estimation of lift-dependent viscous drag, which forms a secondary component of the overall drag polar. Appendix C addresses leading-edge vortex migration and breakdown.

3. METHOD

Section 3.1 introduces the topics of suction force and the factors that model the degree to which the theoretical value is attained. Section 3.2 addresses the location of leading-edge vortex force. These topics are necessary forerunners to Sections 3.3 and 3.4 that discuss the wing drag and lift coefficients.

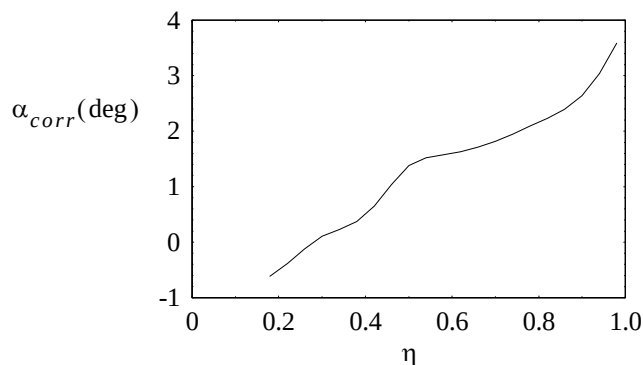
3.1 Leading-edge Suction c_s and Factors σ and $\bar{\sigma}$

Because of leading-edge separation, a wing may not develop in full the maximum possible leading-edge suction at all sections. This behaviour is modelled by applying an empirical factor σ to the theoretical value c_s so that the attained local suction is σc_s .

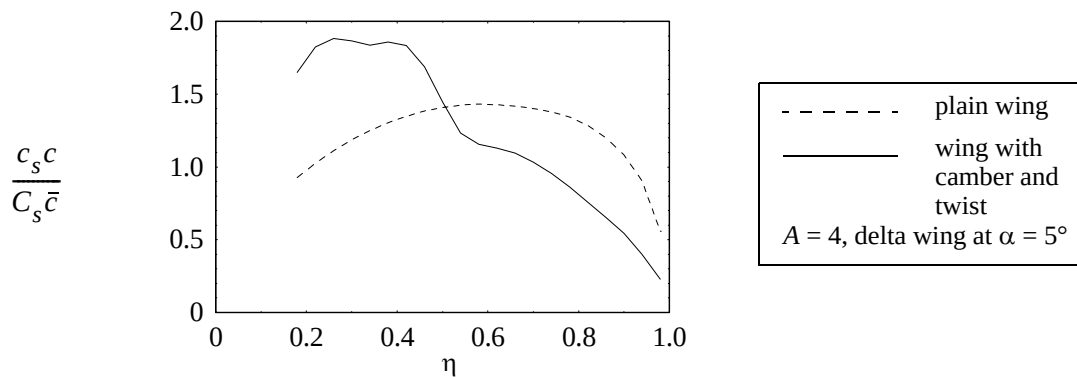
Appendix A shows how the suction factor may be obtained for any streamwise section on a planar wing from a knowledge of the leading-edge radius to chord ratio, ρ/c , the thickness to chord ratio, t/c , the dimensionless position of maximum thickness, ξ_m , the Mach number, M , and Reynolds number, R , and the theoretical value of local leading-edge suction coefficient, c_s .

The value of c_s for a planar wing can be obtained from Item No. 94037 (Derivation 7) or the program in Item No. 94038 (Derivation 8). For a non-planar wing it is shown in Derivation 20 that the value of c_s can be approximated by factoring the value for a section on a planar wing of the same planform at an incidence of one degree by $(\alpha - \alpha_{corr})^2$, where the correction angle, α_{corr} , varies spanwise but is independent of α . In Derivation 20 the incidence at which the leading-edge singularity in a linearised lifting-surface solution disappears, and $c_s = 0$, is used to define α_{corr} . For this Item, α_{corr} is obtained by running the vortex-lattice model described in Derivation 14 for the non-planar and planar cases and separating the loadings due to incidence and camber, see Section 3.3 and Sketch 3.6a. Sketch 3.1a illustrates α_{corr} for a non-planar, aspect ratio 4, delta wing (see Sketch 5.2 for the camber and twist) and those values of α_{corr} apply at all α . Sketch 3.1b compares the normalised theoretical suction distributions for the cambered and twisted wing and a plane wing of the same planform at $\alpha = 5^\circ$. Note that the vortex lattice model is used only to obtain α_{corr} ; the theoretical suction distribution for the planar wing that is to be factored is taken from Item No. 94038 to avoid any irregularities in the vortex lattice solutions (see Appendix A of Item No. 94037 for a discussion on the development of the database of Item No. 94038).

The programmed version of the method performs all the necessary theoretical suction calculations and those for the determination of α_{corr} , running the vortex lattice program of Derivation 14 and the program of Item No. 94038 automatically.

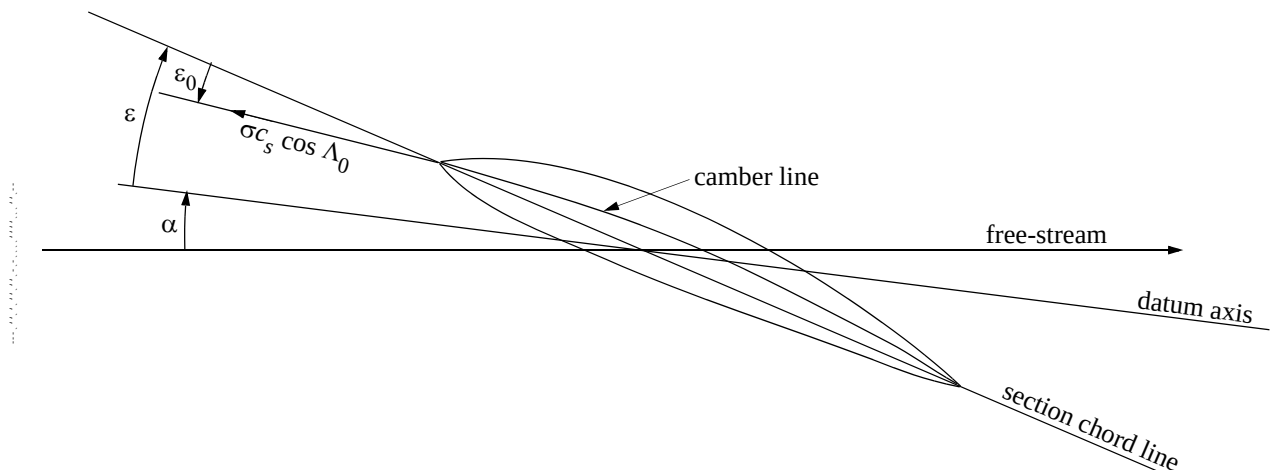


Sketch 3.1a Distribution of α_{corr} (independent of α)



Sketch 3.1b Normalised leading-edge suction distributions for non-planar wing (full suction)

Because the leading-edge suction is predominantly determined by the geometry in the vicinity of the leading-edge, for a cambered section the factor σ is obtained simply by substituting into the equations of Appendix A the cambered-section values of ρ/c , t/c and ξ_m and the value of c_s for the non-planar wing. At each section the attained force coefficient $\sigma c_s \cos \Lambda_0$ acts in the direction of the tangent to the camber line close to its leading-edge, at the centre of the leading-edge radius say. These procedures are suggested in Derivations 17 and 20. The local angle ε_0 defines the leading-edge orientation, see Sketch 3.2.



