

RESPONSE OF STRUCTURES TO VORTEX SHEDDING. Structures of Circular or Polygonal Cross Section

1. NOTATION AND UNITS

		Units	
		SI	British
$\tilde{C}_{L0}$	local r.m.s value of $\tilde{C}_L$ for stationary cylinder ($\eta \rightarrow 0$)		
$\tilde{C}_L$	local r.m.s value of component of fluctuating lift or across-flow force coefficient dependent, in general, on oscillation amplitude; $(\tilde{F}/(\frac{1}{2}\rho V_z^2 D_z H))$. (Note that the force acts normal to both mean flow direction and cylinder axis.)		
$\tilde{C}_{Lj}$	mode-generalised $\tilde{C}_L$ (see Equation (5.9))		
D	cylinder diameter	m	ft
D_m	integrated mean diameter over structure length	m	ft
$\tilde{F}$	fluctuating lift or across-flow force due to vortex shedding	N	lbf
f_ϕ	factor on C_D for yawed structure, see Section 4.3.4		
g_D	ratio of expected maximum value to r.m.s. value of parameter (see Section 5.2)		
H	height or length of structure	m	ft
I_u	intensity of along-wind component of turbulence		
J	number of modes contributing significantly to response		
j	mode number; $j = 1$ corresponds to first or fundamental translational mode		
K_j	mode-generalised stiffness; $(2\pi n_j)^2 M_j$	N/m	lbf/ft
$^x L_u$	longitudinal integral length scale of along-wind component of turbulence	m	m
$^y L_u, ^z L_u$	lateral integral length scales of along-wind component of turbulence	m	ft
M_j	mode-generalised structural mass; $H \int_0^1 m \mu_j^2 d(r/H)$	kg	* slug
m	local mass per unit length of complete structure (see Section 7.2)	kg/m	* slug/ft

For footnotes refer to end of Notation Section

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m_{ej}	equivalent mass per unit length for mode j ; $m_j / \int_0^1 \mu_j^2 d(r/H)$	kg/m	*slug/ft
m_j	mode-generalised mass per unit length for mode j ; $\int_0^1 m \mu_j^2 d(r/H)$	kg/m	*slug/ft
N	index describing mode shape		
n	frequency	Hz	Hz
n_j	natural frequency of mode j	Hz	Hz
n_s	frequency of vortex shedding	Hz	Hz
Q_j	mode-generalised across-flow deflection amplitude for oscillation in mode j (measured from equilibrium position); equal to deflection where $\mu_j = 1$	m	ft
q	local across-flow deflection amplitude	m	ft
q_j	local across-flow deflection in mode j ; $\mu_j Q_j$	m	ft
R	edge radius of polygonal section	m	ft
Re	Reynolds number, VD/ν		
Re_e	effective Reynolds number; $\lambda_r \lambda_R Re$ (see Section A3.1 of Appendix A)		
r	distance along axis of structure; from base for cantilever structure (see Sketch 1.1)	m	ft
r'	distance along axis of structure from tip	m	ft
Sc_j	mode generalised Scruton number; $m_j \zeta_s / (\rho D_m^2)$		
S_{CF}	mode-generalised cross-spectral density of fluctuating side force coefficient	1/Hz	1/Hz
S_{Fj}	mode-generalised cross-spectral density of fluctuating side-force at frequency n	N ² /Hz	lbf/Hz
S_{CL}	spectral density of $\tilde{C}_L$	1/Hz	1/Hz
St	local Strouhal number; $n_s D_r / V_z$		
T	time interval during which expected maximum value occurs	s	s
t	time	s	s
V	flow speed averaged over specified period (see Section 4.1)	m/s	ft/s
V_c	flow velocity corresponding to condition when $n_s = n_j$	m/s	ft/s

For footnotes refer to end of Notation Section

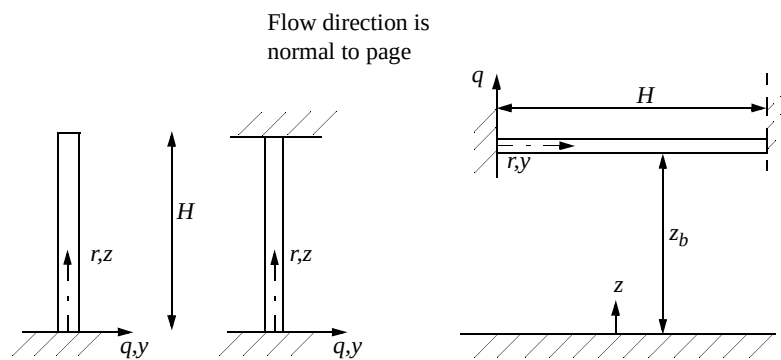
x, y, z	system of right-handed, rectangular, Cartesian co-ordinates with x -axis defined in mean flow direction and z -axis normal to ground or surface		
Y_j, y, y_j	values Q_j, q, q_j respectively for a vertical, cantilever structure	m	ft
z	height above ground or surface [†]	m	ft
γ_{CL}	narrow-band co-coherence of $\tilde{C}_L$ at points r_1 and r_2 at frequency n_s		
ϵ	equivalent roughness height of cylinder surface	m	ft
ζ_{aero}	aerodynamic damping parameter, see Section 5.1.1		
ζ_s	structural damping parameter; ratio of actual value to critical value of absolute damping coefficient		
η	Q_j/D_m or Y_j/D_m		
λ_R	parameter defining effect of surface roughness on Re_e		
λ_T	parameter defining effect of flow turbulence on Re_e		
λ_{Tcrit}	value of λ_T at Re_{crit}		
μ_j	mode-shape function describing deflected shape of structure oscillating in mode j		
ν	kinematic viscosity of air (or gas) flow	m ² /s	ft ² /s
ρ	density of air (or gas) flow	kg/m ³	*slug/ft ³
ϕ	yaw angle (see Section 4.3.4)	degree	degree
ϕ_e	effective yaw angle for inclined structure (see Section 4.3.5)		
ω_j	circular frequency, $2\pi n_j$		
<i>Subscripts</i>			
B	refers to value associated with broad-band, random-amplitude, response (see Section 4.1)		
c	refers to value corresponding to $n_s = n_j$		
H	refers to value at r or z equal to H		
j	refers to value for mode j		
max	refers to expected maximum value occurring in a time interval of T		
N	refers to value associated with narrow-band, constant-amplitude response (see Section 4.1)		

For footnotes refer to end of Notation Section

r	refers to value at distance r along structure axis from base
ref	refers to value at reference point where $\mu_j = 1$
rms	refers to root-mean-square value
z	refers to value at height z
0	refers to value for stationary cylinder (<i>i.e.</i> $\eta \rightarrow 0$)
$2d$	refers to value for two-dimensional flow

* 1 slug = 32.174 lb mass.

† Note that, in Items giving wind-speed information, z is measured from the zero-plane displacement height, d , typically 1 or 2 metres below the general level of surrounding obstacles. See ESDU 82026 for further information.



Sketch 1.1

2. INTRODUCTION

When the wind or any fluid flows past a structure of circular cross section, vortices can be shed alternately from each side and this will induce fluctuating, across-flow forces on the structure. If a natural frequency of the structure is close to the vortex shedding frequency, the structure may respond dynamically and large across-flow oscillation amplitudes can result. Large-amplitude oscillations (sometimes with catastrophic results) have been observed particularly in steel structures such as stacks and structural elements such as tie rods but concrete structures such as stacks may also experience appreciable amplitudes. Even when a structure does not oscillate significantly, vortex-induced forces do occur and they should be taken into account in the overall design process.

This Data Item is concerned with the estimation of the oscillation amplitudes induced by vortex shedding from isolated, flexible structures of circular or polygonal cross section. The overall method is more advanced than that presented in ESDU 85038¹⁷ (now superseded by this Item) in that the new model is developed from a much wider data base and the effects of various parameters are defined in more detail. A single integrated method is provided that can be applied to any type of structure of circular or polygonal cross-section such as parallel-sided, tapered or stepped-diameter type. Because of the detailed nature of the model, hand calculation procedures are not practical and the means of implementation of the overall method is through a computer program provided with this Item (see Section 8).

Interference effects associated with groups of cylinders (*e.g.* a line of chimney stacks) are not considered here but ESDU 79025¹⁰ provides a discussion of these effects together with a guide to data available.

