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Slope of aerofoil lift curve for subsonic two-dimensional flow

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Mr J. Tweedie	— Short Brothers plc, Belfast
Mr A.J. Wells	— Avro International Aerospace, Woodford

* Corresponding Member.

The construction and subsequent development of the Data Item was undertaken by

Mr C.J. Batt — Engineer.

SLOPE OF AEROFOIL LIFT CURVE FOR SUBSONIC TWO-DIMENSIONAL FLOW

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SLOPE OF AEROFOIL LIFT CURVE FOR SUBSONIC TWO-DIMENSIONAL FLOW

1. NOTATION AND UNITS

		<i>SI</i>	<i>British</i>
$(a_1)_T$	inviscid lift-curve slope	degree ⁻¹	degree ⁻¹
$(a_1)_v$	viscous lift-curve slope	degree ⁻¹	degree ⁻¹
$(a_1)_v/(a_1)_T$	lift-curve slope reduction factor		
$\Delta[(a_1)_v/(a_1)_T]$	increment in $(a_1)_v/(a_1)_T$ due to camber		
c	aerofoil chord	m	ft
M	Mach number		
R	Reynolds number based on free-stream conditions and c		
t	maximum thickness of aerofoil section	m	ft
t_ξ	aerofoil thickness at ξ , measured perpendicular to the chord line, see Sketch 3.1	m	ft
x_{tr}	mean chordwise position of upper and lower surface boundary-layer transition, measured aft from leading edge	m	ft
$z_{u\xi}$	aerofoil upper-surface ordinate at ξ , see Sketch 3.2	m	ft
β	$(1 - M^2)^{1/2}$, Prandtl-Glauert compressibility factor		
ξ	chordwise distance aft from aerofoil leading edge as fraction of chord		
ξ_m	position of aerofoil maximum thickness as fraction of chord, measured aft from leading edge		
τ_a	aerofoil trailing-edge parameter as defined in Equation (3.1), see Sketch 3.1	degree	degree
τ_{au}	aerofoil upper-surface parameter as defined in Equation (3.2), see Sketch 3.2	degree	degree

Subscripts

$[\]_{sym}$	denotes value for symmetrical aerofoil
$[\]_{camb}$	denotes value for cambered aerofoil

2. INTRODUCTION

This Item provides a means of predicting the viscous lift-curve slope of aerofoil sections. The method obtains values for the lift-curve slope reduction factor, $(a_1)_v/(a_1)_T$, and requires the inviscid lift-curve slope, $(a_1)_T$, to be taken from Item No. 72024 (Reference 4). When applying the method, values of Reynolds number, and Mach number, and the boundary-layer transition position on the upper and lower surfaces, as well as certain aerofoil geometric parameters, need to be known.

The VGK CFD code (Derivations 1 and 2) was used in conjunction with a variety of NACA aerofoils to obtain the lift-curve slope reduction factor data that form the basis of this Item.

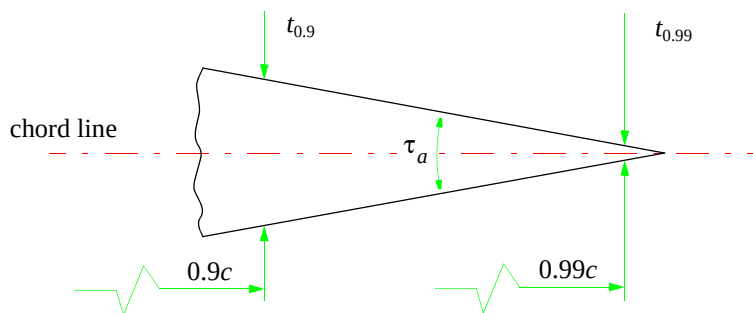
The method accounts for the effect of compressibility but is limited to subcritical flows. Details of the applicability and accuracy of the method are given in Section 7. Worked examples showing the application of the method are given in Section 9.

3. DEFINITION OF TRAILING-EDGE PARAMETERS τ_a AND τ_{au}

The method relies on defining the aerofoil geometry in terms of two trailing-edge parameters.