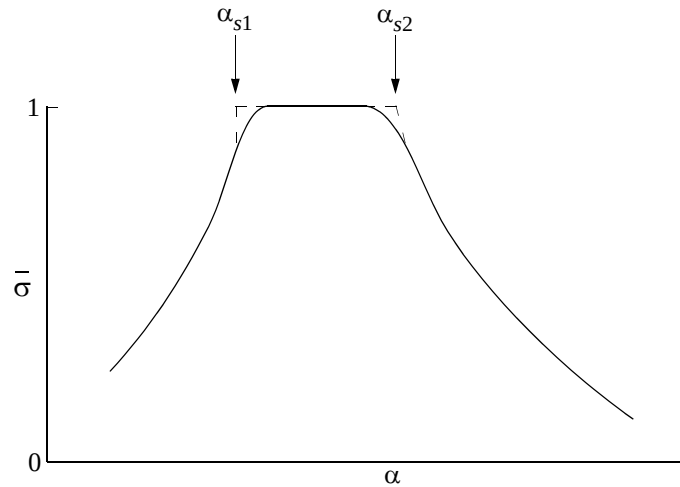
Sketch 3.2 Definition of ε and ε_0

While some components of lift and drag rely on a spanwise integration of locally-attained forces, others are based on overall wing properties for which the mean suction factor, $\bar{\sigma}$, is appropriate. This is given by

$$\bar{\sigma} = \int_0^1 \sigma c_s c d\eta \bigg/ \int_0^1 c_s c d\eta, \quad (3.1)$$

where c_s takes local values appropriate to the non-planar wing.

A knowledge of the range of α over which there is full suction ($\bar{\sigma} = 1$) is also required in some applications. The angles α_{s1} and α_{s2} , see Sketch 3.3, are the limits which define that range. The identification of α_{s1} and α_{s2} requires the extrapolations of the variation of $\bar{\sigma}$ with α from either side of each limit.



Sketch 3.3 Variation of $\bar{\sigma}$ with α

For some extreme combinations of camber and twist it may be possible for $\bar{\sigma}$ to remain below unity at all times, in which case α_{s1} and α_{s2} are both equated to the angle of attack at which $\bar{\sigma}$ reaches its maximum value.

The programmed version of the method makes an initial internal run to find the values of α_{s1} and α_{s2} before beginning the main calculation.

3.1.1 Modification of attainable suction

Provision is made in the programmed version of the method for the user to set a value for a factor $K_{\bar{\sigma}}$ via the input file, see Section 7.3.6. This factor is unity in the correlation of Derivation 24, and a lower positive value will reduce the maximum possible attainable suction at a given angle of attack and narrow the plateau of full suction shown in Sketch 3.3. Section 5.2 discusses the effect of using a value of $K_{\bar{\sigma}}$ below unity in the final paragraph on page 23.

3.2 Location of Leading-edge Vortex Force

For highly-swept wings, where the theoretical leading-edge suction is not fully attained and a leading-edge vortex is formed, the Polhamus analogy (Derivation 11) postulates the appearance of a vortex force equal in magnitude to the unattained suction which acts normal to the wing surface. The location of the centroid of this force is immaterial for a planar wing, provided it is ahead of the trailing edge, but of consequence for a cambered wing because the line of action of the force will be determined by the shape of the local surface.

With increase of incidence the leading-edge vortex on a swept wing is known to migrate aft so that the associated force acts behind the leading-edge. The simplest form for the location of the centroid of the vortex force is that proposed in Derivation 19, where semi-empirical considerations based on momentum conservation give the local chordwise distance aft of the leading-edge as

$$r = c_s c \quad (3.2)$$

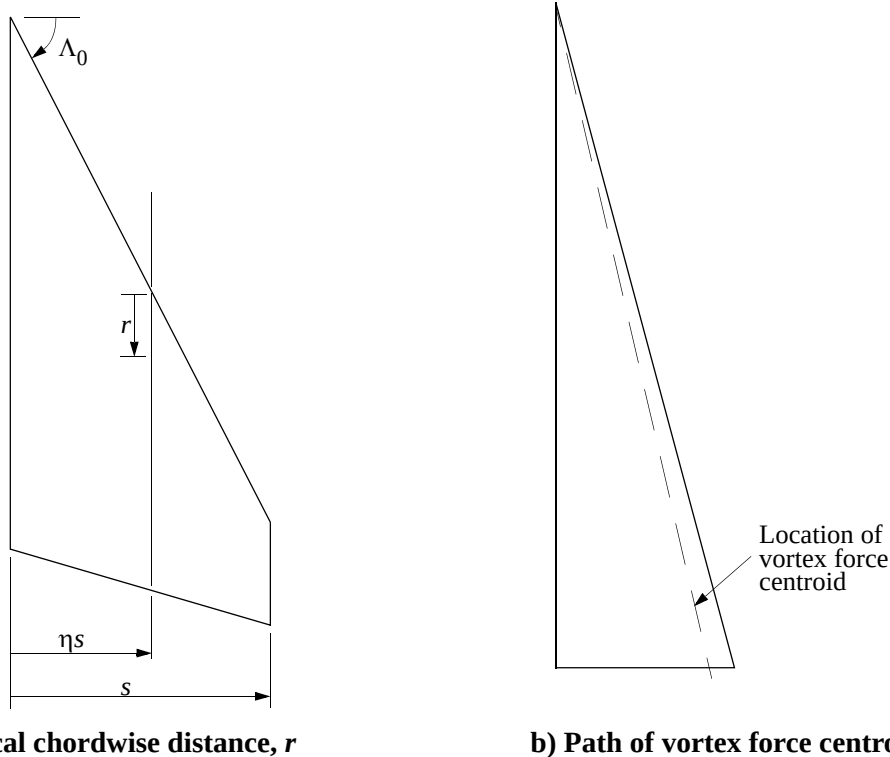
for a sharp-edged section with full loss of leading-edge suction, see Sketch 3.4a.

As an example, Sketch 3.4b shows the locus of the centroid of vortex force for a sharp-edged delta wing of low aspect ratio at an incidence of 12° .

For general application within the method of this Item it is assumed that for sections where there is only a partial loss of suction.