In view of the implementation of the method via a computer program, the purpose of this Data Item is to provide background information on response to vortex shedding and on the derivation of the overall method. The basic applicability and limitations of the methods presented are outlined in Section 3. Some background information on the nature of the response to vortex shedding is given in Section 4. The derivation of the methods of response estimation is outlined in Section 5. The tentative extrapolation of the method to structures such as masts or lighting standards having a polygonal cross-section (*e.g.* octagonal) shape with rounded corners is described in Section 6. Methods of reducing large amplitude responses are described in Section 7 which also explains how the computer program may be used to estimate the response of a structure fitted with an effective helical strake.

The computer program is described briefly in Section 8. A description of the derivation of the data for the various parameters used in the computer model (which includes the effect of buffeting by the lateral component of turbulence) is given in Appendices A and B. A summary of Equation fits to the data is given in Appendix C.

Comparisons of computed and measured response data are given in Section 9.

3. APPLICABILITY AND LIMITATIONS OF THE METHODS

The main application of this Data Item is to structures of circular cross section in the atmospheric wind, such as chimney stacks, circular towers and masts, suspended pipelines and members of composite structures such as tie rods. Other applications may include lighting standards and other structures with eight or more equal sides as described in Section 6. The data may also be applied to isolated tubes in duct flow provided that the blocked area is less than 10 per cent of the duct cross-sectional area. However, the method cannot be applied reliably to structures such as guyed masts or very long-span cables where multiple modes of vibration at closely spaced natural frequencies contribute significantly to the overall response due to vortex shedding. As an approximate guide it is suggested that the frequency ratio, n_{j+1}/n_j , should be greater than about 1.5.

Ancillary components, such as pipes or ladders, attached to circular section structures can cause appreciable asymmetry of the flow around the main structure. Application of the data in this Item to such structures will be less reliable unless the ancillary component lies wholly within the wake of the main structure. Because of the asymmetry caused, addition of ancillary components to circular-section structures may also lead to a response due to galloping.

The method is applicable to cylindrical bodies or structures of circular or polygonal cross section in air or gas flow only. In the case of more dense fluids such as liquids (or more explicitly, when the ratio of the fluid density to the structure density is significantly higher, such as for a steel tube in water flow) the effect of the added mass of the liquid entrained with the oscillating cylinder produces significant forces, out-of-phase with the motion velocity, that must also be taken into account. This information is outside the scope of the present Item.

For wind flows in particular, additional across-flow forces (buffeting) arise due to the transverse components of turbulence and these are included in the computer model provided. In many (but not all) cases, at the critical flow speed at which the vortex-induced forces are a maximum, the across-flow response due to atmospheric turbulence is small. Conversely, at much higher than critical wind speeds, response to turbulence may be more important.

Significant improvements have been made to the method for estimating the off-critical response (at conditions away from that giving the maximum response) over that presented in ESDU 85038.

Cylinders that are yawed (*i.e.* inclined forwards or backwards to the oncoming flow) can be treated (see Section 4.3) but the data on which the method is based are more limited in this case.

Appendices A and B contain a summary of the correlations and sources of data used for each component part of the method. The data on which the method is based cover the Reynolds number range up to about 10^7 and a range of cylinder surface roughness conditions. Extrapolations have been made to cover as wide a range as possible of situations likely to arise in practice. Most of the available test data for oscillating cylinders relate to either rigid spring-mounted ($\mu = 1$) cylinders in two-dimensional flow or cantilever cylinders oscillating in their fundamental translational mode. The methods presented have been extrapolated for other mode shapes.

3.1 Uncertainties in Computed Response

It is not possible to provide simple quantitative guidance concerning the uncertainty of calculated responses. If the response is of the random amplitude type the comparison with those full-scale measurements considered to be the most reliable and other wind-tunnel data suggests that the uncertainty in the peak rms amplitude will be about 15% to 20%. If the response prediction shows that multiple solutions of the narrow-band type (See Section 5.1.2) exist then the structure is more unstable and the uncertainty in the

calculated values is likely to be greater (on average, about $\pm 30\%$). However, from a design point of view it is desirable to avoid configurations that give rise to such instabilities and the use of devices such as helical strakes (see Section 7) is recommended in such cases.

At off-critical wind speeds ($V/V_c < 0.5$ and $V/V_c > 2$) the paucity of reliable measurements makes it difficult to assess the overall reliability of the method and this can only be judged by the comparisons shown in Section 9.

4. THE NATURE OF THE RESPONSE TO VORTEX SHEDDING

The following description applies generally to gas flows but in practice, because most applications relate to air flow, the term wind is used hereafter.

In the (wind) flow about a circular cylindrical structure, except at very low Reynolds numbers, vortices are usually shed alternately* from each side of the body (see Sketch 4.1). This fluctuating asymmetry induces regions of low pressure alternately on each side of the body, resulting in a net across-flow (lift) force which alternates in direction as vortices are shed, and an along-flow (drag) force which fluctuates in magnitude only. The shedding of pairs of vortices (one from each side) may be either almost periodic (*i.e.* of constant frequency and amplitude) or random, depending on the Reynolds number and the turbulence properties of the incident flow and on the amplitude of the across-flow oscillations. The frequency of shedding (n_s) is conveniently represented in non-dimensional form by the Strouhal number, $St = n_s D/V$. For circular cylinders, St is of the order of 0.2 but it varies with Reynolds number and other parameters on which n_s is dependent.

For a cantilever structure the nature of the flow in the tip region is particularly important and this is described in detail in Section A3.2 of Appendix A.



Sketch 4.1 Alternate narrow-band vortex shedding at subcritical Reynolds number

4.1 Across-flow Oscillation

As wind speed, V , increases from zero, the dominant frequency of vortex shedding from a body or structure increases until a critical flow velocity, V_c , is reached when n_s is close to a natural frequency of the body, n_j . At this critical flow speed (when $n_s \approx n_j$), a significant amplification of the across-flow forces occurs and large amplitude across-flow oscillations can result, particularly for lightweight structures and those with low structural damping.

The amplitude of the induced oscillations is dependent on the mass-damping parameter $m_{ej} \zeta_s / (\rho D^2)$ and when this is greater than about 2.5 it is unlikely that the induced motion will exceed a few per cent of the diameter, although fluctuating forces induced by vortex shedding will still be present.

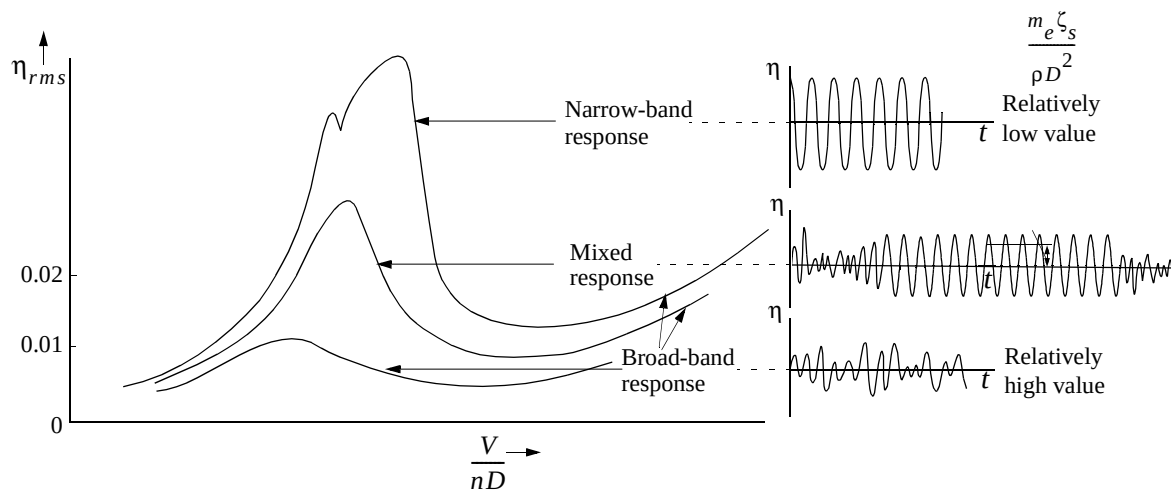
The motion of the body has a significant influence on the strength of the vortices shed and the forces induced by them, which in turn affect the oscillation amplitude. This 'aeroelastic' or 'feed back' effect† controls the form of the response and the methods presented in this Data Item take this phenomenon into account.

