

DESIGN OF LAMINATED PLATES SUBJECTED TO IN-PLANE LOADS AND BENDING MOMENTS

1. INTRODUCTION, OUTLINE OF METHOD AND SOME NUMERICAL RESULTS

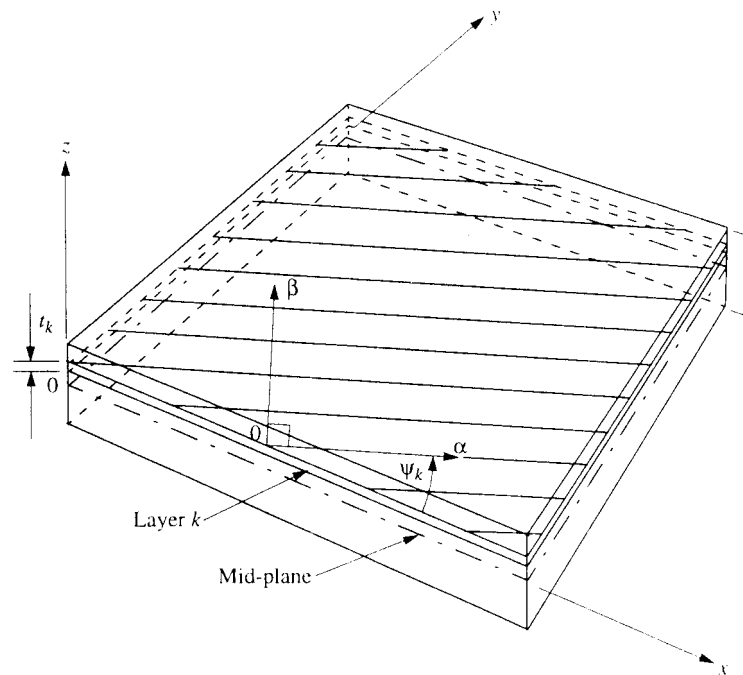
1.1 Introduction

This Item presents a Fortran computer program to determine the required thicknesses of laminated plates to withstand systems of in-plane loads and bending moments. The plates are of the type shown in Sketch 1.1, and are made from a number of thin orthotropic layers of continuous fibre reinforced composite with perfect bonding assumed between the layers. Using classical laminated plate theory, it is relatively straightforward to calculate the separate layer stresses for given layer thicknesses, angles and mechanical properties (see Data Item No. 94004^{*}). However, if the layer thicknesses are not specified at the outset then the problem of finding a set of thicknesses to withstand a given set of loads efficiently is far more difficult. It is this problem that forms the subject of this Data Item.

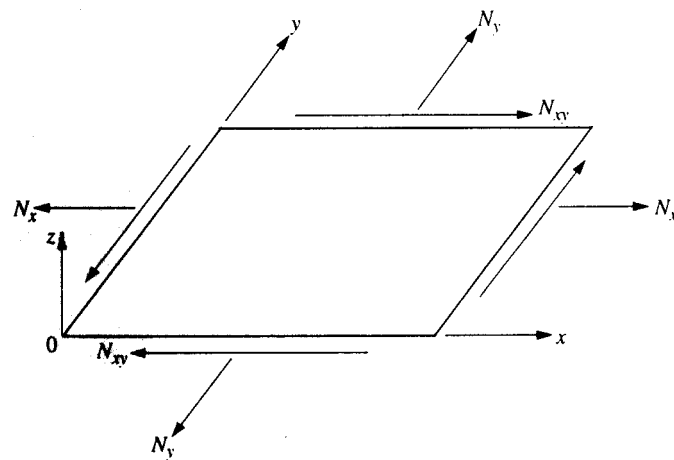
The computer program (described in Section 2) is based on the theory given in Derivation 1 and is restricted to plates that are symmetrical about the mid-plane of the plate. The layers may have different mechanical properties, but layer angles must be limited to 0 degrees, 90 degrees, +45 and –45 degrees. Layers at angles of +45 and –45 degrees occur as adjacent pairs, and the +45 degree layer must have the same mechanical properties as the –45 degree layer. Also, all layers of the same angle have the same thickness. A stacking sequence for the layer angles is specified at the beginning, and a design procedure is used to determine layer thicknesses for this lay-up. In this procedure, layers are thickened or thinned according to certain rules based on expectations of how best the layers will reinforce each other. These are referred to as heuristic design rules. If required the program will provide alternative lay-ups for comparison with the specified lay-up. Also, results are given that take into account practical ply thicknesses, where each layer consists of multiples of a specified standard ply thickness.