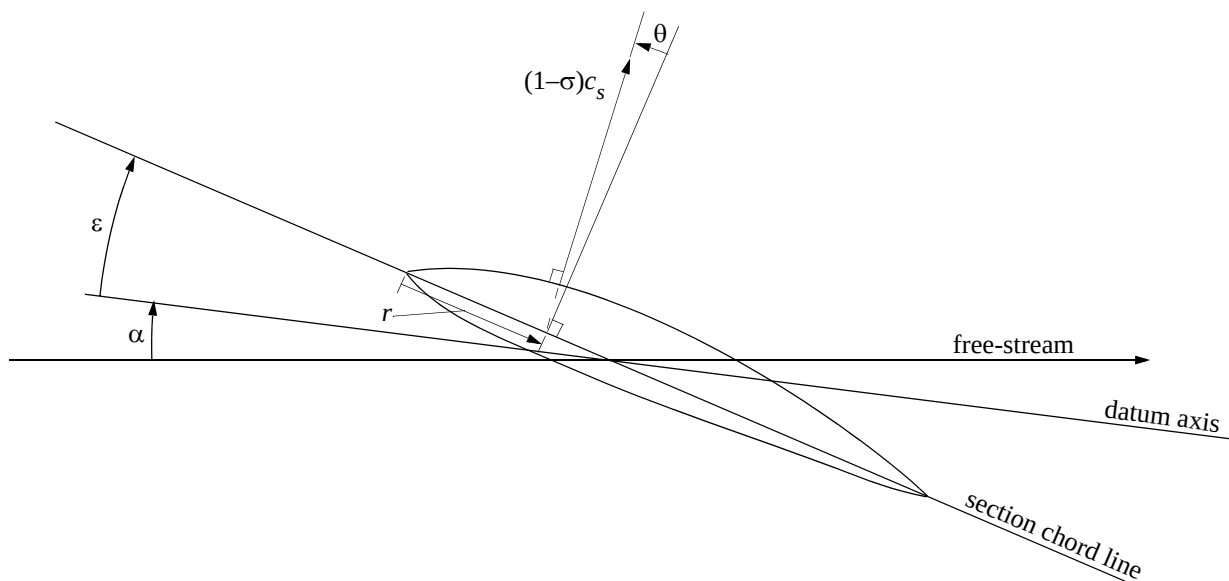
$$r = (1 - \sigma) c_s c. \quad (3.3)$$

Alternative ways of locating the vortex normal force are given in Derivation 22, where the technique of Derivation 19 is one of the options offered. See also Appendix C.



Sketch 3.4 Location of vortex force centroid

With the distance r known the normal to the upper or lower surface at the centroid determines the line of action of the vortex force $(1 - \sigma) c_s$ which makes an angle θ with the normal to the section chord line, see Sketch 3.5. The local force will act on the upper surface if $\alpha \geq (\alpha_{s1} + \alpha_{s2})/2$ and it will act on the lower surface if $\alpha < (\alpha_{s1} + \alpha_{s2})/2$.



Sketch 3.5 Definition of θ

3.3 Drag and Lift Coefficients

3.3.1 Drag coefficient

In Item No. 95025, Equation (3.10) gives the drag coefficient for the planar wing as

$$C_{DW} = C_{AWv} \cos \alpha - \bar{\sigma} C_s \cos \Lambda_0 \sec \alpha + C_{LW} \tan \alpha . \quad (3.4)$$

The first term represents a resolved viscous axial force component such that

$$C_{AWv} \cos \alpha = C_{DPmin} + \Delta C_{Dvisc} , \quad (3.5)$$

where C_{DPmin} is the minimum profile drag coefficient and ΔC_{Dvisc} is a lift-dependent increment.

For a cambered and twisted wing, C_{DPmin} (an input parameter in the program, see Section 7.3.1) occurs at a lift coefficient C_{Lmin} and ΔC_{Dvisc} is modelled in terms of a drag factor K_{visc} and C_{Lmin} (see Appendix B), so that

$$\Delta C_{Dvisc} = \bar{\sigma}^2 K_{visc} \left[a_1 \frac{(\alpha - \alpha_0) \pi}{180} - C_{Lmin} \right]^2 , \quad (3.6)$$

where, following the studies made in the development of Item No. 95025, $\bar{\sigma}^2$ serves as a necessary and satisfactory empirical mechanism to reduce ΔC_{Dvisc} as the angle of attack becomes large and leading-edge suction is lost. (Equation (3.11) gives α_0 .)

The minimum ΔC_{Dvisc} occurs at an angle of attack

$$\alpha_{min} = \alpha_0 + \left(\frac{C_{Lmin}}{a_1} \right) \frac{180}{\pi} . \quad (3.7)$$

The second and third terms on the right hand side of Equation (3.4) are due to the resolved components of the attained leading-edge suction and the lift coefficient. For the non-planar wing corresponding terms involve spanwise integration of section forces for a local incidence $\alpha_l = \alpha + \varepsilon$, with the attained suction force giving rise to a forward-acting chordwise force coefficient $\sigma c_s \cos \Lambda_0 \cos \varepsilon_0$ and a lift coefficient $\sigma c_s \cos \Lambda_0 \sin(\alpha_l - \varepsilon_0)$. If C_{DC} represents the contribution to C_D from the pressure distribution over a cambered wing and if the leading-edge vortex is assumed to contribute C_{Dlev} and the side-edge vortex C_{Dsev} , the total drag coefficient becomes

$$\begin{aligned} C_{DW} = & C_{DPmin} + \Delta C_{Dvisc} - \int_0^1 \sigma(c_s c / \bar{c}) \cos \Lambda_0 \cos \varepsilon_0 \sec \alpha_l d\eta \\ & + \int_0^1 (c_l c / \bar{c}) \tan \alpha_l d\eta + \int_0^1 \sigma(c_s c / \bar{c}) \cos \Lambda_0 \sin(\alpha_l - \varepsilon_0) \tan \alpha_l d\eta \\ & + C_{DC} + C_{Dlev} + C_{Dsev} . \end{aligned} \quad (3.8)$$

In Equation (3.8) $c_l c / \bar{c}$ is the local lift distribution due to incidence, camber and twist, which can be written

$$\frac{c_l c}{\bar{c}} = \left[\left(\frac{c_l c}{C_L \bar{c}} \right)_\alpha a_1 \frac{\alpha \pi}{180} + \left(\frac{c_l c}{\bar{c}} \right)_C + \left(\frac{c_l c}{\bar{c}} \right)_T \right] \frac{\sin \alpha_l \cos^2 \alpha_l}{(\alpha_l \pi / 180)} , \quad (3.9)$$

where the spanwise load grading due to incidence, $(c_l c / C_L \bar{c})_\alpha$, and the loadings due to camber, $(c_l c / \bar{c})_C$, and twist, $(c_l c / \bar{c})_T$, can be obtained from the Multhopp-Richardson lifting-surface program of Item No. 95010 (Derivation 9) or the graphical method of Item No. 83040 (Derivation 4). Comparisons with experimental data show that at high angles of attack the factor $\sin \alpha_l \cos^2 \alpha_l / (\pi \alpha_l / 180)$ that is introduced on the right-hand side of Equation (3.9) provides an improved modelling of lift distribution for use in Equation (3.8) compared to the assumption of a linear variation with incidence*.

As described in Derivation 20 and Reference 43, the drag term C_{DC} arises from summation of the local axial force coefficients, c_{aC} , present as a result of the pressure distribution on a cambered surface.