When the amplitudes of oscillation are small (less than a few per cent of the diameter) the induced

* For structures oscillating in liquid flows, symmetrical vortex shedding has been observed. This will give rise to fluctuating drag forces but no lift forces. References 8 and 9 give some further information.

† This aeroelastic effect is different from that leading to wake galloping or wake flutter response in groups of cylinders. These phenomena are described in ESDU 79025¹⁰.

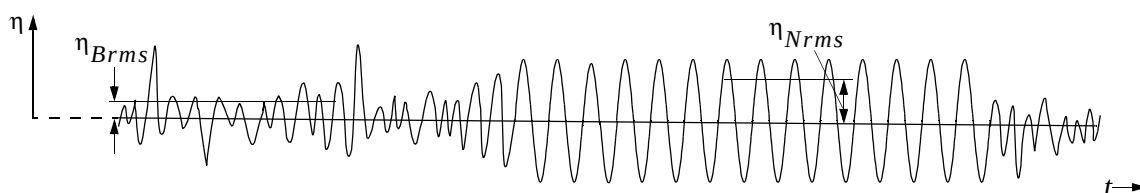
fluctuating forces are generally random in nature, with significant energy distributed over a relatively broad band of frequencies centred on n_s . In this case, the distribution of the vortex-induced fluctuating forces along the length of a cylinder is also disorganised, which means that the maximum value of the fluctuating force does not occur simultaneously at all positions along the cylinder. The degree of coherence of the forces can be characterised by a length scale, L_s (see Section A7. of Appendix A) which is representative of the spanwise distance along the cylinder within which the induced forces are significantly correlated with each other. The smaller the value of L_s , the smaller will be the effective total force acting on the body at any given time. For small-amplitude oscillations, the ratio L_s/H is comparatively small. The resulting response (termed a 'broad-band' response) is of a random amplitude nature at approximately the body frequency n_j (see Sketch 4.2).



Sketch 4.2 Illustration of broad-band and narrow-band response

For larger amplitudes of oscillation, the local forces due to vortex shedding are amplified. In such cases, the motion of the structure tends to cause the shedding frequency, n_s , to “lock-in” to the body frequency, n_j , over a range of local flow velocities, with the effective forces showing a maximum at $V_c = n_j D / St$. The fluctuating forces at various sections along the structure in the lock-in region are in phase with the body motion and thus completely correlated with each other, further enhancing the response. These forces are nearly periodic (sinusoidal) in nature, with significant energy confined to a relatively narrow band of frequencies centred on the body frequency, n_j . The resulting response (termed a “narrow band” response) is of an almost constant-amplitude sinusoidal nature at approximately the frequency n_j (see Sketch 4.2).

To a large extent, therefore, the body motion controls the form of response (either broad-band or narrow-band) that occurs. For sufficiently large oscillation amplitudes, the response will take the form of a constant amplitude sine wave. For smaller amplitudes, the response at the critical velocity is more likely to be of random amplitude at a near-constant frequency. Under some circumstances, the conditions can be such that either form of response can occur. The amplitude may then change, over irregular time periods, from being random and generally small to being sinusoidal and large³⁷ (see Sketch 4.3).



Sketch 4.3 Illustration of switching between broad-band and narrow-band types of response

In the atmospheric wind, and other turbulent shear flows, this change in the form of the response to vortex shedding may also be observed for another reason. The magnitude of the response will vary according to how close the mean flow speed (which is varying with time) is to V_c . In the atmospheric wind there will be gusts of duration of the order of 10 to 20 seconds for which the mean gust speed is equal to V_c and within which the turbulence will be of relatively small-scale nature. A constant-amplitude narrow-band response is possible (depending on the value of $m_{ej}(\zeta_s/(\rho D^2))$ in particular), providing the gust duration is sufficiently long for the oscillation amplitudes to build up to their maximum steady-state values in a time period of approximately $1/(2\pi \zeta_s n_j)$ seconds (Ruscheweyh⁶⁴) and large-scale turbulence can be considered to manifest itself as a slowly varying mean flow speed. For flow speeds different from the critical gust speed the response will be “off-critical” and will change to being small and random in nature.

4.2 Along-flow Oscillation

Alternate vortex shedding produces a fluctuating drag force that, for very lightly damped structures, can induce oscillations in the direction of the mean flow (see, for example, Auger³ and Turnbull¹⁵). The period of this excitation is associated with the shedding of each separate vortex; *i.e.* the response will be a maximum when $n_j D/V = 2St \approx 0.4$. In air (or any other relatively low density fluid) the vortex-induced along-flow oscillations will rarely be significant. In any case, the across-flow excitation will almost certainly be more severe except in the case when the critical flow speed, V_c , for across-flow motion is greater than the design or maximum flow speed; in this case the along-flow motion at a flow speed of $\frac{1}{2}V_c$ may be important. Turnbull¹⁵ discusses the origins and nature of along-flow oscillation due to vortex shedding.

4.3 Calculated Response for Various Types of Structure

The principal types of structures covered by the present method are

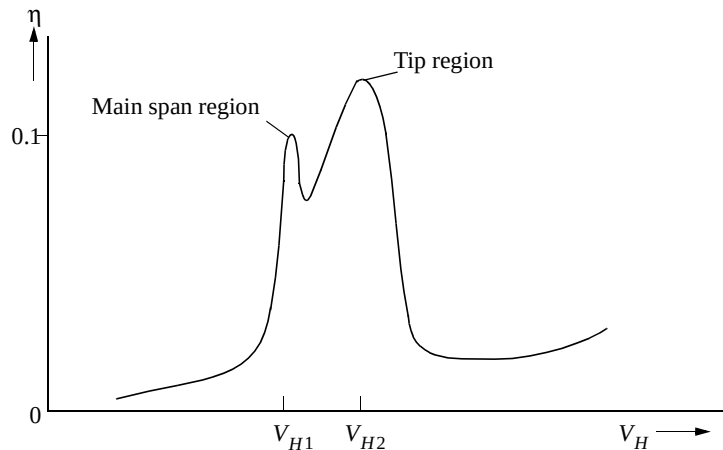
- (i) parallel-sided structures,
- (ii) tapered structures (linear or non-linear in shape),
- (iii) structures with step changes in diameter.

Additionally, each of these types of structure may be essentially two-dimensional with no tip effects (e.g. a structural member of a lattice tower) or a cantilever (e.g. a tall chimney). For a cantilever structure, the effect of the flow around the free end is influential in creating a distinctive region of vortex shedding within the top few diameters; this region exhibits different characteristics from the shedding over the main span region away from the tip. This is described in more detail in Section A3.2 of Appendix A.

In calculating the overall response, the local force spectral density on each element is associated with a local Strouhal number and hence shedding frequency. In the case of a cantilever structure, for a random-amplitude response (Equation A4.4) it is assumed that there is no significant correlation between fluctuating forces occurring on pairs of elements when one element is in the tip region and the other is in the main span region. For a narrow-band response (Equation (5.8)), motion effects cause the shedding frequency to lock into the natural frequency, providing the ratio n_j/n_s is within certain bounds (see Section B5.1 of Appendix B). Separate criteria apply to the tip and main span regions but in this case, because the forces are in phase with the motion, all regions exhibiting lock-in effects are completely correlated with each other.

4.3.1 Parallel-sided structure

In practice, for a parallel-sided structure, it is found that the high values of $\tilde{C}_{L0}$ in the tip region usually

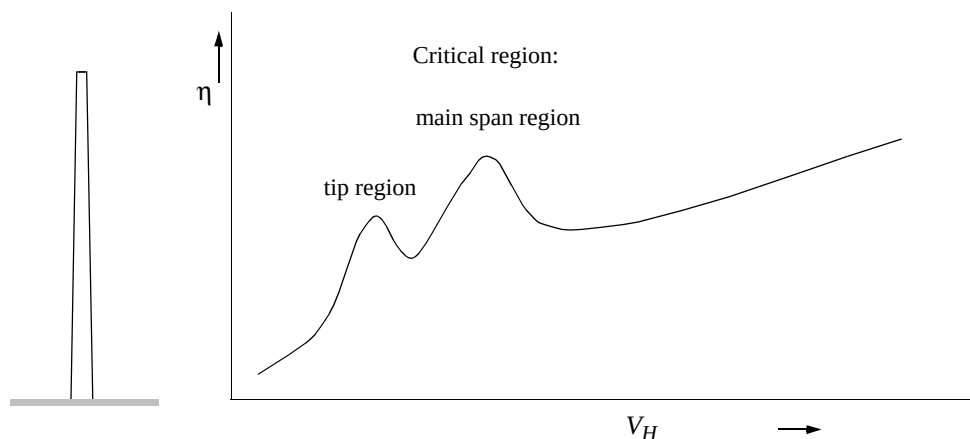


Sketch 4.4 Illustration of possible double-peak response for a parallel-sided cantilever structure

provide the major contribution to the overall response. In some cases, mainly where larger-amplitude lock-in conditions prevail, two peaks in the response occur, one at a value, $n_j D/V_{H1}$, corresponding the Strouhal number over the main span region and the other at a lower value, $n_j D V_{H2}$, corresponding to a lower Strouhal number (St_H) in the tip region.