The trailing-edge parameter τ_a is the angle defined by

$$\tan(\tau_a/2) = \frac{t_{0.9} - t_{0.99}}{0.18c} \quad (3.1)$$

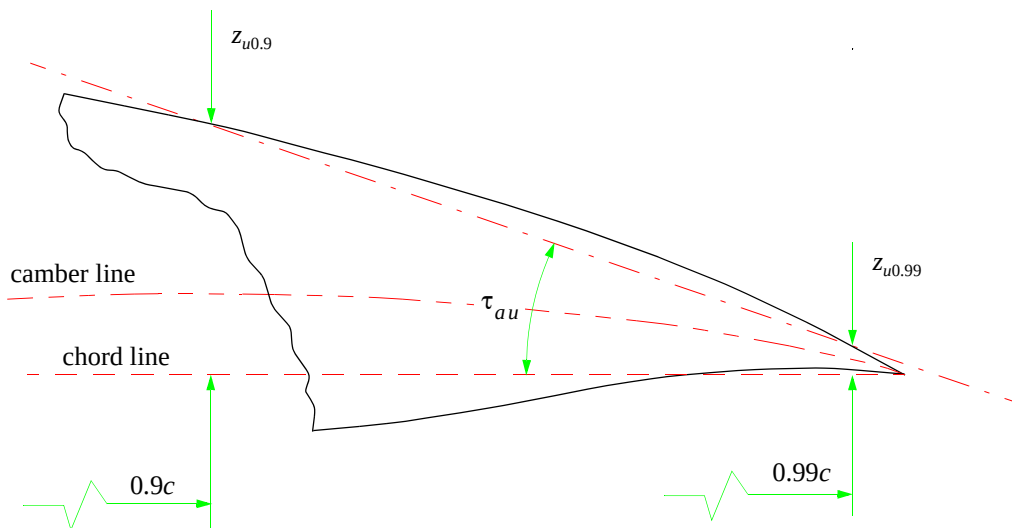


Sketch 3.1 Definition of angle τ_a

This definition provides a suitable analytical parameter for symmetrical aerofoils, even if there is some rounding or finite thickness at the trailing edge.

An additional trailing-edge parameter τ_{au} , an upper-surface angle, is needed for cambered aerofoils. It is defined by

$$\tan \tau_{au} = \frac{z_{u0.9} - z_{u0.99}}{0.09c} \quad (3.2)$$



Sketch 3.2 Definition of upper-surface angle τ_{au}

4. VISCOUS EFFECTS

In an inviscid flow the circulation about an aerofoil, and hence the lift, can be determined from the aerofoil geometry and incidence. An aerofoil in viscous flow develops a boundary layer on each surface with that on the upper surface being, in general, thicker than that on the lower surface when the lift is positive. The boundary-layer displacement thickness causes a reduction in circulation which can be calculated if the boundary-layer profile at the trailing edge is known. One primary viscous effect is that an increase in Reynolds number lessens the reduction in circulation. The boundary-layer transition positions on the upper and lower aerofoil surfaces influence the boundary-layer displacement thicknesses at the trailing edge. Hence variations in boundary-layer transition position also affect the circulation about an aerofoil.

Natural transition positions are not easy to predict and so this Item is concerned only with specified or fixed transition.

The extent of the adverse pressure gradient over the aerofoil surface is a major factor affecting the boundary-layer thickness at the trailing edge, and the boundary layer grows more rapidly over the rear of the aerofoil where the adverse pressure gradient is largest. The aerofoil geometry affects the aerofoil pressure distribution, and the importance of various geometric parameters was considered in the development of the method. These parameters were the angles τ_a and τ_{au} , the maximum thickness to chord ratio, t/c , and the position of the maximum thickness, ξ_m . It was concluded that τ_a was the parameter most affecting the reduction in circulation for symmetrical aerofoils. For cambered aerofoils both τ_a and τ_{au} are significant, the importance of τ_{au} increasing with camber. The other geometric parameters were found to affect the reduction in circulation, but their effect was of secondary importance.

5. EFFECT OF COMPRESSIBILITY

This Item deals only with conditions where the flow over the aerofoil is wholly subsonic. In order to maintain subcritical airflow, the aerofoil incidence range was kept small and the maximum thickness to chord ratio was restricted in the VGK runs performed to establish the theoretical data base for this Item.

The effect of viscosity on circulation is considerably greater at higher Mach numbers due to the more adverse pressure gradient near the trailing edge.

For small incidence ranges the lift-curve slopes of the aerofoils considered were found to be linear whatever the Mach number.