It is estimated as

$$C_{DC} = \int_0^1 c_{aC}(c / \bar{c}) \cos \alpha_l d\eta = \int_0^1 \int_0^1 \{-(d\zeta_C / d\xi) \Delta C_p d\xi\} (c / \bar{c}) \cos \alpha_l d\eta , \quad (3.10)$$

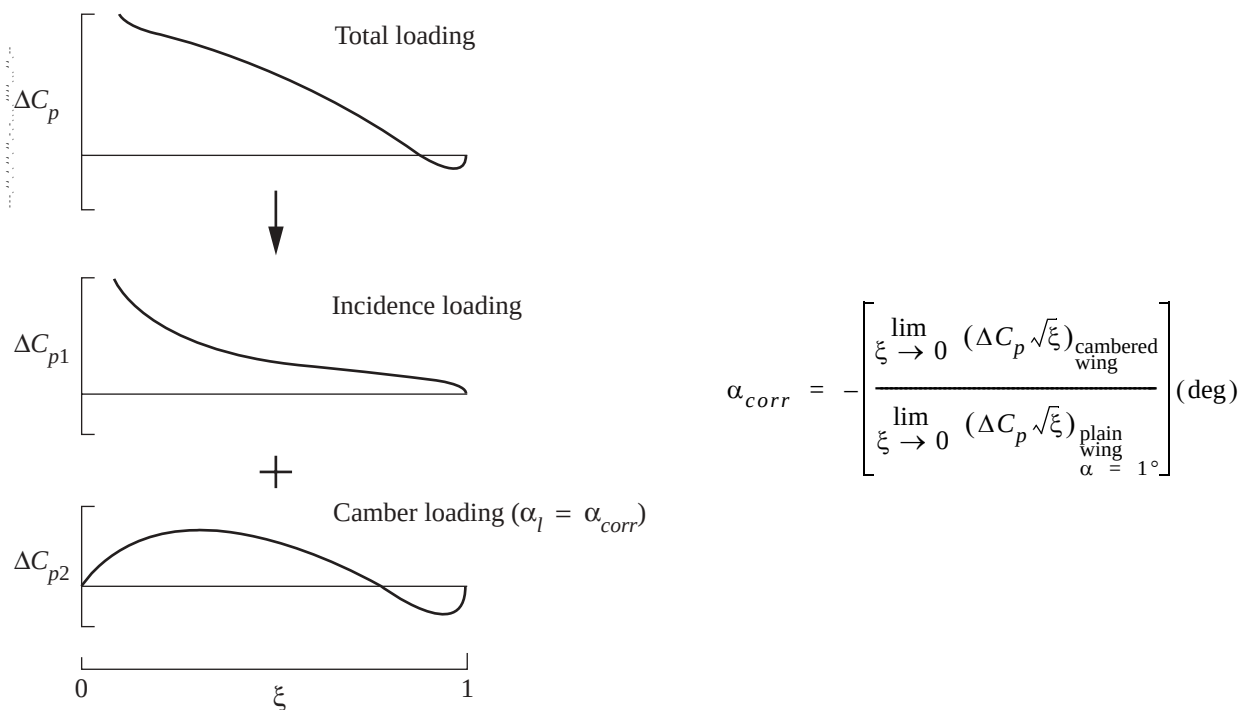
where ΔC_p is the static pressure coefficient increment across a thin cambered section, which in the programmed version of the method is evaluated using the vortex lattice model of Derivation 14. Within the program, c_{aC} is evaluated as in Reference 43, with ΔC_p divided into two components, ΔC_{p1} which arises from the flat plate pressure distribution due to angle of attack and ΔC_{p2} which arises from the pressure distribution due to camber only ($\alpha_l = \alpha_{corr}$), see Sketch 3.6a. The vortex lattice model provides coefficient increments which are constant in each lattice cell. Following an approach similar to that of Reference 43, in order to evaluate Equation (3.10) chordwise distributions of ΔC_{p1} and ΔC_{p2} are assumed across each

* The use of $\alpha - \alpha_0$ in place of α_l in Equation (3.9) produces similar results in most cases but with a bias towards under prediction of C_D .

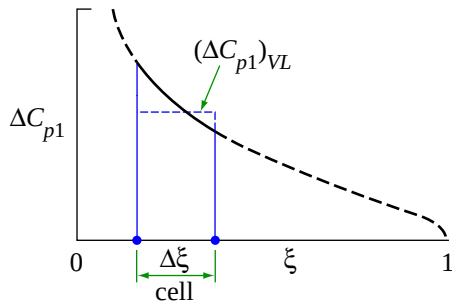
cell, see Sketch 3.6b, with the scaling constants K_1 and K_2 determined so that the integrated values are equal to $(\Delta C_{p1})_{VL} \Delta \xi$ or $(\Delta C_{p2})_{VL} \Delta \xi$, where $(\Delta C_{p1})_{VL}$ and $(\Delta C_{p2})_{VL}$ are the vortex lattice values over the cell of length $\Delta \xi$. The values of ΔC_{p1} and ΔC_{p2} at any ξ are then associated with the local camber-line slope for the integration. Upper and lower surface section co-ordinates are used as input to the program, see Section 7, and the user should ensure that these provide a camber line that is not irregular, particularly near the leading edge, so that the integral in Equation (3.10) is not subject to avoidable error.

With full leading-edge suction, the sum of the third to sixth terms of Equation (3.8) represents the trailing vortex drag. There is some difficulty in a precise numerical evaluation of the sum because it contains drag and suction contributions of opposite sign, with some individual magnitudes in excess of the total sum. This is compounded by particular difficulties in the numerical evaluation of C_{DC} . The effect is that although the sum of the terms behaves as the expected parabola, the displacement in C_L and C_D of the minimum away from $C_D = 0$ at $C_L = 0$, which is to be expected for a twisted wing, is not predicted as accurately as would be expected from direct calculation, using the full-potential method of Item No. 06016 (Reference 46) for example. Consequently, the program input file requires the user to enter values ΔC_{Dtv} and ΔC_{Ltv} to position the minimum of the trailing-vortex drag polar, see Section 7.3.9. Calculation inside the program then bodily shifts the trailing-vortex drag polar so that its minimum occurs at $C_D = \Delta C_{Dtv}$ and $C_L = \Delta C_{Ltv}$. On an empirical basis, the shift in the C_L polar values is factored by $\bar{\sigma}$ and so decreases steadily when there is, eventually, a progressive loss of leading-edge suction as C_L rises in magnitude. Values of ΔC_{Dtv} and ΔC_{Ltv} , which depend on Mach number, wing planform and wing twist distribution, can be estimated for the general case by using the program associated with Item No. 06016 (Reference 46). For simple linear twist, References 44 and 45 give hand calculation methods. The latter also covers linear-lofted twist. Item No. 10022 (Reference 47) is a hand calculation method based on that of Item No. 06016. To obtain best predictions for a wing that is twisted, estimates of ΔC_{Dtv} and ΔC_{Ltv} should always be made. For an untwisted wing $\Delta C_{Dtv} = \Delta C_{Ltv} = 0$ are the appropriate values and these may also suffice for a wing of limited twist ($< 2^\circ$ say), if estimates are unavailable.