4.3.2 Tapered structure

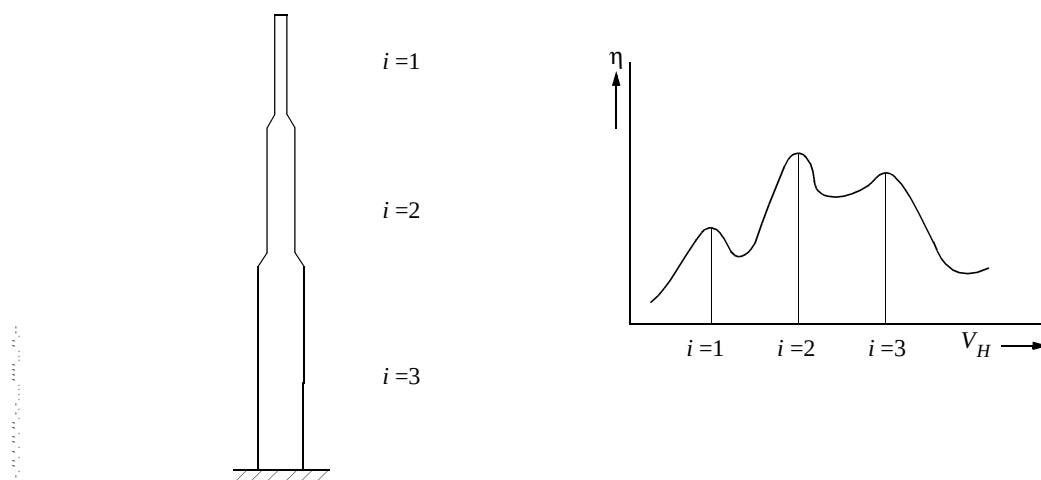
For a tapered structure, the critical region from which the major contribution to the maximum response originates tends to be further from the tip, but this depends on the degree of taper and wind shear. For wind speeds lower than that giving the maximum response, the critical region will move towards the tip as illustrated in Sketch 4.5.



Sketch 4.5 Illustration of possible response for a tapered structure

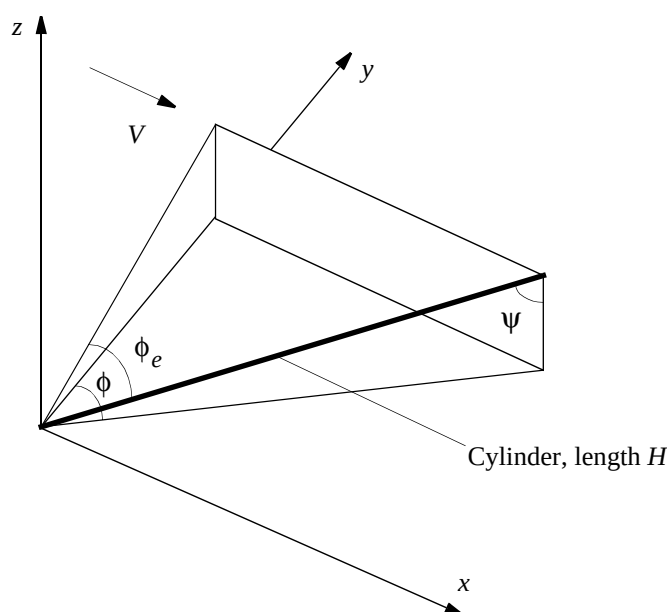
4.3.3 Structure with step changes in diameter

For a structure with step changes in diameter, such as illustrated in Sketch 4.6, a multi-peaked response can be obtained with each peak being associated with critical conditions occurring within each span of constant diameter.



Sketch 4.6 Illustration of response for a stepped-diameter structure

4.3.4 Yawed structures



Sketch 4.7 Yawed structure

In the general case, if the inclination of the cylinder axis to the vertical is defined as Ψ when the wind direction is normal to the cylinder axis (i.e. when $\phi = 0$, see Sketch 4.7) then, as ϕ increases, the effective yaw angle, ϕ_e , will be related to ϕ and ψ by

$$\sin \phi_e = \sin \phi \sin \psi. \quad (4.1)$$

For a cylinder which is yawed at an angle ϕ_e to the oncoming flow, data⁷⁶ show that, for $10^3 < Re < 10^5$, the drag force coefficient can be taken as being dependent on $V \cos \phi_e$, the component of free-stream velocity normal to the cylinder axis, and on the streamwise component of Reynolds number, $VD/(v \cos \phi_e)$. Further data^{69, 83, 95} suggest that the same approximation can be applied for the across-flow response of circular cylinders at subcritical Reynolds numbers. Thus the method can be adapted to apply to yawed,

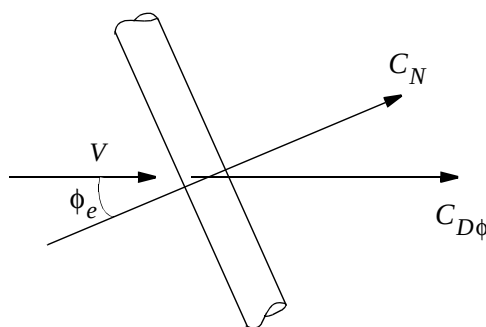
parallel-sided cylinders by substituting

$$V_c = n_j D (St \cos \phi_e) \quad (4.2)$$

$$\text{and} \quad Re = V_c D / (\nu \cos \phi_e). \quad (4.3)$$

For subcritical Reynolds number ($Re \leq 3 \times 10^5$) the calculation procedures in Section 6.1 can be used for yaw angles up to $\pm 45^\circ$. However, there are few data for higher Reynolds numbers with which to check the assumptions of the simple cosine rule. For the mean drag force on a yawed cylinder, ESDU 80025⁷⁶ provides a correction factor, f_ϕ , (based on experimental evidence) for the higher Reynolds number range which is greater than $\cos^2 \phi_e$ for smooth cylinders and tends to $\cos^2 \phi_e$ for rough cylinders (when conditions similar to subcritical Reynolds number prevail). The force coefficient normal to the cylinder axis is given by

$$C_N = (C_D)_{\phi=0} f_\phi \quad (4.4)$$



Sketch 4.8 Yawed cylinder

noting that all force coefficients are based on V^2 and D . Similarly, because there is an approximately linear relationship between $\tilde{C}_{L02d}$ and C_{D2d} in the range of interest (see Sketch 6.2) it is a reasonable assumption, in the absence of other information, that

$$(\tilde{C}_{L0})_\phi = (\tilde{C}_{L0})_{\phi=0} f_\phi. \quad (4.5)$$

In ESDU 80025 a relationship for f_ϕ for $Re_e > 3 \times 10^6$ is given in terms of the cylinder surface roughness parameter, ϵ/D . To generalise this relationship over the whole Reynolds number range, the expression for f_ϕ has been converted to an approximately equivalent expression dependent on C_{D2d} rather than ϵ/D . The resulting expression for f_ϕ is

$$f_\phi = \cos^2 \phi_e - \frac{1}{2} (f_{\phi 1} - \cos^2 \phi_e) f_{cd} \quad (4.6)$$

$$\text{where} \quad f_{\phi 1} = \cos^2 \phi_e [1 + 1.5 (1 - \cos^3 \phi_e)] \quad (4.7)$$

