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A qualitative introduction to static aeroelasticity: controllability, loads and stability

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The Dynamics Committee, which first met in January 1962, has the co-operation of many engineers and scientists in industry, research establishments and universities from whom much assistance and unpublished information is being received.

A QUALITATIVE INTRODUCTION TO STATIC AEROELASTICITY: CONTROLLABILITY, LOADS AND STABILITY

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A QUALITATIVE INTRODUCTION TO STATIC AEROELASTICITY: CONTROLLABILITY, LOADS AND STABILITY

1. INTRODUCTION

Two earlier Items, Nos 94009 and 94045 (References 2 and 3, respectively), dealt with loadings on rigid aeroplanes of classical configuration undergoing steady, or quasi-steady, symmetric manoeuvres at subsonic speeds. The aerodynamic lift distributions and inertial distributions on the wings of two configurations (one unswept and the other swept) were described. The effects of engine nacelles were included. The wing loadings were expressed in terms of spanwise distributions of shear force, bending moment and torque. The lengthwise distributions of the aerodynamic lift and inertial loading along the fuselage were also described. Only overall tailplane loads were given.

In this Item some of the fundamental concepts on the effects of structural flexibility on aeroplanes of classical configuration undergoing steady, or quasi-steady, symmetric manoeuvres at subsonic speeds are introduced.

Virtually all parts of an aeroplane are subject to structural deformations; wings, fuselage, tailplane, engine nacelles' mountings, control surfaces and their linkage systems. The extent to which an airframe component deforms depends on:

- the inertial loading distribution,
- the surface aerodynamic loading distribution,
- the structural stiffness distribution.

Deformations occur because of 'out-of-balance' loadings. For example, in steady level flight the overall lift on the aeroplane must balance the overall all-up weight. Now the wing lift is approximately 90% of the all-up weight while the wing weight is, for a classical configuration, approximately 40% of the all-up weight, so the wing will bend upwards and will also probably twist.

The main difficulty in analysing the effects of structural deformation is the mutual interaction between deformation and aerodynamic lift; a wing deformation changes the wing shape which affects the wing aerodynamic loading, which in turn affects the wing deformation.

The magnitude of structural deformations can be reduced by increasing the structural stiffness, but such a step would invariably lead to an increase of structural weight which would impose serious penalties in performance, primarily a reduction in range. Structural design aims to build a minimum weight structure in which

- the deformations and their effects remain within specified bounds,
- there is adequate strength, with safety margins,

in all phases of flight from take-off to landing, in virtually all weather conditions, throughout the flight envelope.

Static aeroelasticity is concerned with the effects of steady deformations in steady level trimmed flight, or in a quasi-steady pull-out manoeuvre, or on conditions of static stability. These effects are manifested in

- changes in the loadings on the airframe,
- changes in the controllability parameters, such as elevator angle to trim and the manoeuvring incremental elevator angle per g ,
- changes in the static stability characteristics.

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2. NOTATION

Any coherent system of units may be used.

$A_0, A_1, A_2, \dots$	general constants
$B_i, C_i, \dots, G_i$ ($i = 0, 1, 2, 3, \dots$)	general coefficients used for stability characteristic equations in Section 9
$ac, (ac)_T$	aerodynamic centre of wing-fuselage combination and tailplane, respectively
b, b_T	wing span and tailplane span, respectively
C_L	lift coefficient ($= L/(\frac{1}{2}\rho V^2 S)$)
$C_{L\alpha}, C_{L\eta}, C_{L\theta}$	rate of change of lift coefficient with α, η and θ , respectively ($= dC_L/d\alpha, dC_L/d\eta$ and $dC_L/d\theta$)
C_{L0}	lift coefficient due to camber, washout and setting angle
C_M	pitching moment coefficient ($= M/(\frac{1}{2}\rho V^2 S \bar{c})$)
C_{M0}	pitching moment coefficient due to camber, washout and setting angle about aerodynamic centre
$C_{M\eta}, C_{M\theta}$	rate of change of pitching moment coefficient with η and θ , respectively ($= dC_M/d\eta$ and $dC_M/d\theta$)
C_Q	weight coefficient
$\bar{c}, \bar{c}_T$	geometric mean chord of wing and tailplane, respectively
cm	centre of mass of aeroplane
D	aeroplane drag
E_{man}	elevator manoeuvre margin ($= -\Delta\eta_{man}/(\Delta n C_Q)$)
E_{trim}	elevator trim margin ($= -\Delta\eta_{trim}/\Delta C_Q$)
g	acceleration due to gravity
H_m	conventional elevator manoeuvre margin
i	general integer
k_θ	torsional stiffness
L	lift

$l_W, l_W(y)$	distance of aerodynamic centre of overall wing and of wing section, respectively, from origin of axis system
$l_f, (l_f)_T$	distance of origin of wing flexural axis and tail flexural axis, respectively, from origin of axis system
l_T	distance of aerodynamic centre of tailplane from origin of axis system
l_m	distance of mass centre from origin of axis system
M	pitching moment about aerodynamic centre
M_q	rate of change of pitching moment with pitch rate ($= dM/dq$)
M_η	rate of change of pitching moment with elevator angle ($= dM/d\eta$)
M_0	pitching moment about aerodynamic centre due to camber and washout
m_q	pitching moment coefficient derivative due to pitch rate ($= dC_M/d(q\bar{c}/V)$)
n	load factor, L/W
O	axis origin
q	steady pitch rate
S, S_T	wing area and tailplane area, respectively
T	engine thrust
t	time
V	aeroplane velocity in flight direction
W	aeroplane weight
x	general fuselage station measured from cm position
x, y, z	axis directions
y	general span station measured from fuselage centre line
$y_f, (y_f)_T$	flexural axis of wing and tailplane, respectively
z_e, z_{en}	bending deflection due to deformation in y - z plane and y_f - z plane, respectively
α	incidence
γ	flight path angle
$\dot{\gamma}$	rate of change of γ with time ($= d\gamma/dt \equiv q$)