The contributions C_{Dlev} and C_{Dsev} are discussed in Section 3.4.

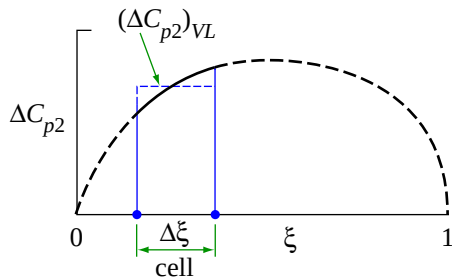


Sketch 3.6a Breakdown of ΔC_p into camber and incidence loadings



Over cell, $\Delta C_{p1} = K_1 \sqrt{\frac{1}{\xi} - 1}$ with K_1 determined

so that $(\Delta C_{p1})_{VL} \Delta \xi = K_1 \int_{cell} \sqrt{\frac{1}{\xi} - 1} d\xi$



Over cell, $\Delta C_{p2} = K_2 \sqrt{\xi(1 - \xi)}$ with K_2 determined

so that $(\Delta C_{p2})_{VL} \Delta \xi = K_2 \int_{cell} \sqrt{\xi(1 - \xi)} d\xi$

Sketch 3.6b Variation of ΔC_{p1} , ΔC_{p2} over cell

3.3.2 Lift coefficient

The angle of attack for zero lift may be found from

$$\alpha_0 = -(C_{LC} + C_{LT}) / (a_1 \pi / 180), \quad (3.11)$$

where C_{LC} and C_{LT} , the lift contributions due to camber and twist, can be obtained from the program of Item No. 95010 or Item No. 83040. The lift-curve slope, a_1 , can be obtained from Item No. 70011 (Derivation 1) but the same value is also provided by Item No. 95010. Again, the programmed method calculates α_0 internally, running the program of Item No. 95010 automatically.

In Item No. 95025, Equation (3.66) gives the lift coefficient for the planar wing as

$$C_{LW} = a_1 \sin \alpha \cos^2 \alpha + \bar{\sigma} C_s \cos \Lambda_0 \sin \alpha + C_{Llev} + C_{Lsev}, \quad (3.12)$$

where C_{Llev} and C_{Lsev} are the contributions from the leading-edge and side-edge vortices respectively, see Section 3.4, and a small contribution from the axial viscous force has been neglected.

For the non-planar wing

$$C_{LW} = a_1 \sin(\alpha - \alpha_0) \cos^2(\alpha - \alpha_0) + \int_0^1 \sigma(c_s c / \bar{c}) \cos \Lambda_0 \sin(\alpha_l - \varepsilon_0) d\eta + C_{Llev} + C_{Lsev}. \quad (3.13)$$

The first term in Equations (3.12) and (3.13) follows the representation by Polhamus (Derivation 11) of the potential lift term in the absence of leading-edge suction.

3.3.2.1 Modification of lift-curve slope

The program input file allows the entry of a factor K_{lcs} which directly multiplies the value of lift-curve slope calculated from Item No. 70011. Values greater or smaller than the default setting of $K_{lcs} = 1$ allow the user to match predicted lift curves to known experimental data, see Section 7.3.7.

3.4 Contributions from Leading-edge Vortex and Side-edge Vortex

The position and orientation of the leading-edge vortex force associated with unattained leading-edge suction have been discussed in Section 3.2. Local axial and normal force coefficients are

$$c_{alev} = -(1 - \sigma) c_s \sin \theta \quad (3.14)$$

$$\text{and} \quad c_{nlev} = \pm (1 - \sigma) c_s \cos \theta \quad (3.15)$$

with the sign in Equation (3.15) to match the sign of $\alpha - (\alpha_{s1} + \alpha_{s2})/2$. Resolution of the section forces gives local contributions to drag and lift.

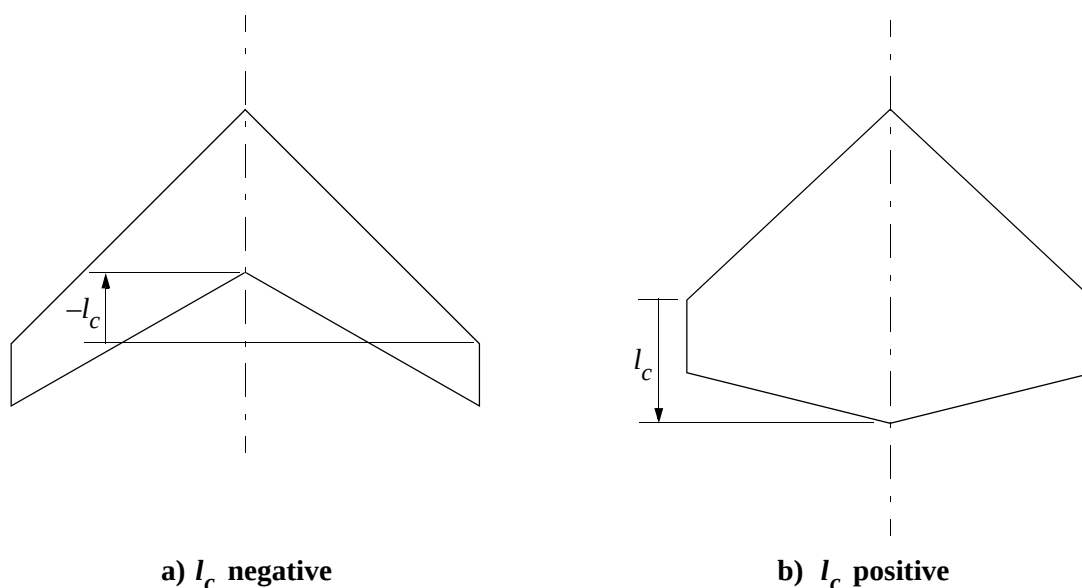
For a planar wing there is also an augmentation of the wing normal force due to the leading-edge vortex passing downstream over an aft part of the wing planform. For a non-planar wing this is expressed through the drag and lift augmentation coefficients,

$$C_{Daug} = \int_0^1 \pm (1 - \sigma)(c_s c / \bar{c})(l_c / s) \cos \Lambda_0 \sin \alpha_l d\eta \quad (3.16)$$

$$\text{and} \quad C_{Laug} = \int_0^1 \pm (1 - \sigma)(c_s c / \bar{c})(l_c / s) \cos \Lambda_0 \cos \alpha_l d\eta \quad (3.17)$$

with the sign in Equations (3.16) and (3.17) to match the sign of $\alpha - (\alpha_{s1} + \alpha_{s2})/2$.

The characteristic length l_c that appears in Equations (3.16) and (3.17) is the streamwise distance from the leading-edge tip to the trailing-edge apex, see Sketches 3.7a and 3.7b; note that l_c is negative in the former sketch.



Sketch 3.7 Illustration of length l_c

In general,
$$\frac{l_c}{s} = \frac{4}{A(1 + \lambda)} - \tan \Lambda_0 . \quad (3.18)$$

Further discussion of l_c is contained in Item No. 95025 and Derivations 15 and 16.