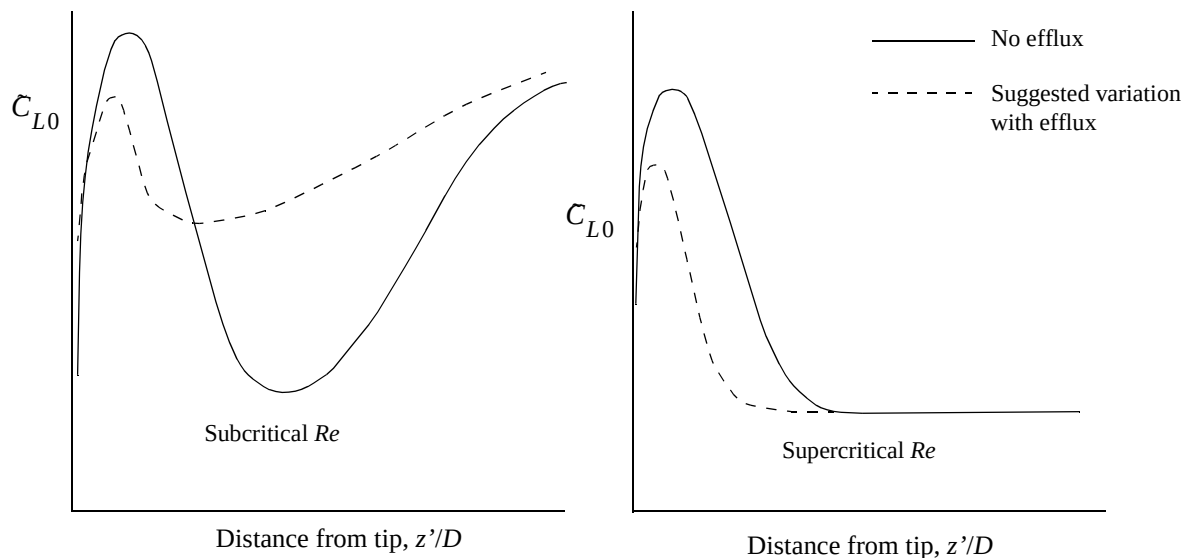
$$\begin{aligned} \text{and} \quad f_{cd} &= 4(C_{D2d} - 1) \\ &= 0 \quad \text{if } C_{D2d} > 1 \\ &= -2 \quad \text{if } C_{D2d} < 0. \end{aligned} \quad (4.8)$$

The local fluctuating side force coefficient normal to the cylinder axis can then be expressed as $\frac{1}{2} \rho V^2 D \tilde{C}_{L0} f_\phi$ but, because of tip effects, it is recommended that this procedure is restricted to structures with H/D greater than about 15 and for values of ϕ_e less than about 45° .

4.4 Effect of Efflux

The present method applies to the case where there is no effluent plume. It is difficult to quantify the effect of efflux from a chimney on its response to vortex shedding. Wind-tunnel tests⁵⁰ show that, at subcritical Reynolds numbers ($Re_e < 10^5$), the effect of efflux is to increase the response. At higher (supercritical*) Reynolds numbers this effect is likely to be reversed, as explained in the following paragraphs.

Gould et al³² suggest that the effluent plume generates another pair of vortices just above the top of the chimney of opposite sign to the original pair present in the tip region (see Sketch A3.2) in the absence of efflux. As the efflux velocity increases, the upper pair of vortices cause a displacement and weakening of the original pair. The apparent effect of an efflux is thus to raise the height at which the peak loading (in relation to C_D and $\tilde{C}_{L0}$) would be experienced to a level closer to the top of the chimney. Sketch 4.9 illustrates this effect at (i) subcritical Reynolds numbers and (ii) supercritical Reynolds numbers.



Sketch 4.9 Effect of efflux on $\tilde{C}_{L0}$ in tip region

At subcritical Reynolds numbers, the hypothesis would suggest that the overall effect of efflux is to increase the response due to the general increases in $\tilde{C}_{L0}$ where the weakening of the tip effect allows the quasi two-dimensional region of relatively high $\tilde{C}_{L0}$ to extend further towards the tip.

At supercritical Reynolds numbers, the opposite effect is likely because $\tilde{C}_{L0}$ is reduced near the tip due to the extension of the quasi two-dimensional region where $\tilde{C}_{L0}$ is much smaller than at subcritical Reynolds numbers. The data of Gould et al³² demonstrate this result for local C_D but no data for $\tilde{C}_{L0}$ are available. However, it is observed in practice that if a chimney is seen to oscillate then increasing the efflux by opening the flues will generally reduce the oscillation amplitude.

Full-scale chimneys are sufficiently large that the supercritical Reynolds number case will nearly always apply. Thus it is likely to be conservative to assume that the no-efflux condition presents the more critical design case (for both along-wind and across-wind response).

* At wind speeds corresponding approximately to the maximum response, supercritical Reynolds numbers will apply at full scale if H/D_{mean}^2 is less than about 20; this will apply to the majority of full-scale chimneys.

5. RESPONSE EQUATIONS

This Section outlines the derivation of the equations used to calculate the across-flow response to vortex shedding. The derivation of the empirical correlations of the parameters used in the equations is described in the Appendices.

The basic assumption of the method is that the forces on the cylinder consist of two contributions.

- (i) A contribution that exists in the absence of motion and is characterised by statistical properties such as spectral density and coherence functions.
- (ii) A contribution that is entirely coherent with the motion of the cylinder. This is a non-linear function of amplitude of motion but, for small amplitudes, it will be assumed proportional to amplitude.

At intermediate amplitudes, the total force is assumed to be obtained by adding these contributions. It seems likely that the random contribution (i) is progressively suppressed as the amplitude increases. This, in essence, represents the vortex shedding 'lock-in' effect. As the amplitude increases, the proportion of the force from the motion-dependent contribution becomes more and more dominant. In the detailed model a factor, which is a function of amplitude, is applied to the random contribution to represent the supposition that it is progressively suppressed.

5.1 The Response Equation

The equation of motion for a cylinder oscillating in mode j can be written in mode-generalised terms as

$$[-\omega_j^2 M_j + i\omega_j \zeta_j 2\sqrt{M_j K_j} + K_j]Q_j = \tilde{F}_j. \quad (5.1)$$

The definition of mode-generalised quantities is explained in ESDU 77032⁷.

5.1.1 Broad-band response

In the case of relatively broad-band forces induced by vortex shedding, the modal force, $\tilde{F}_j$, for small to moderate amplitudes, can be considered to be a combination of components that are in-phase with the motion and those which are independent of the motion. As shown originally by Wootton³⁷ and later by Tunstall⁵⁰ and subsequently adopted by Vickery and Basu^{13, 14}, the local modal force can be represented by

$$\tilde{F}_j = \tilde{F}_0 + \mu_j (\frac{1}{2} \rho V^2 DL) k \frac{\mu_j Q_0}{D} \quad (5.2)$$

where

$$k = \frac{d\tilde{C}_L}{d(Q_0/D)} = \frac{d\tilde{C}_L}{d\eta} \quad (5.3)$$

is a constant (for given conditions) for small to moderate amplitudes. The terms in k consist of real and imaginary components. The real part of k (out of phase with the motion) behaves like a contribution to the stiffness giving a response frequency a little different from the resonant frequency; this is small and is neglected. The in-phase component behaves as a negative contribution to the overall damping in which

form it is denoted by ζ_{aero} . Equation (5.1) can now be re-expressed in the form

$$[-\omega_j^2 M_j + i\omega_j^2(\zeta_s + \zeta_{aero})M_j + \omega_j^2 M_j]Q_j = \tilde{F}_0 \quad (5.4)$$

where ζ_{aero} is usually negative due to aerodynamic excitation from vortex shedding and is given by

$$\zeta_{aero} = -\frac{\rho V^2 \mu_j^2 k}{4m_j \omega_j}. \quad (5.5)$$

In practice, k is found to be a function of the local value of $\tilde{C}_{L0}$ (see Section B3 of Appendix B) and ζ_{aero} is then obtained from an integration of Equation (5.5) over the structure length.