Δ	denotes an increment
δ_m	element of mass
ζ	damping ratio
ε	downwash angle
η	elevator angle
$\theta, \dot{\theta}$	pitch angle and rate of change of pitch angle with time, respectively
θ_e, θ_{en}	twist deflection due to deformation about y-axis and flexural axis, respectively
$\theta_{e0}, \theta_{en0}$	twist deflection due to deformation about y-axis and flexural axis at 75% of semi-span, respectively
$(\bar{\theta}_{e0})_T$	tailplane structural twist due to tailplane lift and weight only (see Equation (4.31))
Λ	wing sweep angle
λ	eigenvalue of stability characteristic equation
μ	relative density parameter ($= (W/g)/(\frac{1}{2}\rho S \bar{c})$)
ρ	air density
ω, ω_0	frequency and natural undamped frequency, respectively

Subscripts

e	denotes elastic deformation
F_{us}	refers to rear portion of fuselage
f	denotes flexural axis
$flex$	denotes flexible
i	general integer
m	relates to mass quantity
man	denotes steady manoeuvre
$rigid$	rigid aeroplane or component
T	tailplane
$trim$	denotes trimmed flight
W	wing

3. GENERAL DESCRIPTION

3.1 Scope and Applicability

In order to highlight the main qualitative features of aeroelasticity in this Item the following simplifications in terms of aeroplane configuration and flight condition are introduced.

- (i) The aeroplane is assumed to be a classically configured transport type aeroplane which restricts the wings to medium and high aspect ratios, typically ≥ 4 , and up to moderate sweep-back angles, typically $\Lambda \leq 35^\circ$.
- (ii) The aeroplane flight condition is restricted to subsonic, symmetric flight at either a steady level flight condition or a quasi-steady manoeuvring condition. All Mach number effects are ignored.
- (iii) All flight angles, including the angle of incidence, the pitch angle and the flight path angle, are assumed small.
- (iv) Wing and tailplane flexibilities are assumed to take the form of bending and twist about an elastic, or flexural axis; chordwise deformations are ignored.
- (v) Weight distributions of all parts of the aeroplane are assumed to be unaffected by deformation effects.

3.2 Reference Shape

All deformations need to be defined relative to some reference, or datum, state.

The basic *datum shape*, known as the '*jig*' shape, is taken in this Item as the shape of the aeroplane when all deformations are zero throughout the structure, *i.e.* when all loads and all internal stresses are zero. It would be the shape of the aeroplane taken up when supported by a jig on the ground in which the upward jig support exactly balances the weight distribution of the aeroplane. This jig shape can be alternatively defined mathematically as the aeroplane shape as the airframe structural stiffness tends to infinity.

A *reference shape* can be defined as the shape of the aeroplane at a specified flight condition; deformations at other flight conditions are then estimated relative to this reference shape. For the case of a subsonic transport aeroplane, the high Mach number cruise condition is often chosen as the flight condition where the flight shape is specified to meet optimum cruise efficiency. For a combat aeroplane, steady level flight at a high subsonic Mach number at a given altitude may be taken as the reference flight condition, but other flight conditions may be specified, for example, a maximum sustained turn.

Another *reference shape* can be taken with the aeroplane standing on the ground supported by its undercarriage.

3.3 Axis System

The flight dynamicist uses a number of axis systems for aeroplane motions; all systems are right handed Cartesian, Oxyz, with the x and z axes chosen in the plane of symmetry of the aeroplane, the x axis pointing forward and the z axis downwards, and the y axis pointing to starboard.

For dynamic studies of a rigid aeroplane it is usual for the aeroplane shape to be defined as the reference datum shape, as discussed Section 3.2, with the origin, O, of the axis system chosen at the overall centre

of mass of the aeroplane. The origin of such an axis system is dependent upon the amount and distribution of fuel and payload carried by the aeroplane. A number of axis systems used for rigid body analysis are described within Data Item No. 67003 (Reference 1).