The effects of the leading-edge vortex are only partially realised if the vortex migrates past the wing trailing edge or the vortex breaks down ahead of the wing trailing edge and this is modelled through a factor F_{VB} which was developed from a correlation for sharp-edged delta wings shown in Reference 41. See also the discussion in Appendix C.

The generalised extension of F_{VB} to non-planar wings of any planform is tentative and so warrants a treatment only in terms of overall wing characteristics as reflected through the variation of $\bar{\sigma}$ with incidence. Figure 1 shows F_{VB} as a function of leading-edge sweep and an effective incidence α_{FVB} , where

$$\alpha_{FVB} = |\alpha - \alpha_{s1}| \quad \text{for} \quad \alpha < \alpha_{s1} , \quad (3.19)$$

$$\alpha_{FVB} = |\alpha - \alpha_{s2}| \quad \text{for} \quad \alpha > \alpha_{s2} \quad (3.20)$$

and
$$\alpha_{FVB} = 0 \quad \text{for} \quad \alpha_{s1} \leq \alpha \leq \alpha_{s2} . \quad (3.21)$$

With F_{VB} introduced, Equations (3.14) to (3.17) are used to derive the values of C_{Dlev} and C_{Llev} as

$$C_{Dlev} = F_{VB} \int_0^1 [(c_{alev}c/\bar{c}) \cos \alpha_l + (c_{nlev}c/\bar{c}) \sin \alpha_l] d\eta + F_{VB} C_{Daug} \quad (3.22)$$

and
$$C_{Llev} = F_{VB} \int_0^1 [(c_{nlev}c/\bar{c}) \cos \alpha_l - (c_{alev}c/\bar{c}) \sin \alpha_l] d\eta + F_{VB} C_{Laug} . \quad (3.23)$$

Hence

$$\begin{aligned} C_{Dlev} &= F_{VB} \left\{ \int_0^1 -(1 - \sigma)(c_s c/\bar{c}) \sin \theta \cos \alpha_l d\eta + \int_0^1 \pm(1 - \sigma)(c_s c/\bar{c}) \cos \theta \sin \alpha_l d\eta \right. \\ &\quad \left. + \int_0^1 \pm(1 - \sigma)(c_s c/\bar{c})(l_c/s) \cos \Lambda_0 \sin \alpha_l d\eta \right\} \\ &= F_{VB} \left\{ \int_0^1 (1 - \sigma)(c_s c/\bar{c}) \{ \sin(\pm \alpha_l - \theta) \pm (l_c/s) \cos \Lambda_0 \sin \alpha_l \} d\eta \right\} \end{aligned} \quad (3.24)$$

and

$$\begin{aligned} C_{Llev} &= F_{VB} \left\{ \int_0^1 \pm(1 - \sigma)(c_s c/\bar{c}) \cos \theta \cos \alpha_l d\eta + \int_0^1 (1 - \sigma)(c_s c/\bar{c}) \sin \theta \sin \alpha_l d\eta \right. \\ &\quad \left. + \int_0^1 \pm(1 - \sigma)(c_s c/\bar{c})(l_c/s) \cos \Lambda_0 \cos \alpha_l d\eta \right\} \\ &= F_{VB} \left\{ \int_0^1 (1 - \sigma)(c_s c/\bar{c}) \{ \pm \cos(\pm \alpha_l - \theta) \pm (l_c/s) \cos \Lambda_0 \cos \alpha_l \} d\eta \right\} . \end{aligned} \quad (3.25)$$

The side-edge vortex normal force is given for a planar wing in Item No. 95025 as

$$C_{st} = F_t \left(\frac{\lambda}{\beta A} \right)^{0.75} \sin^2 \alpha, \quad (3.26)$$

where F_t is given in Figure 2 as a function of βA and Λ'_0 .

For a non-planar wing the tip incidence is inserted in place of α and the resolved drag and lift contributions are

$$C_{Dsev} = \pm F_t \left(\frac{\lambda}{\beta A} \right)^{0.75} \sin^3(\alpha + \varepsilon_t) \quad (\text{always positive}) \quad (3.27)$$

and

$$C_{Lsev} = \pm F_t \left(\frac{\lambda}{\beta A} \right)^{0.75} \sin^2(\alpha + \varepsilon_t) \cos(\alpha + \varepsilon_t) \quad (3.28)$$

with the signs in Equations (3.27) and (3.28) matching the sign of $\alpha + \varepsilon_t$.

3.4.1 Modification of contributions

The improvement in the prediction of drag polars by the inclusion of the leading-edge vortex and side-edge vortex contributions has been established in many studies following Derivations 11 and 13, perhaps reported more often for the former than the latter. However, the program input file allows the user to enter the factors K_{lev} and K_{sev} to weaken or remove their respective contributions, independently, by setting pairs of values below the recommended default settings of $K_{lev} = 1$ and $K_{sev} = 1$, see section 7.3.8. The factors multiply the lift and drag contributions directly so that values of zero will remove their effect completely.

4. WING-BODY COMBINATION

The presence of a body requires that the lift and drag calculated for an isolated wing be corrected for a number of effects.

Firstly there are the direct contributions of the body to drag and lift. As in Item No. 95025 these are approached through the body normal force and axial force.

The body normal force coefficient, based on the data in Item Nos 87033 and 89014 (Derivations 5 and 6) for forebody-cylinder combinations with or without conical boat-tails is

$$C_{NB} = C_{NB\alpha}(S_B/S_W)\sin(\alpha + \alpha_B)\cos(\alpha + \alpha_B) + C_{NBC}(S_p/S_W) + \Delta C_{NB}(S_B/S_W), \quad (4.1)$$

where the first two terms come from Item No. 89014 and the boat-tail term is from Item No. 87033.

For present purposes, for angles of attack up to 30° it is sufficient to take

$$C_{NB\alpha} = 2 + 0.001386e^{5.376M}, \quad (4.2)$$

and, for $M \geq 0.5$,

$$C_{NBC} = 0.58 \sin^2(\alpha + \alpha_B) + p[(\alpha + \alpha_B - 17.5)/12.5][0.02 + 0.008265e^{5.2(M - 0.5)}] \quad (4.3)$$

with C_{NBC} taking the sign of $\alpha + \alpha_B$, and where $p = 1$ if $(\alpha + \alpha_B) > 17.5^\circ$

and $p = 0$ otherwise. The values of C_{NBC} at $M = 0.5$ hold for $0 \leq M < 0.5$.

Finally,

$$\Delta C_{NB} = -2 \left(1 - \left(\frac{D_b}{D} \right)^2 \right) [1 - \sin^{0.6}(\alpha + \alpha_B)] \sin(\alpha + \alpha_B) \cos(\alpha + \alpha_B) - \frac{l_a}{D} \left(1 - \frac{D_b}{D} \right) \sin(\alpha + \alpha_B) |\sin(\alpha + \alpha_B)|. \quad (4.4)$$

Within the program the cross-flow normal force coefficient C_{NBC} , as plotted in Figure 3, is stored in digitised form for greater accuracy than that provided by Equation (4.3).