The solution of the equation of motion for small to moderate amplitude is given by

$$\sigma_y^2 = \sum_{j=1}^J \frac{\pi \mu_j^2}{4(\zeta_s + \zeta_{aero})} \frac{n_j S_{Fj}}{(M_j \omega_j^2)^2} \quad (5.6)$$

where J is the number of modes contributing significantly to the response and S_{Fj} is the spectral density of the fluctuating force due to vortex shedding. In turbulent flow, an additional low frequency component to S_{Fj} arises due to buffeting by the lateral component of turbulence and this is included in the present method as described in Section A9. of Appendix A. Equation (5.6) can be re-expressed in terms of non-dimensional parameters, based on conditions at the reference point (where $\mu_j = 1$), as

$$\left(\frac{\sigma_{yB}}{D}\right)_{ref} = \eta_B = \frac{(n_j S_{CF})^{1/2}}{16\pi^{3/2}} \frac{\rho D_{ref}^2}{(\zeta_s + \zeta_{aero})^{1/2}} \left(\frac{V_{ref}}{n_j D_{ref}}\right)^2. \quad (5.7)$$

For a vertical cantilever structure, D_{ref} is the integrated mean diameter and V_{ref} corresponds to V_H . The derivation of the spectral density of the fluctuating side force coefficient, S_{CF} , is described in Section A4. of Appendix A.

5.1.2 Narrow-band response

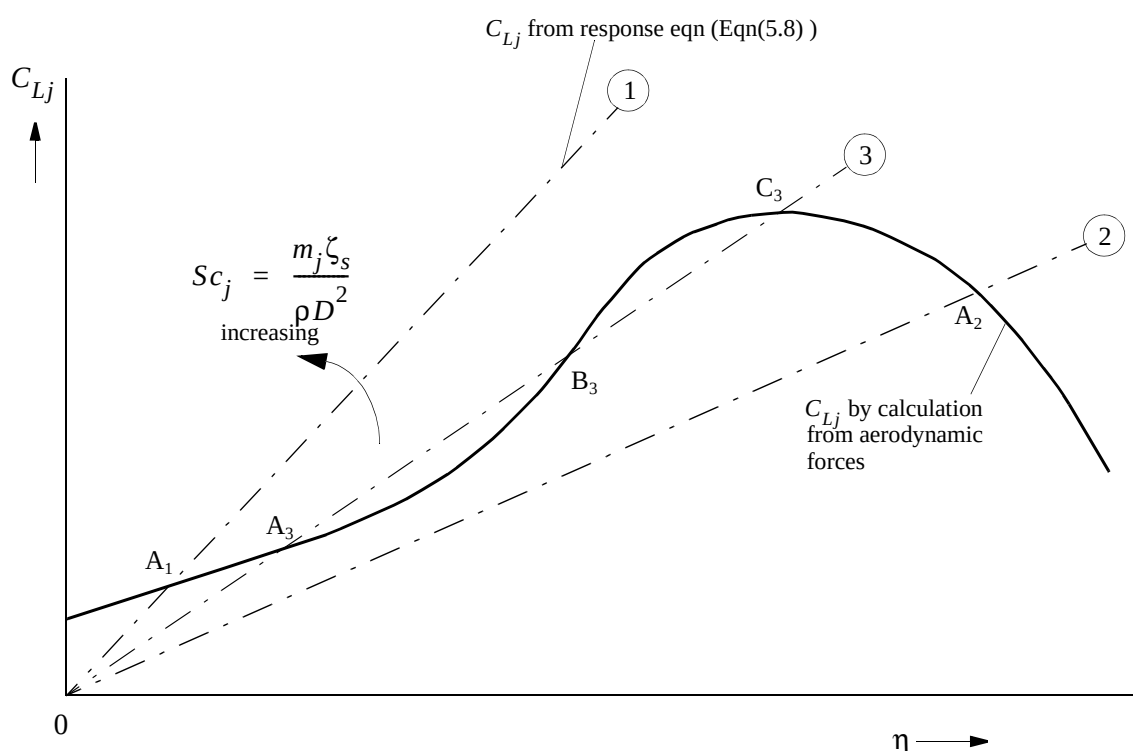
For larger amplitudes, when the condition of 'lock-in' (described in Section 3) occurs, the spanwise forces due to vortex shedding within the lock-in region are in phase with each other and a sinusoidal, constant amplitude response at $n = n_j$ is assumed from which

$$\begin{aligned} \sigma_{yN}^2 &= \frac{S_{Fj}}{4\zeta_s^2 K_j^2} = \frac{\int_0^H (\frac{1}{2}\rho V^2 D \tilde{C}_{Lj} \mu_j)^2 dr}{4\zeta_s^2 (2\pi n_j)^4 M_j^2}, \\ \left(\frac{\sigma_{yN}}{D}\right)_{ref} &= \eta_N = \frac{1}{16\pi^2} \frac{\rho D_{ref}^2}{m_j \zeta_s} \left(\frac{V_{ref}}{n_j D_{ref}}\right)^2 \tilde{C}_{Lj} \end{aligned} \quad (5.8)$$

where C_{Lj} is the mode-generalised fluctuating force coefficient (a non-linear function of η) given by

$$\tilde{C}_{Lj} = \frac{1}{H} \int_0^H \frac{D_z}{D_{ref}} \left(\frac{V_z}{V_{ref}} \right)^2 \mu_j \tilde{C}_L dr. \quad (5.9)$$

The derivation of $\tilde{C}_{Lj}$ is described in Section B5 of Appendix B. In general, $\tilde{C}_{Lj}$ is a non-linear function of the oscillation amplitude so that an iterative method of solution is required to solve Equation (5.8). A graphical solution is illustrated in Sketch 5.1. Note that, at large oscillation amplitudes, $\tilde{C}_L$ decreases with increasing amplitude. The process is thus self limiting, with the maximum attainable oscillation amplitude being of the order D .



Sketch 5.1 Narrow-band (sinusoidal) response

The full line in Sketch 5.1 represents the variation of $\tilde{C}_{Lj}$ with η using the model of the aerodynamic forces due to vortex shedding derived in Section B5 of Appendix B. The initial slope of this curve is linear for two-dimensional flow but this is modified for a cantilever structure when the spanwise extent of the lock-in region is taken into account as noted later. The chain dotted lines represent the linear relation of Equation (5.8) for different values of mode-generalised Scruton number, $Sc_j = m_j \zeta_s / (\rho D^2)$ and the intersection of those lines with the full curve gives possible solutions for the resulting amplitude, η .

For a large value of Sc_j , the resulting amplitude (A_1) is small. For a small value of Sc_j , the resulting amplitude (A_2) is large. For the intermediate value of Sc_j , three solutions are possible (A_3 , B_3 , C_3). In practice it is usually found that the intermediate solution (B_3) is unstable and the alternative solution, C_3 , is more likely to be triggered. In this range of Sc_j it is clear that small variations of the structural parameters can cause large changes in the resulting response. **Thus, where more than one solution is obtained by the present method, the oscillation must be considered unstable and there will be large uncertainties in the solutions obtained.** The safest procedure is to assume that the largest of the narrow-band amplitude

solutions prevails and this criterion is assumed in the computerised method.

In the case of three-dimensional flow (*e.g.* cantilever structures in a shear flow) the spanwise extent of any lock-in region is dependent on η , tending to zero as η tends to zero. In this case, the variation of C_{Lj} given by the curve A1 A3 B3 C3 will decrease to zero as $\eta \rightarrow 0$ and the narrow-band solutions at the intersections such as A1 and A3 may not arise. This provides a practical criterion for determining whether low-amplitude narrow-band solutions occur (in addition to a broad-band solution). The computerised method automatically takes this consideration into account.

5.2 Computed Response, Maximum Values

The maximum response is given by

$$\eta_{max} = g_D \eta \quad (5.10)$$

where η is the rms response amplitude and g_D is a peak factor which for a random-amplitude (broad-band) response is given by

$$g_D = \sqrt{2 \log_e(n_j T)} + \frac{0.577}{\sqrt{2 \log_e(n_j T)}} \quad (5.11)$$

$$\approx 4$$

and for a narrow-band (sinusoidal) response

$$g_D = \sqrt{2}.$$

As noted in Section 4.1, the variation of response amplitude with time may contain a mixture of random-amplitude and narrow-band types of response. In such cases, the method will predict at least two solutions, η_B (random amplitude) and η_N (a larger sinusoidal response amplitude). The overall rms response in this case will be given by an expression of the form

$$\eta = [f_t \eta_N^2 + (1 - f_t) \eta_B^2]^{1/2} \quad (5.12)$$

where the factor f_t represents the proportion of time during the period T for which the response in time T will be of the narrow-band type, so that f_t tends to unity as η_N increases. From the data of Wootton³⁷ (smooth flow) and that of Kwok and Melbourne⁷⁸ (turbulent flow), approximate values of f_t are given by the following equations which allow for the fact that, with increasing turbulence intensity, the increased unsteadiness in the flow will reduce the tendency to lock-in conditions.