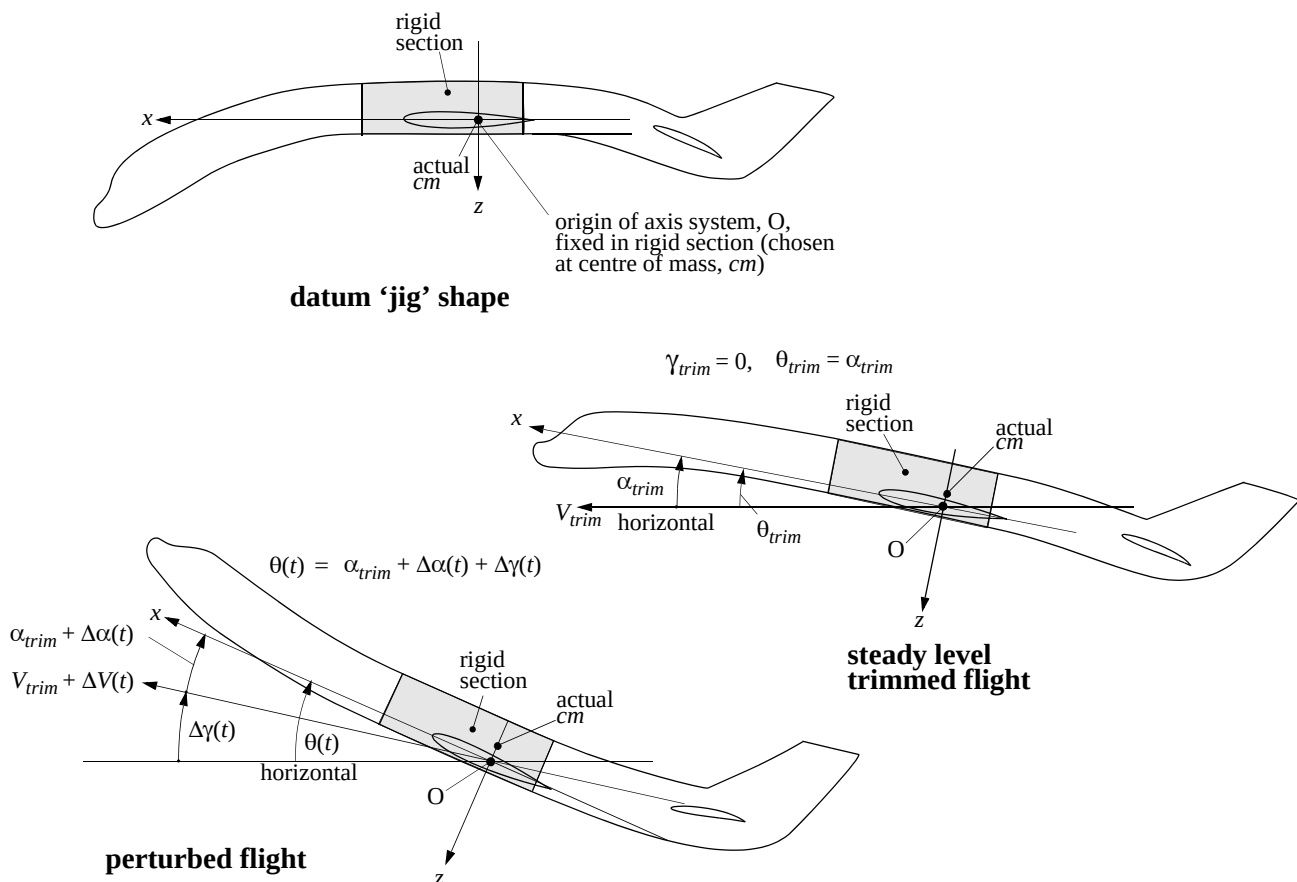
For flexible aeroplanes there are a wider number of options for defining moving axis systems: they include Datum axes, Attached or Structural axes, Generalised Attached axes, Mean Axes and Aeroelastic axes.

For static aeroelasticity, Attached or Structural axes are the most straightforward and are used in this Item.

Attached or Structural axes can be defined when a section of fuselage in the region of the wing-fuselage junction is assumed to be rigid. For such an arrangement the two halves of the wing are assumed built-in either side of the rigid fuselage section, and flexible front and rear portions of fuselage are assumed to be built-in to either end of the central rigid structure.

The origin of the axis system lies at the overall centre of mass when the mass is distributed through the datum 'jig' aeroplane shape, in the plane of symmetry on the fuselage with the Ox axis pointing forward along the fuselage fixed in the rigid section. It is assumed that this overall centre of mass lies within the rigid fuselage section.

In flight the axis origin for the flexible aeroplane will not usually coincide with the overall centre of mass because the mass is now distributed throughout the deformed aeroplane shape; however, it may be assumed that the overall centre of mass is close to the origin displaced slightly along the Oz axis. Sketch 3.1 illustrates this axis system showing the aeroplane in its 'jig' shape, in steady level trimmed flight and in perturbed flight. The aeroplane deformed shapes are shown exaggerated to illustrate their effects.



Sketch 3.1 Attached or Structural axes

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Three angles are used for longitudinal motion, are shown in Sketch 3.1 and are defined as follows.

The angle of incidence or angle of attack, α , of the aeroplane is the angle between the Ox axis fixed in the rigid section of the fuselage and the direction of flight, taken as positive in the nose-up direction.

The angle of pitch of the aeroplane, θ , which is the angle between the Ox axis and the horizontal, defined as positive in the nose-up direction.

The flight path angle, γ , is the angle of the flight path from the horizontal, defined as positive in the nose-up direction.

For steady level trimmed flight

$$\gamma_{trim} = 0$$

and
$$\theta_{trim} = \alpha_{trim} .$$

For time varying perturbed flight

$$\alpha(t) = \alpha_{trim} + \Delta\alpha(t) ,$$

$$\gamma(t) = \Delta\gamma(t)$$

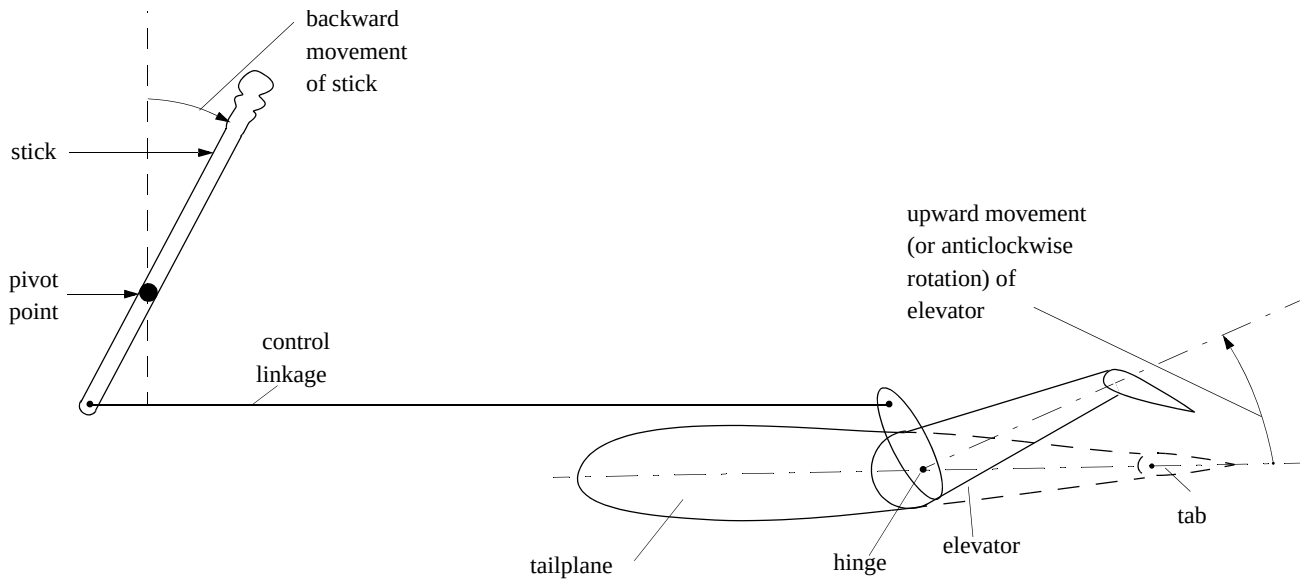
and
$$\theta(t) = \alpha_{trim} + \Delta\alpha(t) + \Delta\gamma(t) .$$