The positive directions for the applied in-plane loads and moments are shown in Sketches 1.1a and 1.1b and the design analysis can be carried out with any combination of these loads and moments. Plates can be designed to withstand up to 20 different load cases. A maximum allowable strain criterion is used to determine the position and mode of failure. The design iteration procedure is stopped when the most critical layer just reaches one of the allowable strain values of the material. It is not possible to ensure that all of the layers fail at the same time because of the complicated interaction between them when the thicknesses are changed. Generally there is no guarantee that the final design will be optimal, but Section 1.4.3 shows that the heuristic method agrees well with the optimisation techniques used in References 3 and 4.

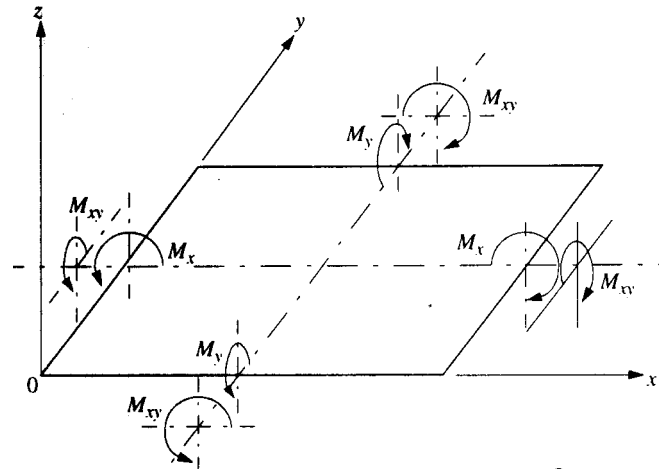
^{*} Data Item No. 94004 “Stress analysis of laminated flat plates”.



Sketch 1.1a Laminated plate geometry



Sketch 1.1b Positive applied force resultants



Sketch 1.1c Positive applied moment resultants Note that this convention is different from that used in Data Item No. 94003*

1.2 Notation

Both SI and lbf in units are quoted but any coherent system of units may be used.

C	"layer type" number in program input		
C_{max}	total number of "layer types"		
E_{α}, E_{β}	moduli of elasticity of layers in α - and β -directions respectively (E_{α} is modulus in fibre direction)	N/m ²	lbf/in ²
$G_{\alpha\beta}$	elastic shear modulus of layers in α - and β -directions	N/m ²	lbf/in ²
h	total plate thickness	m	in
k	layer identification number		
I_o	output from indicator		
L	load case number in program input		
L_{max}	number of load cases		
M_x, M_y	bending moments per unit length about y-axis and x-axis respectively (see Sketch 1.1c)	N m/m	lbf in/in
M_{xy}	twisting moment per unit length (see Sketch 1.1c)	N m/m	lbf in/in
N_x, N_y	in-plane direct loads per unit length (see Sketch 1.1b)	N/m	lbf/in

* Data Item No. 94003 "Stiffnesses of laminated flat plates".

N_{xy}	in-plane shear load per unit length (see Sketch 1.1b)	N/m	lbf/in
N	total number of layers in laminate		
n	number of alternative lay-ups		
R_1, R_2	minimum and maximum values of L for program output		
RF	strain failure reserve factor (= allowable strain/applied strain)		
t_k	thickness of k th layer	m	in
t_s	standard thickness of layer type	m	in
$O(x, y, z)$	plate main reference axes (see Sketch 1.1a)		
$O(\alpha, \beta, z)$	principal axes of orthotropy of a layer (see Sketch 1.1a)		
X	material properties test factor (see Table 2.3)		
γ	shear strain		
ϵ	direct strain		
κ	curvature of laminate	m^{-1}	in^{-1}
$\nu_{\alpha\beta}$	Poisson's ratio for layer resulting from load applied in α -direction with induced strain in β -direction		
σ	direct stress in layer, tensile positive, compressive negative	N/m^2	lbf/in^2
τ	shear stress in layer	N/m^2	lbf/in^2
ψ_k	angle of major principal axis of orthotropy, α_k , of k th layer measured from plate reference axis, Ox , (see Sketch 1.1a)	degree	degree
Suffixes			
a	allowable strain values		
c	compression strain values		
k	refers to k th layer		
0	refers to mid-plane of laminate		
t	tension strain values		
x, y, z	relates to x -, y - and z -directions respectively		
α, β	relates to α - and β -directions respectively		

1.3 Outline of Method

Details of the algebraic equations used in the computer program can be found in Derivation 1, and only an outline of the method of analysis is given here.