The body axial force coefficient, C_{AB} , will typically take the form $C_{AB0} \cos^2(\alpha + \alpha_B)$ where C_{AB0} is the value when $(\alpha + \alpha_B) = 0$. The value of C_{AB0} may be estimated using Item Nos 68020, 76033, 77020, 78019, 78041, 80021 and 96012 (References 32 to 39) for example, or an experimental value may be used. No allowance is made for the effect of wing-body interference and any axial force arising on a tapering afterbody at high incidence as a result of suction due to trailing forebody vortices is also neglected.

The direct body contributions to lift and drag are then

$$C_{LB} = C_{NB} \cos(\alpha + \alpha_B) - C_{AB} \sin(\alpha + \alpha_B) \quad (4.5)$$

and

$$C_{DB} = C_{AB} \cos(\alpha + \alpha_B) + C_{NB} \sin(\alpha + \alpha_B). \quad (4.6)$$

In Item No. 95025 for the plane wing, on the basis of acceptable agreement with experimental data, the body contributions are simply added to the gross wing contributions, subject to some modification of the wing viscous drag terms. The spanwise integrations employed for the cambered and twisted wing facilitate some further amendment of the gross wing contribution prior to addition. In Equations (3.8) and (3.13) for C_{DW} and C_{LW} the local incidence α_l is taken as $\alpha + \alpha_B$ across the wing-body intercept. To allow for the loss of suction across the intercept the lower limit of integration is set at η_B instead of zero in the fifth term of Equation (3.8) for C_{DW} , in the second term of Equation (3.13) for C_{LW} , and in the integrals in Equations (3.24) and (3.25) for C_{Dlev} and C_{Llev} . The lower limit of zero is *retained* in the third term of Equation (3.8) as a result of theoretical and empirical studies that showed this to be essential.

The final allowance to be made concerns the lift-curve slope. In the calculation of $c_l c / \bar{c}$ (Equation (3.9)), α_0 (Equation (3.11)) and C_s (Equation (A3.8)) it is preferable to retain use of the gross wing lift-curve slope because loading and suction distributions are made initially for the gross wing, with subsequent allowance for the body as described above. But for C_{LW} (Equation (3.13)), there is a direct use of the wing lift-curve slope and it is more suitable there to replace a_1 by an effective lift-curve slope, a_{1eff} , based on the net wing, which is the normal practice for the low aspect ratio wings. Therefore the lift-curve slope, a_{1N} , is calculated for the net wing which has aspect ratio

$$A_N = \frac{(1 + \lambda)(1 - \eta_B)A}{1 - (1 - \lambda)\eta_B + \lambda} \quad (4.7)$$

and taper ratio

$$\lambda_N = \frac{\lambda}{[1 - (1 - \lambda)\eta_B]}, \quad (4.8)$$

the wing sweep being unaltered.

This is then multiplied by $(1 + \eta_B)^2$ to apply slender-body interference factors and is corrected to the gross wing reference area, so that

$$a_{1eff} = a_{1N} (1 + \eta_B)^2 S_N / S_W \quad (4.9)$$

where

$$\frac{S_N}{S_W} = \frac{(1 - \eta_B)(1 - (1 - \lambda)\eta_B + \lambda)}{1 + \lambda}. \quad (4.10)$$

With the modifications above, Equation (3.8) for the wing drag coefficient becomes

$$\begin{aligned} C_{DW} = & C_{DPmin} + \Delta C_{Dvisc}(S_N/S_W) \\ & - \int_0^1 \sigma(c_s c / \bar{c}) \cos \Lambda_0 \cos \varepsilon_0 \sec \alpha_l d\eta + \int_0^1 (c_l c / \bar{c}) \tan \alpha_l d\eta \\ & + \int_{\eta_B}^1 \sigma(c_s c / \bar{c}) \cos \Lambda_0 \sin(\alpha_l - \varepsilon_0) \tan \alpha_l d\eta \\ & + C_{DC} + C_{Dlev} + C_{Dsev}. \end{aligned} \quad (4.11)$$

In Equation (4.11), following the treatment in Item No. 95025, the value of C_{DPmin} is to be estimated as the minimum profile drag of the wing outboard of the body but based on the gross wing area S_W ; ΔC_{Dvisc} is calculated from Equation (3.6) and is corrected, as shown, by the area ratio S_N/S_W to form a value appropriate to the wing outboard of the body.

For the wing lift coefficient, Equation (3.13) becomes

$$C_{LW} = a_{1eff} \sin(\alpha - \alpha_0) \cos^2(\alpha - \alpha_0) + \int_{\eta_B}^1 \sigma(c_s c / \bar{c}) \cos \Lambda_0 \sin(\alpha_l - \varepsilon_0) d\eta + C_{Llev} + C_{Lsev} \quad (4.12)$$

These contributions to wing drag and lift are added to the body contributions to form the wing-body lift and drag coefficients

$$C_D = C_{DW} + C_{DB} \quad (4.13)$$

and

$$C_L = C_{LW} + C_{LB} \quad (4.14)$$

The factor K_{lcs} which is entered in the program input file adjusts the lift-curve slope in C_{LW} in the way described in Section 3.3.2.1. See Section 7.3.7.

5. ACCURACY AND APPLICABILITY

5.1 Accuracy

Predictions of the method have been compared with experimental data from Derivations 25 to 31. Sketches 5.1 to 5.8 illustrate results for a number of the configurations examined. At the top left and top right of each sketch, respectively, the camber and the twist distributions are shown. At bottom left the drag polar of C_D against C_L is shown and at bottom right the lift curve of C_L against α is displayed.

Sketch 5.1 shows results for a delta wing of aspect ratio 4 with camber and twist distributions designed to achieve a trapezoidal distribution of spanwise loading. Sketch 5.2 shows results for a delta wing of aspect ratio 2 with the same camber and twist distributions. Sketch 5.3 shows a second delta wing of aspect ratio 2 with camber and twist distributions designed to achieve an elliptical spanwise loading. In each case the prediction of C_D and C_L is good.

Sketch 5.4 shows an aspect ratio 2.5 swept wing with a taper ratio 0.4, where again the design spanwise loading is elliptical. The prediction of the drag polar is good until there are noticeable inaccuracies in the prediction of C_L .

Sketch 5.5 shows an aspect ratio 6 swept wing of taper ratio 0.4 with a constant camber distribution but without twist. Again the prediction of the drag polar is good until the prediction of C_L is at fault.

Sketch 5.6 shows results for a conically-cambered, slightly-cropped delta wing of aspect ratio 2.3; here the predicted lift coefficient falls below the experimental value at high C_L with a consequence that C_D is over-predicted in that region.

Sketch 5.7 shows results for a straight-tapered wing of aspect ratio 3 with conical camber and a leading-edge sweep of 45° . Once more the prediction is good whilst the C_L prediction remains satisfactory.

A final illustration is provided in Sketch 5.8, which compares results for an aspect ratio 2.8 wing with a leading-edge sweep of 50° and a taper ratio of 0.2. Good results are obtained for both a planar wing and a non-planar wing, with camber and twist distributions which give, at a Mach number of 0.5, a lift coefficient decrement of about 0.065 at constant α .