3.4 Controllability

Aeroplanes must be capable of being flown throughout the flight envelope in all but the severest of weather conditions by a pilot, human or mechanical. The pilot must be able to manoeuvre and to retain control of the aeroplane at all times; if in the rare event control is lost a safe recovery is mandatory. A conventional aeroplane is manoeuvred in longitudinal flight using the two primary controls of *throttle*, which alters the thrust setting of the aeroplane, and *stick displacement*, which rotates the elevator. In addition to the two primary controls there is a secondary trim control.

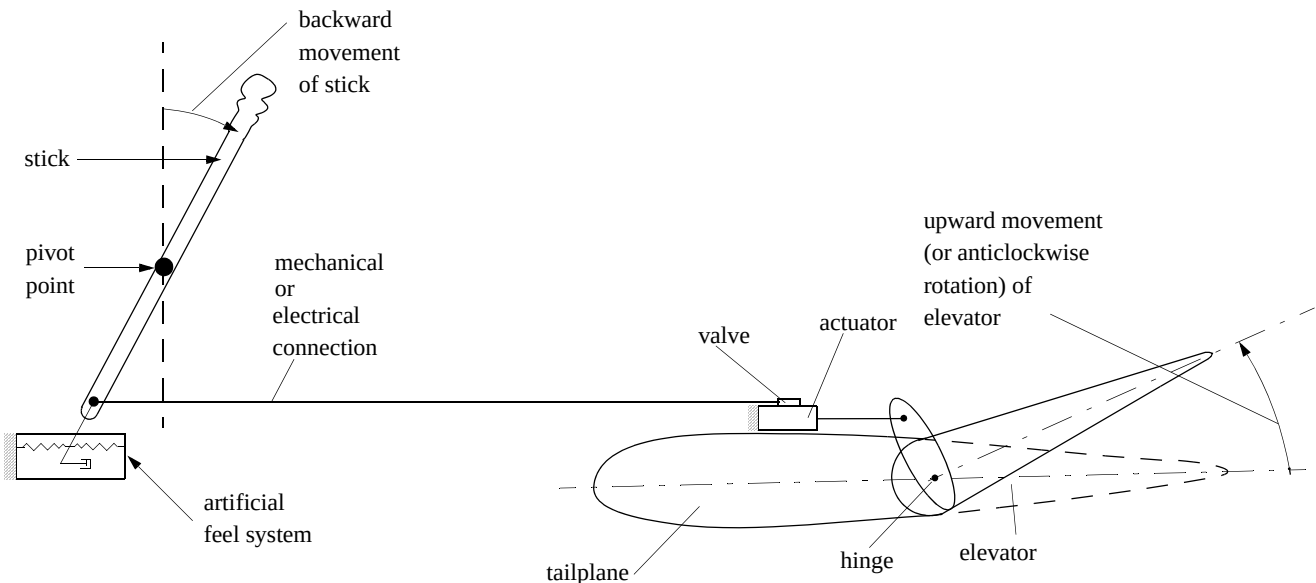
There are two main types of control system, *manual control systems* for smaller aeroplanes and *powered control systems* for both larger aeroplanes and for high speed aeroplanes. In both systems, when it is assumed that the control system is rigid, a simple gearing ratio exists between stick displacement and change in elevator angle.

- (i) In a **manual control system** the stick is connected to an elevator via a mechanical linkage; a schematic of a manual control system is shown in Sketch 3.2. A backward displacement of the stick rotates the elevator counter-clockwise, or in an upwards direction, while a forward stick displacement rotates the elevator clockwise or downwards. Sometimes a bob weight is attached to the bottom of the stick; such bob weights are ignored in this Item and then the force applied to the stick by the pilot opposes the elevator hinge moment. The pilot also has independent control of a trim tab which lies at the trailing edge of the elevator; changes in the tab angle alter the elevator hinge moment, and hence the stick force, without altering the tailplane lift.



Sketch 3.2 Manual control system

- (ii) Powered controls are used for aeroplanes when the stick forces required by the pilot to move the elevator are too large. In these cases a valve operates a hydraulic actuator which provides the power needed to rotate the elevator. The stick force required to move the valve is small. In order to provide the pilot with a traditional feel for the aeroplane response in terms of stick force applied, an artificial feel system is used which, based on flying aeroplanes with manual control systems, aims to give a realistic stick force versus stick displacement characteristic. In such systems a trim tab is unnecessary but a separate trim control is still available to the pilot to reduce the stick force to zero; the stick force in this case is reduced to zero within the artificial feel system while the actuator holds the elevator in the demanded position. A schematic of a powered control system is shown in Sketch 3.3.



Sketch 3.3 Powered control system

Controllability as measured by a pilot are the stick displacement and stick force required to execute a manoeuvre.

3.4.1 Trim control

An aeroplane is said to be trimmed when it maintains a steady level flight condition with a zero stick force. The external forces and moments acting on the airframe must then be in static equilibrium and for a manual control system with no bob weights the elevator hinge moment must be zero. A trimmed state can only be maintained if the aeroplane in question is inherently stable; a trimmed state therefore implies a stable aeroplane.