The laminated plate is assigned a particular stacking sequence for the layer angles, ψ_k , and an initial set of layer thicknesses is determined using a netting analysis where only the fibres in certain specified directions are assumed to carry a given set of in-plane loads and moments as described in Sections 1.3.1 and 1.3.2. The plate is then tested for failure. Depending on the mode of failure and where it occurs, specific heuristic re-design rules are invoked that either thicken or thin some layers to a certain extent. This new design is then tested for failure and the process is repeated until the set of thicknesses is such that a layer (or layers) is just on the point of failure at the prescribed loads (see Section 1.3.3).

For multiple load cases, an initial set of thicknesses is calculated for each separate load case. The lowest thickness value for each layer orientation is then selected from this initial set to form the starting point for re-design using the heuristic rules. A parallel design strategy is used in the analysis (Derivation 2) wherein the reserve factors for all load cases are considered together. In each redesign cycle the lowest reserve factor is found, and the appropriate layers are either thickened or thinned according to the heuristic rules. Like the single load case, this procedure is repeated until the lowest reserve factor is close to unity.

A corresponding design having the same lay-up but with the layer thicknesses rounded up or down (as chosen) is then provided (see Section 1.3.4). Alternatives having the same number of layers or fewer than a specified lay-up can also be obtained (see Section 1.3.5) as can a full layer-by-layer stress and strain analysis for each design (see Section 1.3.6).

1.3.1 Netting analysis for in-plane loads

This is a simplification of laminated plate theory where the individual layers are assumed to withstand the applied loads independently in their respective fibre directions. Thus, the 0° layers withstand the N_x load and the 90° layers withstand the N_y load. Additionally, since a pure shear load can be represented by direct tension and compression loads at $\pm 45^\circ$ to the direction of shear, the $\pm 45^\circ$ layers are assumed to withstand the shear load. In each case, account is taken of the different fibre strengths in tension and compression. Since, at this stage, no bending moments are applied, the thicknesses so obtained are independent of the stacking sequence and can be distributed in any way throughout the plate. In cases where layers of the same orientation have different mechanical properties, arithmetic mean values are used for each set of 0° , 90° and $\pm 45^\circ$ degree layers.

1.3.2 Initial analysis for moments

The situation with applied moments is more complicated than that with applied in-plane loads. This is because the strain in any ply depends on its distance from the neutral plane of the laminate as well as on the magnitude of the moments. The analysis is carried out in two stages. Firstly, an initial thickness is obtained for each of the 0° , 90° and $\pm 45^\circ$ layers by considering, in turn, single layers of the 0° and 90° fibre orientations and two of the $\pm 45^\circ$ layer pairs, and finding the thickness required for the relevant moment. Thus, in a manner similar to the load netting analysis, the 0° and 90° layers relate to the bending moments M_x and M_y , respectively, and the $\pm 45^\circ$ layers to the twisting moment M_{xy} .

In the second stage, the resulting thicknesses for the 0° , 90° and $\pm 45^\circ$ layers are used as the initial values of an iteration scheme which takes into account the layers' distances from the neutral plane, as well as the stiffnesses of all layers, in the evaluation of the layer strains. This is based on laminated plate theory. In general, these initial values will be too large because the bending stiffness of the resulting lay-up is increased when some layers are moved to their actual positions further away from the neutral plane. Since the thickness

of a layer determines the distance from the neutral plane of all others further from the neutral plane, an iterative procedure must be followed in order to derive the final moment analysis thicknesses. Details of this analysis are given in Section 2.2.2 of Derivation 1, but in essence three algebraic cubic equations are obtained for the three thicknesses corresponding to 0° , 90° and $\pm 45^\circ$ layers, and since they are interdependent they have to be solved in an iterative manner until a converged set of thicknesses is obtained. Due to the fact that the plate is now being analysed as a whole, it is sometimes found that a layer angle type (as determined from the first stage moment analysis) is no longer required. This is because the associated moment is being reacted adequately by the remaining layers. This situation arises with the numerical example discussed in Section 1.4.1 where, in the second stage of the moment analysis, the 90° layers that correspond with the M_y moment are found to have zero thickness and therefore play no further part in the plate design procedure. Finally, the layer thickness values that were obtained from the in-plane load netting analysis are then distributed evenly over the appropriate layer thicknesses from the moment analysis to form the initial plate design.