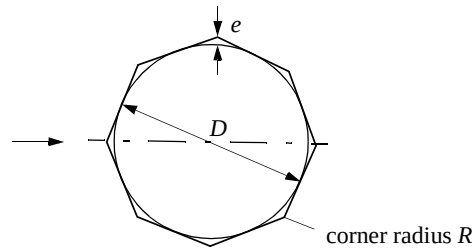
$$f_t = c_t \left(\frac{\eta_N}{\eta_B} - 2 \right) \text{ or } 0 \text{ whichever is greater,} \quad (5.13)$$

$$\text{where } c_t = 0.125 [1 + \exp(-7.6 \times 10^4 I_{uH}^{3.6})]. \quad (5.14)$$

These relationships are built into the computer model.

In the case of a mixed response, the maximum value is given by the greater of $4\eta_B$ or $\sqrt{2} \eta_N$.

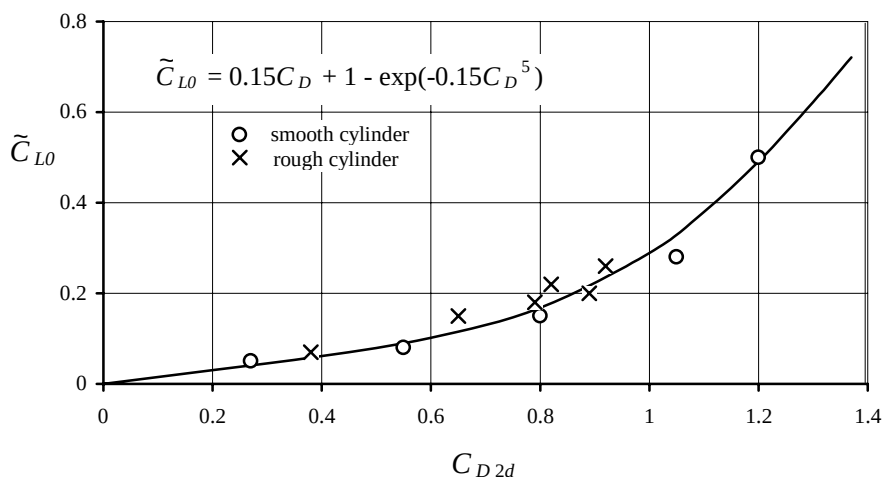
6. APPLICATION TO STRUCTURES OF POLYGONAL CROSS SECTION



Sketch 6.1 Polygonal section

In ESDU 79026, values of C_D are presented for cylinders with regular polygonal cross-sectional shapes with one corner pointing symmetrically into the oncoming flow. These data were derived by equating the polygonal shape (with rounded corners) to an equivalent circular cylinder with added roughness. A relationship is derived to calculate the effective roughness ratio, e/D , from the number of sides and the corner radius ratio, R/D , where D is the diameter of the inscribed circle. The effect of turbulence on the effective Reynolds number, Re_e , is based on that used for a plain circular cylinder. The method is calibrated using measured data from various sources for polygonal cross-sectional shapes.

To predict the response to vortex shedding of structures with polygonal cross sections, values of $\tilde{C}_{L0}$ are required. There are no known systematic data from which such values can be deduced. However, it is known that, for particular classes of shapes, a general relationship can be established between $\tilde{C}_{L0}$ and C_D for two-dimensional flow. For circular cylinders, a general relationship between C_D and $\tilde{C}_{L0}$ is given in Sketch 6.2 based on data from this Data Item and ESDU 80025⁷⁶.



Sketch 6.2 Correlation between $\tilde{C}_{L0}$ and C_D for two-dimensional circular cylinders

Because of the similarity in cross-sectional shape, values of $\tilde{C}_{L0}$ for polygonal sections could be expected to collapse onto the same general correlation curve as in Sketch 6.2. This is assumed in the present method but the applicability is limited to polygonal sections with eight or more equal sides.

For structures of polygonal cross section, the computer program for calculating the response to vortex shedding requires as additional input the number of sides and the corner radius. Parameters such as Re_e , the effective surface roughness and C_{D2d} are calculated by the program based on relationships derived in ESDU 79026. All other parameters in the computer model, with the exception of $\tilde{C}_{L02d}$ which derives from the relationship shown in Sketch 6.2, are then assumed to be the same as for a circular cylinder of the same effective surface roughness.

It must be emphasised that there are no measured data with which to calibrate this procedure but it is expected to give reasonable results. However, the uncertainty in the calculated response cannot be established.

It is recommended that the method should be used only for cross sections with eight or more sides and for aspect ratios, H/D , greater than 10.

7. METHODS OF REDUCING RESPONSE DUE TO VORTEX SHEDDING

If the response calculation indicates the possibility of occurrence of vortex-induced oscillations of unacceptable magnitude, consideration must be given to methods of reducing those oscillations. The following Sections summarise various methods of reducing the severity of oscillations due to vortex shedding. They can be classified under two main headings. Firstly, changes in the structural properties (natural frequency, mass and damping) and, secondly, changes in shape or the addition of aerodynamic spoilers (such as strakes or shrouds).

7.1 Changes in Natural Frequency, Mass or Damping

An increase in the natural frequency (or diameter D) will increase the critical flow speed, $V_c = n_j D / St$, since St is approximately constant (although it depends on Re and the oscillation amplitude). If V_c is greater than the maximum flow velocity expected to occur (e.g. the maximum design wind speed), oscillations due to vortex shedding will not generally be a problem. The natural frequency can be increased by increasing the stiffness but this should be used with caution since often the mass will be increased also, with the net result that there is no significant change in n_j . Although an increase in the diameter of a stack will also increase the natural frequency, this method will often be insufficient to put the critical wind speed above the maximum design wind speed.

An increase in non-structural mass (*i.e.* mass that does not change the stiffness of the structure) can be used to decrease the amplitude of oscillation. This technique has been used, for example, to reduce oscillations of a tubular railway gantry tower¹ and also to inhibit vibration in pin-jointed tie rods by filling them with sand⁶ (in this case the sand would have tended also to increase the damping). Note that although the structural damping coefficient C_j is unchanged by an increase in non-structural mass, the damping parameter ζ_j is decreased since $\zeta_j = C_j / (4\pi n_j M_j)$, which is proportional to $1/\sqrt{M_j}$, assuming that the stiffness remains unchanged.

It is important to note that the reduction of natural frequency brought about by increasing the mass may reduce the frequency of a higher mode of oscillation. The critical flow speed for this mode may then come within the range of flow speeds which the structure will experience. However, if the mass is incorporated at the nodal point* of, say, the second mode, the natural frequency of that mode is unaffected whilst the first mode natural frequency is reduced.

An increase in structural stiffness can be obtained using guy wires. However, this method may not eliminate oscillations completely and may introduce new problems through oscillations of the guys themselves. The use of slack guy wires can increase the damping of the structure considerably.

Other devices for increasing the damping are described in Section 4.9 of ESDU 77032⁷. Those methods suitable for decreasing vortex-induced oscillations of stacks, towers and masts include direct viscous dampers, impact dampers, tuned dampers^{11,16} and structural dampers such as gunite linings and resilient damping pads between joints at the base between the steel work and foundations, or at the primary support point. The use of high-hysteresis ropes for guys will increase the effective damping as will also the insertion of viscous dampers (shock absorbers) in the guy wires. The guys must be connected away from the nodal points of the mode of interest.

7.2 Changes in Shape, Aerodynamic Spoilers

Modifications to the shape are usually acceptable only at the early design stage of a structure whereas the addition of aerodynamic spoilers can, if necessary, be considered even after a structure is in service.

* A nodal point is a point along a structure at which the oscillation amplitude in a particular mode is always zero.