A pilot flies an aeroplane in the longitudinal plane with the use of two controls, a throttle, which alters the aeroplane thrust and a stick, fore and aft movements of which change the elevator setting. A rearward displacement of the stick produces an upwards (negative) elevator angle change.

To increase aeroplane speed while maintaining constant altitude the pilot will gradually push the stick forward, introducing a downward (positive) change of elevator angle, which increases the tailplane lift thereby reducing the aeroplane incidence, while at the same time gradually increasing the engine thrust in order to increase speed and hence to maintain constant lift. Once the new steady-state is reached the aeroplane can be retrimmed to zero the stick force. A pilot learns to co-ordinate changes in each of the two controls, throttle setting and stick displacement, to achieve a given change of trim flight condition.

Consider an aeroplane in an initial trimmed steady state flight condition defined by a forward speed of V_{trim} and an associated elevator angle to trim of η_{trim} . At a nearby steady state condition the speed is assumed equal to $V_{trim} + \Delta V_{trim}$ and the elevator angle equal to $\eta_{trim} + \Delta \eta_{trim}$.

One criterion for positive trim control is that

'a forward stick displacement results in an increase in trim speed'

or that *'a backward stick displacement results in a decrease in trim speed',*

both of which may be expressed as

$$\text{positive trim stick displacement control} = \frac{\Delta \eta_{trim}}{\Delta V_{trim}} > 0 . \quad (3.1)$$

The non-dimensional variable that represents the trim dynamic pressure is the **weight coefficient**, C_Q , defined by

$$C_Q = \frac{W/S}{\frac{1}{2} \rho V_{trim}^2} . \quad (3.2)$$

This coefficient, C_Q , is inversely proportional to $(\frac{1}{2} \rho V_{trim}^2)$ and so decreases as trim dynamic pressure increases.

An incremental change in C_Q corresponding to an incremental change in trim speed is

$$\Delta C_Q = \frac{-2 C_Q \Delta V_{trim}}{V_{trim}} . \quad (3.3)$$

The **elevator trim margin**, denoted by E_{trim} , is defined as

$$E_{trim} = - \frac{\Delta \eta_{trim}}{\Delta C_Q} ; \quad (3.4)$$

hence for positive trim control, by reference to Equation (3.1),

$$E_{trim} > 0 . \quad (3.5)$$

A further requirement for positive trim control is that

‘an increase in trim speed must be associated with a forward force on the stick before retrimming’

or that *‘a decrease in trim speed is associated with a backward force on the stick before retrimming’*,

both of which may be expressed as

$$\text{positive trim stick force control} = \frac{\Delta(\text{stick force})}{\Delta V_{trim}} > 0 . \quad (3.6)$$

In this Item the main emphasis on trim control is taken to be the stick displacement criterion (Equation (3.5)) which applies to aeroplanes with either manual control systems or powered control systems. However the stick force criterion is important and must be satisfied. For a manual control system the criterion given by Equation (3.6) effectively defines the sign of the incremental elevator hinge moment, while for a powered control system this criterion must be satisfied via the artificial feel system.

The incremental stick force and displacement that satisfy the above qualitative criteria must be neither too small, which would make control too sensitive, nor too large, which could result in excessive pilot workload.

Structural deformations in trim flight affect the magnitude of the trim control parameters; in extreme cases their sign can change, resulting in loss of trim control.

3.4.2 Manoeuvre control in quasi-steady pull-outs

Consider an aeroplane initially in steady level trimmed flight at a speed of V_{trim} with an elevator angle of η_{trim} . Suppose that the pilot applies a rapid rearward stick change which is then held, leaving the throttle unchanged. The upward or anticlockwise rotation of the elevator induces a downward change of lift on the tailplane, which in turn rotates the aeroplane nose-up. The overall lift on the aeroplane is increased and so the aeroplane climbs. A short period transient motion of the aeroplane is initiated, which quickly decays (for a stable aeroplane) and then the aeroplane closely follows an upward circular flight path in a quasi-steady state in which the loads can be assumed steady and the speed constant at the initial flight speed, V_{trim} . The aeroplane is subject to a constant normal acceleration towards the centre of a circular flight path. This quasi-steady state will, after a while, degenerate into a slow time varying motion as the aeroplane speed begins to change. Sketch 3.4 illustrates the various phases of a pull-up manoeuvre.