1.3.3 Redesign heuristics

The netting and initial moment analyses above provide a starting point for an iterative design procedure. Each iteration uses reserve factors obtained by classical laminate theory to establish a redesign using the various heuristic rules. The rules are represented by a set of conditions followed by an action. The conditions specify the fibre orientation of the layer (or layers) that has the lowest reserve factor, as well as some subsidiary conditions to do with the existence of layers of particular fibre orientations, whilst the action specifies the fibre orientation of the layers to be thickened or thinned. When the lowest reserve factor is less than unity, certain layers are thickened according to the reserve factor, in order to increase the strength of the plate, and vice-versa when the lowest reserve factor is greater than unity. Table 1.1 gives a list of the heuristic redesign rules that have been developed from Derivation 1. For example, Rule 1A states that if the lowest reserve factor occurs in a 90° layer in the transverse fibre direction and if 0° layers exist, then thicken or thin all the 0° layers.

Alternatively, Rule 1B states that if for the same condition as Rule 1A there are no 0° layers present, then thicken or thin all the 90° layers.

These rules were developed for plates made from a single material. For plates made from a number of different materials it is expected that the rules will still be effective providing the materials are not too dissimilar. It should be noted that if different materials are used, but with the same fibre orientations, then all layers at that orientation will have the same thickness irrespective of their stiffnesses. In this case the analysis may give results that deviate from minimum thickness values.

1.3.4 Rounding layers to standard ply thicknesses

When the laminate thickness for the applied loading has been derived using the method described in Sections 1.3.1 to 1.3.3, a corresponding laminate of the same lay-up but comprising layers that are each multiples of a specified standard ply thickness is determined. This is obtained by rounding the thickness of each layer of a particular orientation up or down, as specified, to the nearest multiple of a chosen standard ply thickness. When rounding down is specified for a layer with a derived thickness less than the standard ply this layer will be removed. The laminate with such rounded layers is not optimised to give a strain failure reserve factor of unity in any layer but the resulting laminate is analysed to give the stresses, strains and reserve factors in each layer. Guidance on the use of this facility is given in Section 2.4.

1.3.5 Lay-up alternatives

The program will, if required, consider additional lay-ups to that specified, for thickness selection under the same loading conditions. For specified lay-ups comprising up to 100 layers the program will obtain

alternatives having that number of layers or less. The method of choosing these alternatives is detailed in Section 2.3.

1.3.6 Strain and stress analysis

The layer strains and stresses for the specified lay-up and the alternatives can be provided. These are obtained by applying conventional elastic, small displacement, laminate theory to the laminates.

TABLE 1.1 HEURISTIC REDESIGN RULES

Rule No.	Conditions					Action
	Fibre angle (α) of layer with lowest reserve factor (degrees)	Associated failure mode	Fibre angle of layers present or otherwise (degrees)			
			0	90	± 45	
1A	90	transverse	✓	✓	*	0
1B	90	transverse	X	✓	*	90
2A	0	transverse	✓	✓	*	90
2B	0	transverse	✓	X	*	0
3A	90	shear	*	✓	✓	± 45
3B	90	shear	✓	✓	X	0
3C	90	shear	X	✓	X	90
4A	0	shear	✓	*	✓	± 45
4B	0	shear	✓	✓	X	90
4C	0	shear	✓	X	X	0
5	± 45	transverse	*	*	✓	± 45
6A	± 45	shear	✓	✓	✓	0 and 90
6B	± 45	shear	X	X	✓	± 45
6C	± 45	shear	✓	X	✓	0
6D	± 45	shear	X	✓	✓	90
7A	0	fibre	✓	*	*	0
7B	t90	fibre	*	✓	*	90
7C	± 45	fibre	*	*	✓	± 45

KEY

✓ The layer is present.

X The layer is not present.

* The layer may or may not be present

1.4 Some Numerical Results

1.4.1 Applied moments

Consider the case of a laminated plate where ply angles are laid up in the sequence $[0/45/-45/90]_s$ with the following mechanical properties which are typical values for a carbon fibre epoxy composite.

$$\begin{aligned}
 E_{\alpha} &= 135\,000 \text{ N/mm}^2 & E_{\beta} &= 9000 \text{ N/mm}^2, \\
 G_{\alpha\beta} &= 4000 \text{ N/mm}^2, & \nu_{\alpha\beta} &= 0.33, \\
 \varepsilon_{\alpha at} &= 0.0125, & \varepsilon_{\alpha ac} &= -0.0125, \\
 \varepsilon_{\beta at} &= 0.01, & \varepsilon_{\beta ac} &= -0.02, \\
 \gamma_{\alpha\beta a} &= 0.02.
 \end{aligned}$$

Suppose the in-plane applied loads N_x , N_y and N_{xy} are all zero, and the applied moment resultants are $M_x = 2000 \text{ N mm/mm}$, $M_y = 1000 \text{ N mm/mm}$ and $M_{xy} = 1000 \text{ N mm/mm}$.