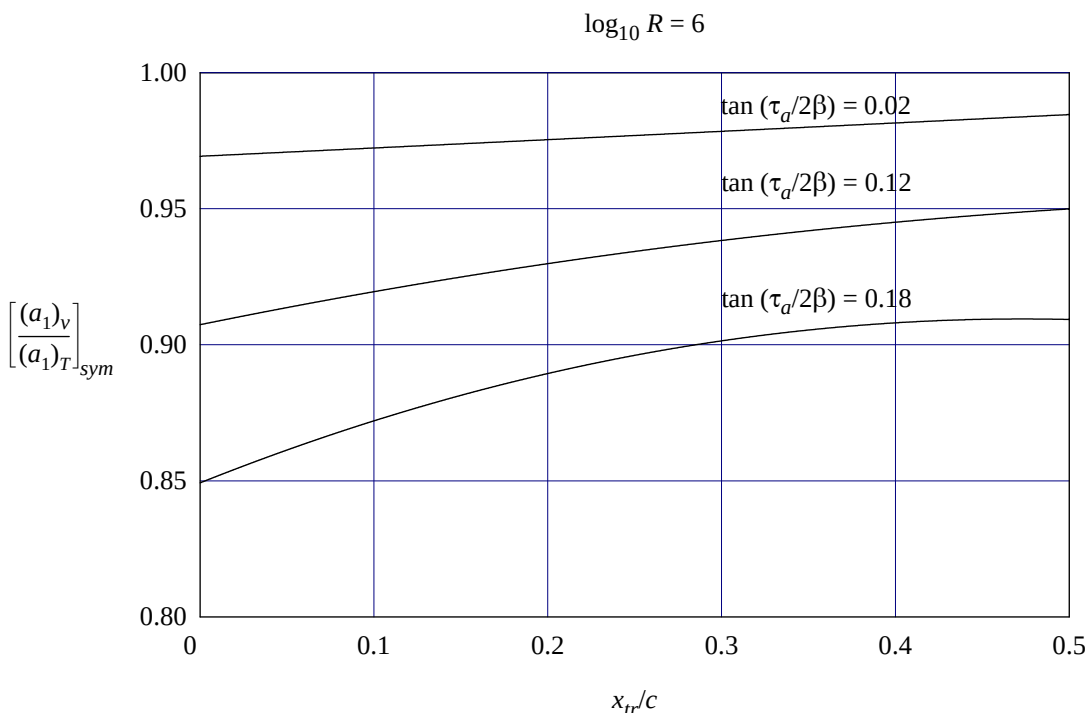
6. METHOD

6.1 Symmetrical Aerofoils

6.1.1 Incompressible flow, $M = 0$

Figures 1 to 6 give charts for the variation of the lift-curve slope reduction factor with Reynolds number and trailing-edge parameter τ_a , for symmetrical section aerofoils for specified boundary-layer transition positions of $x_{tr}/c = 0, 0.1, 0.2, 0.3, 0.4$ and 0.5 , respectively.

The variation of the lift-curve slope reduction factor with transition position is sufficiently linear to permit linear interpolation at intermediate fixed-transition positions between those covered by the charts, see Sketch 6.1.



Sketch 6.1 Variation of $(a_1)_v/(a_1)_T$ with x_{tr}/c .

Natural transition occurs on an aerofoil surface at a chordwise position dependent on the aerofoil geometry and the flow conditions. However, in the VGK program transition may be specified upstream of its natural position, and this was done in the construction of Figures 1 to 6.

The charts take into account differing boundary-layer transition positions on the upper and lower surfaces by using the average of the two positions, x_{tr} .

The viscous lift-curve slope is obtained by applying the lift-curve slope reduction factor from this Item to the inviscid lift-curve slope obtained from Item No. 72024; a check on values of the inviscid lift-curve slope from Item No. 72024 showed them to be in good agreement with values determined by using VGK.

6.1.2 Compressible Flow

To allow for the effect of compressibility, in Figures 1 to 6 the charts are presented in terms of τ_a/β rather than τ_a . The effect of the inclusion of the Prandtl-Glauert factor, $\beta (= (1 - M^2)^{-1/2})$, is that $(a_1)_V/(a_1)_T$ decreases with rising Mach number. However, the viscous lift-curve slope, $(a_1)_V$, still rises with Mach number as does the inviscid lift-curve slope, $(a_1)_T$, albeit at a slower rate than $(a_1)_T$.

6.2 Cambered Aerofoils

6.2.1 Incompressible flow, $M = 0$

The effect of camber is to reduce the value of the viscous lift-curve slope. As for the symmetrical aerofoil the method requires a knowledge of the aerofoil geometry in the trailing-edge region; it is applicable to complex as well as simple camber lines.

For camber lines that depart from the chord line by less than $0.02c$ the increment in $(a_1)_V/(a_1)_T$ due to camber, $\Delta[(a_1)_V/(a_1)_T]$, is small, negative and hardly significant. However, for larger cambers $\Delta[(a_1)_V/(a_1)_T]$ can exceed -0.005 in magnitude and so a camber correction method becomes necessary.

The additional trailing-edge angle parameter that is needed to account for aerofoil camber variation is τ_{au} , see Equation (3.2) and Sketch 3.2. The parameter τ_{au} suffices to account for variations in the amount of camber and the chordwise position of maximum camber.

For incompressible flow Figures 7 to 9 for $\log_{10} R = 6, 7$ and 8 can be used with $M = 0$, to obtain the change in lift-curve slope reduction factor due to camber in terms of the composite trailing-edge parameter, $\tan \tau_{au} - \tan(\tau_a/2)$. (This parameter is an approximation to the tangent of the angle of intersection of the chord line and the line passing through the camber-line ordinates at $\xi = 0.9$ and $\xi = 0.99$.)

The first stage in the method is to find the lift-curve slope reduction factor due to the aerofoil thickness distribution, using the symmetrical aerofoil method.

The next stage is to obtain the change in lift-curve slope reduction factor due to camber, using Figures 7 to 9. Linear interpolation may be employed between Figures 7 to 9 for intermediate values of $\log_{10} R$.