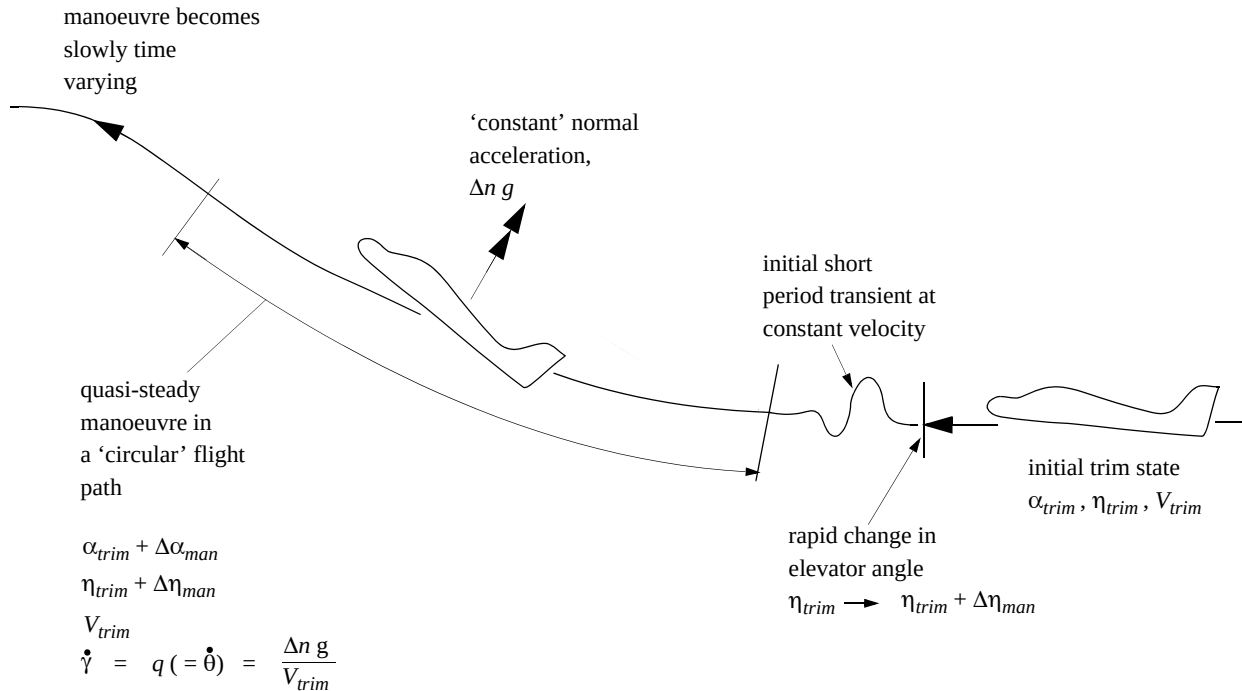
The general term for the quasi-steady motion in a circular flight path, upwards or downwards, is known as a pull-out manoeuvre, an upward circular flight path being a pull-up manoeuvre and a downward circular flight path being a push-over manoeuvre. The push-over manoeuvre is initiated by a rapid forward stick change similar to that described for the pull-up manoeuvre but in the reverse sense.

The quasi-steady pull-up manoeuvre, as described, in which an aeroplane can be assumed to be flying at constant normal acceleration along the arc of a circle at constant speed, V_{trim} , constant angular velocity,

$\dot{\gamma}$, and constant normal acceleration, may be expressed in terms of normal acceleration as

$$\text{normal acceleration} = \Delta n g, \quad (3.7)$$

in which Δn is zero in steady level flight.



Sketch 3.4 Pull-up manoeuvre

In the pull-up manoeuvre, since the angle of incidence, α , and the angular velocity or rate of change of flight path angle, $\dot{\gamma}$ (which is equal to the pitch rate, q), are assumed constant, so the normal acceleration and angular velocity are related by

$$\dot{\gamma} = q = \frac{\Delta n g}{V_{trim}}. \quad (3.8)$$

The criteria for positive pull-out manoeuvrability are that

'a backward stick displacement, and backward applied force, results in an upward (positive) normal acceleration (i.e. a pull-up manoeuvre)'

and *'a forward stick displacement, and forward applied force, results in a downward (negative) normal acceleration (i.e. a push-over manoeuvre).'*

These criteria may be expressed as

$$\frac{\Delta\eta_{man}}{\Delta n g} < 0 \quad \text{and} \quad \frac{\Delta(\text{stick force})_{man}}{\Delta n g} < 0, \quad (3.9)$$

where $\Delta\eta_{man}$ is the change in elevator angle used to initiate the manoeuvre and $(\Delta n g)$ is the resulting normal acceleration.

The main parameter used to quantify longitudinal manoeuvrability is the **elevator manoeuvre margin**, E_{man} , defined by

$$E_{man} = - \frac{\Delta \eta_{man}}{C_Q \Delta n}, \quad (3.10a)$$

where, from reference to Equation (3.9), for positive manoeuvre control

$$E_{man} > 0. \quad (3.10b)$$

For rigid aeroplanes E_{man} is independent of trim speed but for flexible aeroplanes E_{man} varies with trim speed.

In the literature on the dynamics of rigid aeroplanes a conventional manoeuvre margin, H_m , is used, and for rigid aeroplanes the criteria

$$E_{man} > 0 \quad \text{and} \quad H_m > 0$$

are synonymous. However the definition of H_m becomes obscure for flexible aeroplanes whereas the definition of E_{man} is unambiguous, so E_{man} is used in this Item.

The main emphasis on manoeuvre control in this Item is on the stick displacement criterion (Equation (3.10b)). However the stick force criterion is extremely important; while with a manual control system it is necessary to ensure that the (*stick force/g*) which, in the absence of any bob-weights, is equivalent to the (*incremental elevator hinge moment/g*) is negative, with a powered control system the artificial feel system must satisfy the stick force criterion.

3.5 Quasi-static Loads

The overall loads on an aeroplane and their distributions are needed in order to ensure that the structural strength requirements are satisfied.

In steady level trimmed flight the aeroplane loading consists of the wing, fuselage and tailplane trim lift distributions and the weight distributions throughout the airframe. Engine nacelles and external stores contribute primarily to the weight distributions.

In quasi-steady pull-out manoeuvres with a normal acceleration of $(\Delta n g)$, the incremental changes in the aerodynamic lift and inertial loading distributions are all proportional to Δn . These incremental loadings need to be added to the trim loadings to obtain the total loads in the pull-out manoeuvre.

3.6 Static Stability

The effect of aeroplane flexibility on static stability is traditionally an important feature of static aeroelasticity. However it is necessary to clarify what is meant by static stability. This is covered in Section 9.