The solution was found to correspond with the maximum positive roots from the solution to the cubic equations for the moment analysis, and was achieved after 6 iterations of the heuristic redesign rules. Table 1.2 gives the final layer thickness values and the strain failure reserve factors at the top and bottom of each layer. It is interesting to note that the 90° layers do not appear in the output as they have zero thickness. This is because there are no positive roots to the cubic equation for the 90° layers. Referring to the strain failure reserve factors, it is seen that all of them are greater than or equal to unity as required. Failure is predicted when the reserve factor is equal to unity, and occurs at the top of layer number 2 in the fibre direction. Also, there is a corresponding failure at the bottom of layer number 5 because of symmetry. The next lowest reserve factor is 1.11 and occurs in layers 1 and 6 with a matrix shear mode of failure. In fact, of the thirty possible failure modes, half of them have reserve factors that lie between 1.0 and 2.12, inclusive. Thus the heuristic redesign rules are attempting to bring the reserve factors in each ply to unity and so probably the final design is fairly close to an optimum.

Fibre failure, as predicted in layers 2 and 5, will lead to catastrophic failure of the whole plate. However, it is often the case that transverse matrix failure will occur first, due to the relatively low strength across the fibres, and the analysis will produce a design that corresponds to whichever mode of failure first reaches a reserve factor of unity. If it is desirable to establish when fibre failure is likely to occur then transverse matrix failure and shear failure can be by-passed by artificially raising their allowable strains to large values.

TABLE 1.2

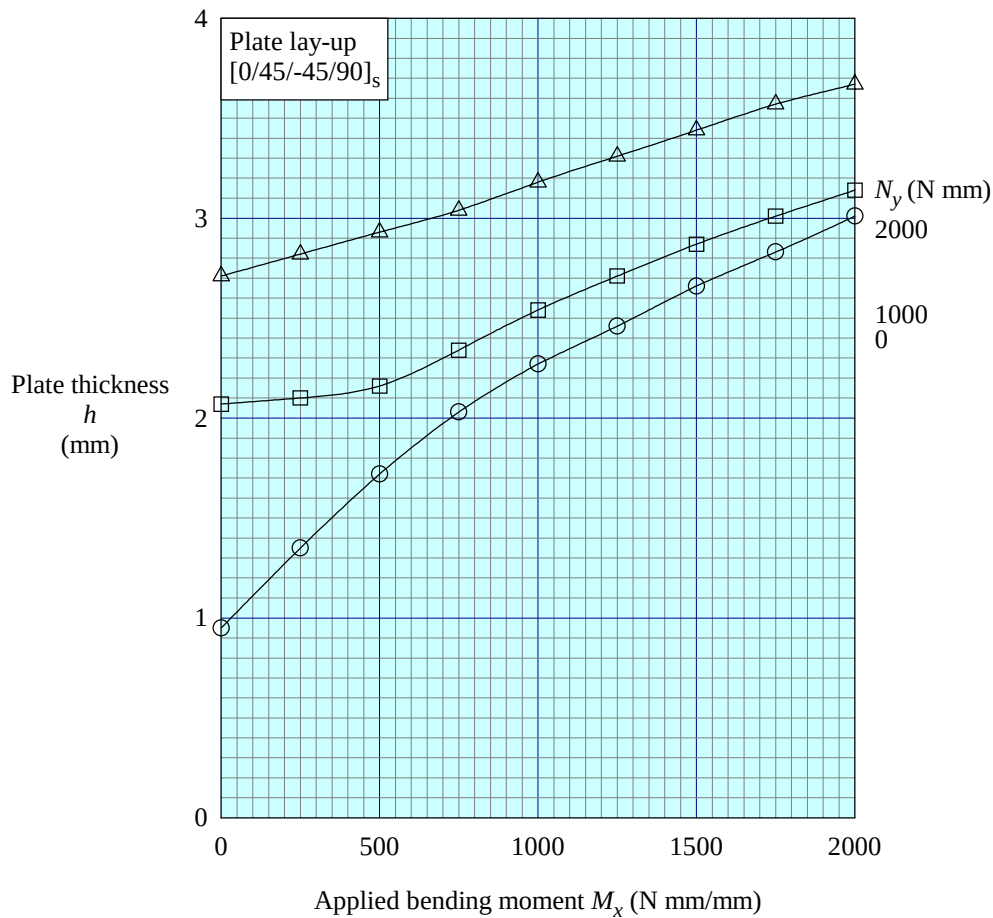
Final Ply Thicknesses and Strain Failure Reserve Factors for [0/45/–45/90]_s Plate with Zero In-plane Loads and $M_x = 2000$, $M_y = 1000$ and $M_{xy} = 1000$ N mm/mm