From the above and other comparisons it is concluded that there is usually a reasonable prediction of drag polars up to $C_L = 0.7$ provided that C_L is predicted accurately. Sketch 5.9a illustrates the overall accuracy of prediction for non-planar wings by plotting the difference between predicted and experimental values of C_D against C_L for $C_L = 0.3, 0.5$ and 0.7 . Generally the errors in C_D are only slightly worse than $0.01 C_L$ in magnitude for $0.3 \leq C_L \leq 0.7$, the same accuracy level that was achieved for planar wings using the method of Item No. 95025, see Sketch 5.9b reproduced from that Item. Application of the method of this Item to the planar-wing data base produces very similar results to those in Sketch 5.9b.

The model predictions of the individual drag contributions from C_{DC} and the suction terms were also checked against the theoretical thin-aerofoil and lifting-line predictions for an elliptically loaded wing with chordwise-symmetric parabolic camber in Reference 42 in order to examine their relative contributions.

5.2 Applicability

Table 5.1 shows the ranges of parameters covered in the data base of experimental results for non-planar wings. Some of the ranges are more restricted than those for planar wings in Item No. 95025. This is due to the smaller number of experimental data available, but the method is expected to apply in the parameter ranges for which good results have been obtained for planar wings, which are given in Table 5.2 and may be used to extend those of Table 5.1. The method makes use of the 'equivalent wing' concept when applied to wings that are not straight-tapered but it should not be applied in cases where the wing leading-edge or trailing-edge sweep angles change by more than 20° at a crank.

Note also that when establishing the database for predicting attainable suction the work in Derivation 24 included theoretical studies for a wide range of leading-edge radius index, $0 \leq r_{i6} \leq 1.2$, supported by experimental aerofoil section data for a range of Reynolds number $10^6 \leq R \leq 30 \times 10^6$. This suggests that the method can be used with confidence for normal-section parameters r_{in} and R_n within those ranges.

The method was aimed at application to configurations with wings of low to moderate aspect ratio, $A \leq 6$ say, and thin section where there is a significant loss of leading-edge suction from low to moderate values of lift coefficient. It is not primarily intended for application to high aspect ratio cambered wings of thicker section designed to develop almost all of the theoretical leading-edge suction over a wide range of lift coefficient, although acceptable results have been obtained. For wings with $\Lambda_{1/4} \leq 45^\circ$ and $0.1 \leq t/c \leq 0.2$, ESDU 07002 (Reference 40) provides a prediction method for wing profile drag in such cases, which is suitable for hand calculation, and this may be combined with a traditional trailing-vortex drag contribution to form the total drag coefficient. Item No. 88030 (Reference 38) may be used to make an initial estimate of the range of linear characteristics for a cambered and twisted wing, within which the method of ESDU 07002 will apply.

For wings with highly-swept leading edges that lead to the generation of vortex lift the point of action of the centroid of that force, and hence its orientation, θ , as determined by the local surface slope of the wing, see Sketch 3.5, can significantly affect the drag polar. The point of action of the vortex lift is modelled through Equation (3.3) which has proved satisfactory for the database studied. Ignoring the vortex lift orientation can lead to poor predictions in some cases, as illustrated in Sketch 5.10 which compares the prediction for the configuration of Sketch 5.3 with that which would result if the vortex lift was assumed always to act normal to the local chord line ($\theta = 0$). See also the comments in Appendix C on the effects of vortex migration and vortex breakdown ahead of the wing trailing edge.

As explained in Section 3.2 of Item No. 95025, the limiting pressure coefficient, C_{plim} , plays a crucial role in determining the accuracy of prediction of the drag polar. The correlation established in Derivation 24 was achieved by fitting a lower bound (as opposed to a mean line) to a database of limiting pressure coefficients inferred from experimental axial force data on aerofoil sections. This bound would correspond to the maximum possible attainable suction and Derivation 24 advises that predictions which use the fit tend to represent idealised conditions, *i.e.* smooth surfaces and stable uniform flow with little turbulence ahead of the wing. The programmed version of the method has provision for the limiting pressure parameter, $F (= -\gamma C_{plim})$ to be decreased by the use of a factor K_σ set in the input file, see Section 7.3.6. A value of unity will produce the lowest C_D at a given C_L . A lower value of K_σ , perhaps as low as 0.5, might be needed in the interpretation of some experimental data (where it is available), as part of an investigation into minor configuration alterations. This could allow calibration of the changes in the predicted drag polar against the effects of a reduction in attained suction caused perhaps by boundary-layer transition at low Reynolds number or Mach number. Sketch 5.11 shows the improvement in prediction that can be achieved, using as an example data from Derivation 26 for a plane wing. In general, it may always be advisable for the user to investigate some small change ($\sim 10\%$) in K_σ to test the sensitivity of the drag polar to the attainable suction model.

The programmed version of the method also provides for the inclusion of a factor K_{lcs} in the input file, see Section 7.3.7. This factor multiplies the lift-curve slopes a_1 and a_{1eff} that are calculated within the program and provides a mechanism whereby the user may match the predicted lift characteristics at low α to known experimental data, if they are available.

Similarly, factors K_{lev} and K_{sev} may be entered via the input file to weaken or remove the lift and drag contributions of leading-edge and side-edge vortices and so investigate their influence, see Section 7.3.8.

TABLE 5.1

$2 \leq A \leq 6$	$0.17 \leq M \leq 0.6$	$0.085 \leq M_n \leq 0.45$
$38^\circ \leq \Lambda_0 \leq 64^\circ$	$1.6 \times 10^6 \leq R \leq 8 \times 10^6$	$0.5 \times 10^6 \leq R_n \leq 8 \times 10^6$
$0 \leq \lambda \leq 0.4$	$0.03 \leq t/c \leq 0.08$	$0.06 \leq t_n/c_n \leq 0.11$
$-12^\circ \leq \varepsilon \leq 2^\circ$	$0.0006 \leq \rho/c \leq 0.006$	$0.005 \leq \rho_n/c_n \leq 0.013$
$-20^\circ \leq \varepsilon_0 \leq 0$	$0.27 \leq r_i \leq 0.32$	$0.27 \leq r_{in} \leq 0.36$

TABLE 5.2

$1.7 \leq A \leq 6$	$0.05 \leq M \leq 0.6$	$0.035 \leq M_n \leq 0.45$
$0 \leq \Lambda_0 \leq 64^\circ$	$1.2 \times 10^6 \leq R \leq 14 \times 10^6$	$0.5 \times 10^6 \leq R_n \leq 7 \times 10^6$
$0 \leq \lambda \leq 1$	$0.03 \leq t/c \leq 0.10$	$0.03 \leq t_n/c_n \leq 0.19$
	$0.0005 \leq \rho/c \leq 0.0076$	$0.001 \leq \rho_n/c_n \leq 0.055$
	$0.15 \leq r_i \leq 0.90$	$0.15 \leq r_{in} \leq 0.47$