4. TRIM CHARACTERISTICS OF AEROPLANES WITH UNSWEPT WINGS

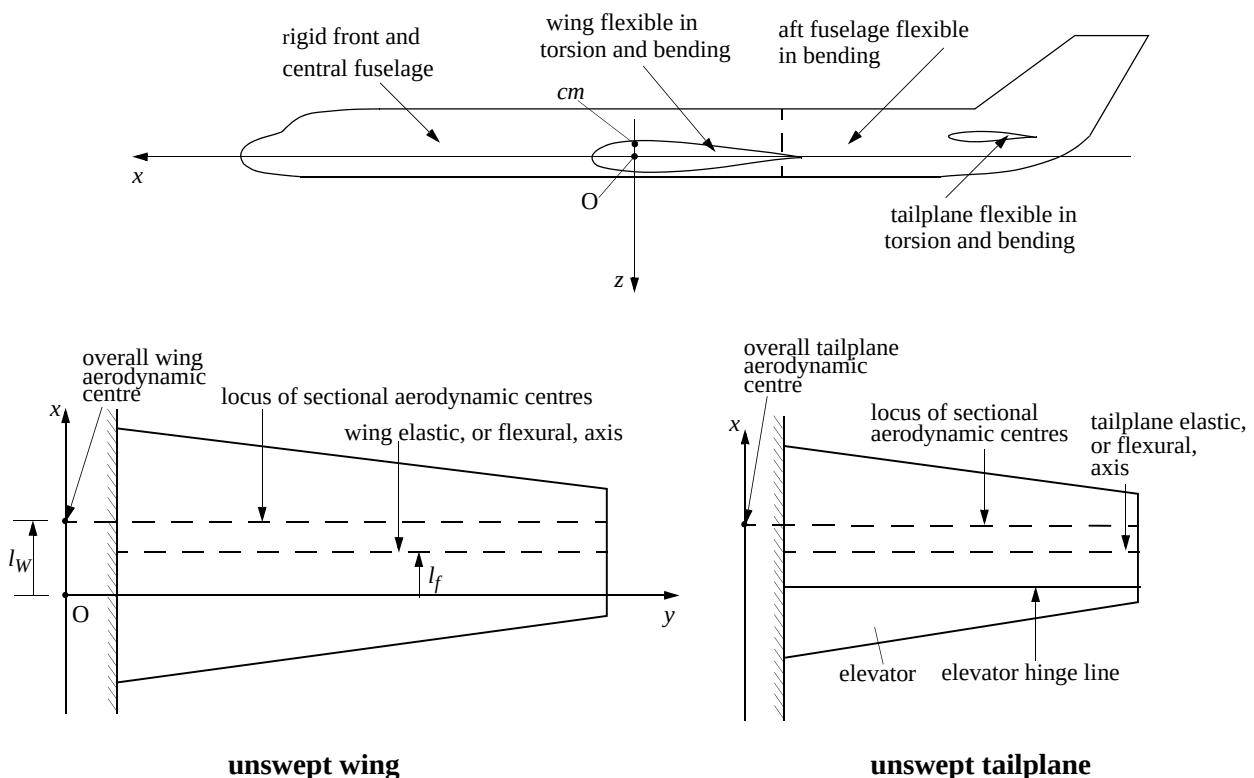
To build up an understanding of the effects of structural deformation on the trim characteristics of flexible aeroplanes a number of simplified aeroplane models are considered in sequence. Wings and tailplanes are unswept in this section.

4.1 Flexible Aeroplane

The flexible aeroplane to be considered and which is illustrated in Sketch 4.1 is a conventional transport-type which is assumed to possess:

- (i) a rigid front and central fuselage,
 - (ii) an unswept flexible wing which can twist about an unswept elastic, or flexural, axis and bend about the wing root,
 - (iii) an aft fuselage flexible in longitudinal bending
- and (iv) an unswept flexible tailplane, with a full span elevator, which can twist about an unswept tailplane flexural axis and bend about the tailplane root.

The control system can be either a manual system or a powered system.



Sketch 4.1 Model of a flexible aeroplane

This page Amendment A

The wing, in general, has built-in camber and washout (a built-in nose down twist which increases towards the wing tip) and is set at an angle, the 'wing setting angle', to the fuselage. The tailplane may be assumed to have zero camber and washout but is set at a fixed angle, the 'tailplane setting angle', to the fuselage.

The axis system, $Oxyz$, taken for the aeroplane are Attached axes, as defined in Sketch 3.1, fixed within the rigid front and central fuselage with the origin, O , taken at the centre of mass position, cm , of the aeroplane when in its 'jig' shape.

A sequence of models is considered, featuring an aeroplane with a flexible wing alone, a flexible aft fuselage alone, a flexible tailplane alone and then these flexibilities are combined. Their effects on overall wing and tailplane loads, on control angles to trim and on trim control characteristics are described.

4.2 Overall Trim Loads

Consider the flexible aeroplane as defined in Section 4.1 flying in steady level trimmed flight. The overall loads acting on the aeroplane as shown in Sketch 4.2 are assumed to consist of:

T	a thrust taken to act along the Ox axis,
L_W	a wing lift taken to act vertically upwards through the wing aerodynamic centre,
M_W	a wing moment, defined as positive when acting to rotate the nose of the aeroplane upwards and taken to act about a pitch axis through the wing aerodynamic centre,
L_T	a tailplane lift taken to act vertically upwards through the tailplane aerodynamic centre,
M_T	a tailplane moment, defined as positive when acting to rotate the nose of the aeroplane upwards and taken to act through the aerodynamic centre of the tailplane (M_T is due to the elevator),
D	an overall drag taken to act in the horizontal direction
and W	a weight which acts vertically downwards through the centre of mass, cm , of the total aeroplane.