Layer No.	Layer angle (deg)	Layer thickness (mm)	Top or bottom of layer	Strain failure reserve factors (= Allowable strain/Actual strain)		
				α -direction	β -direction	$\alpha\beta$ -direction
1	0	0.3725	Top	1.67	1.57	1.11
			Bottom	2.12	2.01	1.42
2	45	0.6803	Top	1.00	12.30	22.21
			Bottom	2.00	24.60	44.43
3	– 45	0.6803	Top	15.38	1.60	44.43
			Bottom	∞	∞	∞
	90	0	Top	–	–	–
			Bottom	–	–	–
	90	0	Top	–	–	–
			Bottom	–	–	–
4	– 45	0.6803	Top	∞	∞	∞
			Bottom	15.38	3.20	44.43
5	45	0.6803	Top	2.00	12.30	44.43
			Bottom	1.00	6.15	22.21
6	0	0.3725	Top	2.12	4.01	1.42
			Bottom	1.67	3.15	1.11

1.4.2 In-plane loads and a moment

Consider the same plate as in Section 1.4.1 but with a different loading. $N_x = 1000$ N/mm, $N_{xy} = 250$ N/mm, $M_y = 0$, $M_{xy} = 0$, and allow both N_y and M_x to vary. For these cases, many program runs were made, and the final total plate thicknesses are shown in Sketch 1.2 for various values of N_y and M_x . The curves are approximately linear in character and show interesting trends. Each of the three curves shows that the plate thickness increases significantly as the applied bending moment M_x is increased from zero to 2000 N mm/mm. This effect becomes less pronounced as N_y is increased. Also, for a given value of M_x , the plate thickness always increases as N_y is increased from zero to 2000 N/mm.

Figure 9 of Derivation 1 applies to exactly the same cases as those considered above, and so it would be expected to be in agreement with Sketch 1.2 of this Data Item. However, the curves in Derivation 1 for $N_y = 2000$ N/mm and 1000 N/mm give plate thicknesses that are much greater than those obtained using the present analysis. This is because only a maximum positive root solution was used in the initial moment analysis, and also the ply thicknesses have been rounded up to practical values at the end of each iteration.



Sketch 1.2 Total thickness of iteratively derived plate designs for varying values of M_x and N_y

1.4.3 Multiple load cases

Some limited results have been published in References 3 and 4 for multiple load cases using traditional mathematical methods. The examples considered are for a [0/45/-45/90]_s plate subjected to in-plane loads only. In Reference 3 three load cases are considered and Reference 4 considers four cases. Values for layer thicknesses obtained using the present method are found to agree very well with those from References 3 and 4, with total plate thicknesses differing by no more than 5.5 per cent.

1.4.3.1 Comparisons with Reference 3

Material properties (for all layers)

$$E_\alpha = 137\,900 \text{ N/mm}^2, E_\beta = 8963 \text{ N/mm}^2, G_{\alpha\beta} = 4480 \text{ N/mm}^2, \nu_{\alpha\beta} = 0.304,$$

$$\epsilon_{\alpha at} = 0.00825, \epsilon_{\beta ac} = -0.00575, \epsilon_{\beta at} = 0.00615, \epsilon_{\beta ac} = -0.02306,$$

$$\gamma_{\alpha\beta a} = 0.0246.$$

Applied Loadings (all in N/mm)

Case	N_x	N_y	N_{xy}
1	700	175	350
2	- 525	175	- 350
3	350	175	525

Results of minimum thickness (mm)

Ply angle (degrees)	0	+45	-45	90	Total
Reference 3	0.598	0.748	0.748	0.058	2.152
Program A9636	0.536	0.742	0.742	0.154	2.174

1.4.3.2 Comparisons with Reference 4

Material properties (for all layers)

$$E_\alpha = 137\,900 \text{ N/mm}^2, E_\beta = 8963 \text{ N/mm}^2, G_{\alpha\beta} = 4480 \text{ N/mm}^2, \nu_{\alpha\beta} = 0.304, \\ \epsilon_{\alpha at} = 0.00857, \epsilon_{\alpha ac} = -0.00857, \epsilon_{\beta at} = 0.00471, \epsilon_{\beta ac} = -0.00176 \text{ and } \gamma_{\alpha\beta a} = 0.0184.$$

Applied Loadings (all in N/mm)

Case	N_x	N_y	N_{xy}
1	3500	0	0
2	2625	-2625	875
3	- 2625	1750	1750
4	0	0	3500

Results of minimum thickness (mm)

Ply angle (degrees)	0	+45	-45	90	Full Total
Reference 4	4.064	4.572	4.572	1.524	14.732
Program A9636	4.224	4.856	4.856	1.612	15.548

1.5 Derivation and References

This section lists sources that have assisted in the preparation of this Data Item.