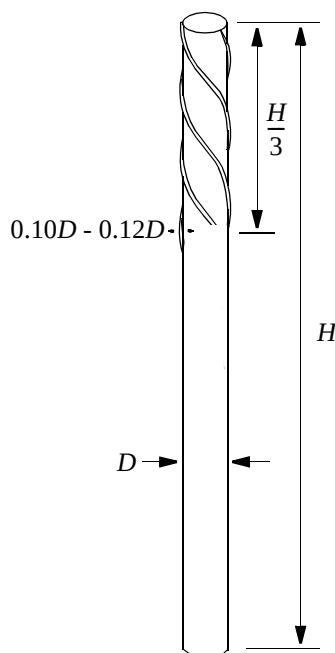
Increasing the diameter of a structure will increase the critical flow velocity (since $V_c = n_j D/St$) and, for cantilever structures, will decrease the aspect ratio, H/D . A decrease in aspect ratio can often be accompanied by an increase in stiffness but will, in any case, reduce the severity of vortex-induced oscillations. A decrease in aspect ratio to a value less than about 8 will tend to suppress vortex-induced oscillations.

Because of the effects of taper reducing the coherence of vortex-induced forces, tapered cantilever structures tend to experience smaller responses to vortex shedding than do parallel-sided structures.

Aerodynamic spoilers are intended to reduce the strength of the vortex-shedding forces. In many cases, optimum design is critical and non-optimum devices will be ineffective or may even aggravate the problem. Some spoilers increase the mean drag coefficient which will lead to increased buffeting response. Those which introduce asymmetry into the structure cross-section may lead to problems with response due to galloping.

An extensive review of aerodynamic spoiler devices intended to reduce response to vortex-shedding, including some unsuccessful designs, is given in Reference 12. There are three main types that can be effective in reducing vortex-induced oscillations. They can be described in general terms as strakes, shrouds and fins.

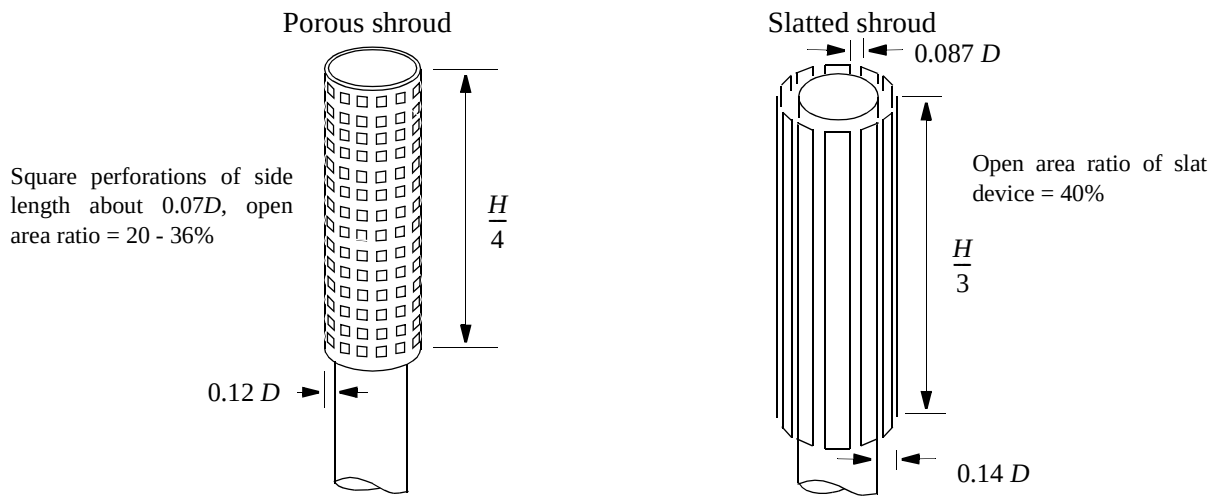
The first type of spoiler is the helical strake, which is widely used in preventing vortex-induced oscillations of steel stacks. An effective design²⁵ is illustrated in Sketch 7.1. This is effective in reducing the correlation



Sketch 7.1 Helical Strake

of the vortex-induced forces along the upper part of the stack, thereby reducing the excitation forces. The use of wire ropes wound helically around a stack is often proposed. Data reviewed in Reference 12 suggest that wires with diameter of not less than about $0.06D$ are required, with a pitch of $8D$ to $16D$, rather than

the $5D$ for sharp-edged strakes. An extent of at least $H/2$ is required. Some non-optimum helical wire windings were shown to drastically increase the response, so the method should be used with caution.



Sketch 7.2 Porous shroud and Slatted shroud

The second type of spoiler is the porous shroud. An effective design³³ of a porous shroud in which the perforations must be square is shown in Sketch 7.2. There is evidence⁴ that this design of shroud also improves the efflux dispersal when the ratio of efflux velocity to free stream velocity is less than unity. For some shell-like structures, perforations can be conveniently incorporated into the shell (e.g. the upper 25 per cent of the outer shell of a multi-flue stack). An alternative type of shroud design, consisting of a number of slats disposed around the cylinder⁷⁰, is shown in Sketch 7.2.

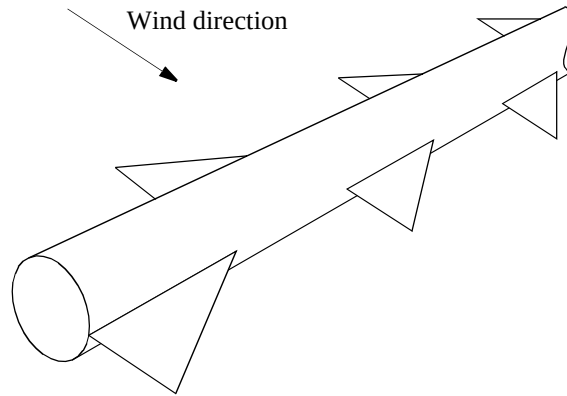
It should be noted that, to be effective, the strake or shroud must extend into the region where the maximum excitation originates. For tapered structures this may require it to extend lower than $H/3$ from the tip and, for stepped structures, strakes or shrouds may be required on the lower sections as well as, or sometimes instead of, the upper, smallest-diameter section.

A comparison of the drag coefficient (C_D) for the strake and shroud systems is shown in Table 7.1 (further information may be found in References 12 and 76.) It should be noted that although at subcritical Reynolds numbers ($Re_e < 3 \times 10^5$), C_D for the shrouded cylinders is less than for a plain cylinder, at higher Reynolds numbers the reverse is true. At high Reynolds number, therefore, the response due to turbulence buffeting will be increased when any of these devices is used.

TABLE 7.1

Device	C_D (based on D) for infinitely long device	
	$Re < Re_{crit}$	$Re \approx 3 \times 10^6$
Plain smooth cylinder	1.2	0.6
Helical strake	1.3	1.3
Porous shroud	0.9	0.8
Slatted shroud	1.1	1.1

The third type of spoiler device takes the form of saw-tooth fins, fins or splitter plates that reduce vortex-induced forces by interfering with the exchange of fluid across the wake. Such devices are essentially uni-directional and are therefore effective only when the flow from one particular direction needs to be considered, *e.g.* heat exchanger tubes⁵ and pipe-line suspension bridges² (see Sketch 7.3). These devices may lead to problems with galloping by causing asymmetry of the flow around the tubes or pipes.



Sketch 7.3 Saw-tooth fins on a pipe-line suspension bridge

8. COMPUTER PROGRAMS

The methods described in this Data Item can, in practice, only be used via a computer program. Two versions are being made available in the form of

- (1) a module running in Microsoft Excel (v5.0 or later),
- (2) a FORTRAN code.

In developing the present overall method it was found convenient to use Microsoft Excel which provides the facility of linking with program modules in Visual Basic. All input and output data are written to an interactive spread sheet and the majority of the calculations are performed via the Visual Basic modules running in the background. It is easy and convenient to use because input data, intermediate calculations and final results can all be viewed (and printed out) at the same time. Input data can be changed within the spreadsheet and either single runs made (giving intermediate results of imported parameters) or batch runs made giving only the final response. The batch calculation is linked dynamically with a plotting facility so that, as results are computed (or re-computed), they are immediately plotted onto a linked graph.

The second program (yet to be issued) will be a FORTRAN code running in the ESDUview interface which provides a user-friendly front end for the handling of input/output files. The calculations are similar to those provided in the Excel version but a somewhat greater range of optional input data schemes are provided. The source code for this program will also be provided.

Instructions for running the Excel module, together with worked examples, are detailed in ESDU 96031¹⁸. A similar Item will be issued to support the FORTRAN program.