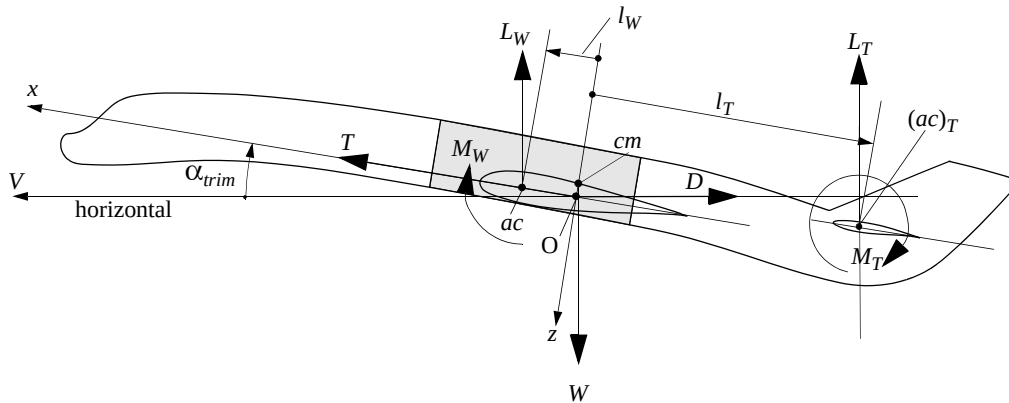
In this Item the lifts on the front fuselage and aft fuselage are neglected.

The aerodynamic centre of a wing or tailplane is defined as the centre of pressure of the incremental loading due to an incremental change of its angle of incidence. According to this definition the aerodynamic centre is independent of camber, washout and all deformations. The aerodynamic centre position of unswept wings and tailplanes lie close to their respective $1/4$ -chord points.

The following distances are defined:

l_W	=	the distance of the wing aerodynamic centre <i>ahead</i> of the axis origin of the aeroplane,
l_T	=	the distance of the tailplane aerodynamic centre <i>aft</i> of the axis origin of the aeroplane.

Since the fuselage aerodynamic lifts are ignored in this Item, the aerodynamic centre position of the wing-fuselage combination is identical with the position of the aerodynamic centre of the wing.



Sketch 4.2 Loads on flexible aeroplane in steady level trimmed flight

Throughout this Item representative numbers for a transport aeroplane will be used to illustrate the typical variation of quantities.

The distances of the aerodynamic centre of a wing and of a tailplane from the origin, O, are typically

$$l_W = 0.2\bar{c} \text{ and } l_T = 3.8\bar{c} ,$$

where $\bar{c}$ is the mean chord of the wing defined by wing area, S , over wing span, b .

Steady bending of an unswept wing or tailplane does not induce any significant loading, since steady bending does not change the local incidence along the span. Consequently for an unswept configuration in steady motion the only wing and tailplane deformation that result in changes to the aerodynamic loading is twist.

For a rigid unswept wing the pitching moment about the aerodynamic centre, M_W , is due to wing camber, since the loads due to washout and setting angle act on the locus of sectional aerodynamic centres across the span and do not contribute to M_W .

For a flexible unswept wing, the spanwise aerodynamic loading induced by structural spanwise twist deformation acts along the line of aerodynamic centres across the wing and so this loading does not contribute to M_W .

The moment, M_W , on an unswept wing is independent of the loading due to wing deformation and depends only on built-in wing camber.

In trimmed flight, assuming a small trim angle of incidence, $\cos(\alpha_{trim}) \approx 1$, and neglecting the contribution to lift from the thrust, the following equilibrium equations are:

$$T \approx D, \tag{4.1}$$

$$L_W \approx \frac{W l_T - M_W - M_T}{l_W + l_T} \tag{4.2}$$

$$\text{and } L_T \approx \frac{W l_W + M_W + M_T}{l_W + l_T} ; \tag{4.3}$$

to obtain the last two equations, moments are taken about the aerodynamic centre of the tailplane and wing, respectively. It is assumed in these equations that the difference between taking the weight, W , through the cm and the origin, O , is negligible.

$$\text{Typically} \quad \frac{M_T}{W} \approx 0.005 \bar{c},$$

which is small compared with l_T ($\sim 3.8 \bar{c}$) and l_W ($\sim 0.2 \bar{c}$). Hence, expressions for the wing and tailplane trim lifts, using Equations (4.2) and (4.3), may then be simplified to

$$L_W \approx \frac{W l_T - M_W}{l_W + l_T} \quad (4.4)$$

$$\text{and} \quad L_T \approx \frac{W l_W + M_W}{l_W + l_T}. \quad (4.5)$$

According to Equations (4.4) and (4.5) the overall wing and tailplane trim lifts for an unswept aeroplane configuration are independent of airframe flexibility.

4.3 Rigid Aeroplane with Zero Wing Camber, Zero Wing Wash-out and Zero Wing and Tailplane Setting Angles

The rigid aeroplane treated in this section is used as a reference for comparison with the various flexible configurations considered later.

The simplified model of a rigid aeroplane with an unswept wing possessing zero built-in camber and twist and with zero setting angles of the wing and tailplane to the fuselage is treated in this section. For this configuration

$$M_W = 0, \quad l_W = 0.2 \bar{c} \quad \text{and} \quad l_T = 3.8 \bar{c}$$

and so the wing and tailplane lifts from Equations (4.4) and (4.5) are

$$L_W \approx 0.95 W \quad (4.6)$$

$$\text{and} \quad L_T \approx 0.05 W. \quad (4.7)$$

For this model the wing angle of incidence is identical with the aeroplane angle of incidence, α_{trim} . Denoting a non-dimensional lift curve slope as $C_{L\alpha}$, the total wing lift may be expressed as

$$L_W = \frac{1}{2} \rho V_{trim}^2 S (C_{L\alpha})_W \alpha_{trim}. \quad (4.8)$$

The trim incidence for the rigid aeroplane, $(\alpha_{trim})_{rigid}$, with the use of Equation (4.4), may then be written as

$$(\alpha_{trim})_{rigid} = \frac{C_Q}{(C_{L\alpha})_W} \left(\frac{l_T}{l_W + l_T} \right), \quad (4.9)$$

where C_Q is the weight coefficient defined in Equation (3.2).

Typically, neglecting Mach number effects,

$$(C_{L\alpha})_W \approx 4.5 \text{ per rad} \quad \text{and} \quad \frac{l_T}{l_W + l_T} \approx 0.95.$$