1.5.1 Derivation

1. MORTON, S.K.
WEBBER, J.P.H. Heuristic methods in the design of composite laminated plates. *Composite Structures*, Vol. 19, pp. 207-265, 1991.
2. MORTON, S.K.
WEBBER, J.P.H. Heuristic design of composite laminates for strength, stiffness and multiple load cases. University of Bristol, Department of Engineering Mathematics and Aerospace Engineering. Internal Report, 1994.

1.5.2 References

The references given are recommended sources of information supplementary to that in this Item.

3. SCHMIT, L.A.
FARSHI, B. Optimum laminate design for strength and stiffness. *Int. Journal for Numerical Methods in Engineering*, Vol. 7, pp. 519-536, 1973.
4. VANDERPLAATS, G.N.
WEISSHAAR, T.A. Optimum design of composite structures. *Int. Journal for Numerical Methods in Engineering*, Vol. 27, pp. 437-448, 1989.

2. USE OF ESDUpac A9636

2.1 Introduction

ESDUpac A9636 is a Fortran computer program to determine the required thicknesses of laminated plates made from thin orthotropic layers of fibre reinforced composite material to withstand systems of in-plane loads and bending moments. The program accepts values that define the problem for a specific laminate in terms of layer material properties of up to 20 different types including their individual orientations and a standard ply thickness (to multiples of which the layer type thicknesses may be rounded up or down), the laminate in-plane direct loading and moment loading for up to 20 loading combinations, the number of layers (up to 100) and the choice of output form.

The program provides layer thicknesses for the specified laminate lay-up and the percentage of layers of particular orientations. It details, if required, the design strain ratios and the actual strains and stresses of individual layers. These data are repeated for a laminate of the same lay-up made up of layers which, for a particular layer type, may each be rounded up or down to a thickness that is a multiple of a chosen standard ply thickness. The standard ply thickness may be different for each layer type. The program will also give data for a chosen number of alternative lay-ups of 2 layers up to the number of layers in the specified laminate. These additional lay-ups represent a practical selection of those for which it is possible to obtain solutions with the analytical procedure and heuristic rules set out in Section 1.3. The system by which these alternatives are selected is detailed in Section 2.3.

This Data Item relates to Version 1.0 of ESDUpac A9636 and Fortran program A9636V10. Both the ESDUpac number and the program number contain the version number encoded in the three characters V10. Throughout this Data Item the ESDUpac and its program are referred to by the number A9636 and Version 1.0 is taken to apply. Note that the Contents page of the Sub-series Software Volume states the current version number.

The ESDUpac Fortran source code is provided on the Compact disk. Information on how to install and run the program is contained in the introduction to the Software Volume. In addition to the program the ESDUpac contains example inputs and the resulting outputs.

This part of the Data Item describes the program input data and explains the output and all messages. For file input and output, Section 2.4 explains the input file data requirements and construction. Examples illustrating the use of the program are provided. A description of the theory on which the program is based is given in Section 1.3 and further information is given in Derivations 1 and 2.

Every reasonable effort has been made to ensure that the program of ESDUpac A9636 performs the intended calculations satisfactorily and reference to the range of the tests made is provided. No representation as to the suitability or fitness of the program for any particular purpose is made and no liability for any loss occasioned by any persons as a direct or indirect result of use of the program whether arising from negligence or otherwise can be accepted. In no event shall ESDU or any individuals associated with the development of the program be liable for any damages, including loss of profit or consequential loss, arising out of or in connection with the program.

2.2 Program Limitations

The analysis to which the heuristic rules are applied and which is used to determine the layer stresses and strains is based on elastic, small deflection, orthotropic laminate theory as set out in ESDU Data Items Nos 94003 and 94004. The program will deal with a specified laminate of 2 up to 100 layers of up to 20 different layer material types that are laid up in any symmetric combination of the angles 0, 90, 45 and -45 degrees only.

If a selection of alternative lay-ups is requested the number of solutions output may be less than the number specified. This is because some of the solutions, using the system in Section 2.3, give near identical lay-ups, and may therefore be ignored. In these cases the layer orientations are the same and laminate thicknesses differ in ratio by less than a factor of 10^{-4} .

If none of the layer orientations coincides with the direction of loading (for example $\psi_k = 0$ and ± 45 degrees only with M_y alone), the netting analysis would yield zero thickness layers and determination of the thickness is not possible. In this event an error message is also printed, see Section 2.5.2.

2.3 Method of Selecting Alternative Lay-ups

The program will, if required, provide the thicknesses for alternative lay-ups to the one initially specified. The method is designed to give a large number of different lay-ups derived from the specified lay-up.

The method takes account of the following aspects:

- (i) there are up to 20 different layer types as specified,
- (ii) there are only 3 layer orientations 0, 90, +45 and -45 degrees, the last two being treated in pairs,
- (iii) layers of the same orientation are of the same unrounded thickness regardless of layer type,
- (iv) under in-plane loads the layer thicknesses derived are not dependent on their position through the thickness of the laminate and
- (v) under moment loading the layer thicknesses derived are dependent on their position through the thickness of the laminate.

The procedure involves five stages which are detailed in an example below. It uses the specified lay-up as a starting point, stage (1), on the assumption that the user has a practical reason for choosing that lay-up. From consideration of other lay-ups having the same number of layers it then works its way down to consider lay-ups with only two layers, stage (5).

The selection process starts by taking the specified lay-up first and then produces 5 additional lay-ups with the same number of layers in a way that gives each of the three layer orientations a reasonable chance to be the outer layer, stages (2) and (3). This is based on the assumption that the applied moments are the most significant loads in determining the best lay-up and layer thicknesses (aspects (iv) and (v) above). It is assumed that if certain orientations do not appear close to the outside of the specified laminate then the designer does not consider them to be of particular significance under the given moment loading. It then produces lay-ups with progressively reduced numbers of layers in them by removing the thickest layers likely to have the least influence on the moment loading (aspects (iv) and (v) above), that is those nearest the mid-plane (stages (4) and (5)).

During the selection process the number of lay-ups considered will be reduced because identical lay-ups will have been produced. This occurs for several reasons, (a) because repetitive orientation patterns appear in the lay-ups (aspects (i), (ii) and (iii) above), (b) certain orientations take no load and are removed (this occurs in particular when the initial netting analysis finds no applied loading in the direction of a particular orientation), and (c) later lay-ups with less layers repeat earlier lay-ups from which layers have been removed.

The stages in the lay-up selection process are illustrated with the aid of the following 16 layer example.

- 1) The program first takes the lay-up specified by the user:

$$+ [0/0/-45/45/90/45/-45/90]_s .$$

- 2) Two further lay-ups are created by moving the two outer layers (on both sides) to the mid-plane in turn:

$$\begin{aligned} & [0/-45/45/90/45/-45/90/0]_s . \\ & [-45/45/90/45/-45/90/0/0]_s . \end{aligned}$$

- 3) Three further lay-ups are created by reversing the stacking sequence (from the surface to the mid-plane) of the first three:

$$\begin{aligned} & [90/-45/45/90/45/-45/0/0]_s , \\ & [0/90/-45/45/90/45/-45/0]_s , \\ & [0/0/90/-45/45/90/45/-45]_s . \end{aligned}$$

- 4) When the six laminates with the specified number of layers have been analysed, the number of layers are reduced to create six further lay-ups with one less layer in the half-thickness (two in the whole thickness). This is done by removing the thickest layer (on each side of the mid-plane) in that half of the layers closest to the mid-plane.

If the specified lay-up is considered as an example and the thickest layer in the half closest to the mid-plane is assumed to be the 90 degree layer then the new lay-up is:

$$[0/0/-45/45/90/45/-45]_s .$$

If the third lay-up above is considered as an example and it is assumed that the -45 degree layer is the thickest in the half nearest the mid-plane then, remembering that $+45$ and -45 degree layers are treated in pairs, the new lay-up is:

$$[-45/45/90/90/0/0]_s .$$

- 5) This process is repeated until the laminate consists of only one layer in the half (two in the whole) laminate.

The following subsidiary rules are applied during the above process.

- 1a) In stage (2) above $45/-45$ layers are moved in pairs.
- 4/5a) In stages (4) and (5) $45/-45$ layers are removed in pairs.
- 4/5b) Where there is an odd number of layers in the half laminate the "half" of this considered for removal of the thickest layer is the integer number below the real half value.
- 4/5c) Since layers of the same orientation have the same thickness in the analysis, where several layers of the same orientation are the thickest in the "half" rule the layer nearest the mid-plane will be the one removed. For example if the 45 degree layers are the thickest in the sixth laminate above (see stage (3)) then the reduced laminate would be:

$$[0/0/90/-45/45/90]_s .$$