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Similarity rules for application in aircraft performance work

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THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item which was monitored and guided by the Performance Committee. This Committee first met in 1946 and now has the following membership:

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SIMILARITY RULES FOR APPLICATION IN AIRCRAFT PERFORMANCE WORK

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SIMILARITY RULES FOR APPLICATION IN AIRCRAFT PERFORMANCE WORK

1. NOTATION

		SI	British
A	aspect ratio, b^2/S	—	—
B	free parameter	—	—
b	span	m	ft
C_D	pressure drag coefficient	—	—
$\hat{C}_D$	transformed pressure drag coefficient (see Sections 4.2 and 5)	—	—
C_L	lift coefficient	—	—
C_m	pitching moment coefficient	—	—
C_p	pressure coefficient	—	—
c	chord	m	ft
$F_i[\]$	arbitrary function of the quantity in $[\dots]$, $i = 0 \dots n$	—	—
f_D	pressure drag function	—	—
f_L	lift function	—	—
f_m	pitching moment function	—	—
k	factor used in representation, $\tau = k\tau_0$	—	—
l	length of body	m	ft
M	Mach number	—	—
r	radial coordinate	m	ft
S	wing reference area	m ²	ft ²
t	thickness	m	ft
V	freestream velocity	m/s	ft/s
x, y, z	Cartesian coordinates	m	ft
γ	ratio of specific heats	—	—
θ	angular measure defined in text	—	—
τ	thickness parameter	—	—
τ_0	reference value of τ in $\tau = k\tau_0$	—	—
ϕ	(velocity) potential function	—	—
χ_j	arbitrary function, $j = 1 \dots n$	—	—

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2. INTRODUCTION

In order to formulate generalised databases from *ad hoc* experimental data it is convenient to have a simple theoretical framework within which to work. Once constructed, such databases permit the speedy and economical evaluation of, for example, an initial design concept.

To provide such a framework, this Data Item presents some aspects of the rules of *subsonic*, *transonic* and *supersonic* similarity[†]. These are derived from approximate forms of the differential equations that govern the flow fields around various shapes of interest.

Airframes may be characterised as relatively thin planar surfaces attached to bodies of revolution, so it is possible, in formulating a data base for initial design studies, to make use of predictions relating to these simple forms. The relationships presented here suffice to describe empirical data over a considerable range of flight speeds and geometries even though the underlying approximations would not suggest this outcome. However, it is important to be aware of the limitations and range of applicability of the theoretical forms so that the merit of any empirical correlation can be properly assessed.

The Data Item presents methods of representing the solutions to the relevant linear, inviscid, differential equations for (potential) flow about a two-dimensional (thin) aerofoil and an axisymmetric body. A formulation of the solution is suggested that extends the range of Mach number over which the parameters may be used to describe experimental information. The extension of the two-dimensional case to three-dimensional flow is summarised. The formulation suggested for the simpler instance is shown to be equally applicable in this case. In addition, there is a discussion of the non-linear equation that is more nearly applicable to the strict transonic case where the local velocities are close to the speed of sound. The form of the dependence of aerodynamic coefficients on thickness is illustrated for one case.

The use of the rules in relation to Performance studies is further illustrated in a series of examples issued as separate Data Items.

3. THE SIMILARITY RULES

3.1 General

The reader is referred to the References in Section 6 for details of the procedures involved in the formulation of, and the methods used to solve, the differential equations. Sections 3.2 to 3.7 contain statements of the similarity rules as they apply to pressure coefficient on a surface and this is of interest in its own right. However, in the absence of viscous effects, forces and moments due to the whole flowfield can be assumed to be related in a simple manner to the pressure distribution.

3.2 Two-Dimensional[‡] Linearised Flow

It is assumed that the perturbation velocities in the co-ordinate directions due to the presence of an object in the flow are small in comparison with the speed in the freestream. This is nearly the case for subsonic and supersonic flow when the freestream Mach number is not too close to unity. The relevant differential

[†] A comparable concept, from elsewhere in aeronautical engineering, is the notion of *dynamical similarity*, which is based on *dimensional analysis* and has been used both to design geometrically scaled models for testing and to formulate methods for correlating the results with full-scale data. The outcome of that process is to enable interpolation and, sometimes, extrapolation of full-scale data that are often limited in scope because of the expense and difficulty of testing.

[‡] This does not include the case of axisymmetric flow – see Section 3.6

equation is

$$\frac{\partial^2 \phi}{\partial x^2} + \left(\frac{1}{1-M^2} \right) \frac{\partial^2 \phi}{\partial z^2} = 0. \quad (3.1)$$

When the freestream Mach number approaches unity the first term in Equation (3.1) becomes comparable with those that have been disregarded; the equation is therefore not applicable in that case – see Section 3.3.

Equation (3.1) may be solved for shapes which can be characterised by

$$\frac{z}{c} = \tau F_0 \left[\frac{x}{c} \right] \quad (3.2)$$

where τ can be thought of as a thickness parameter for two-dimensional shapes, such as aerofoils, at zero angle of attack. For other cases, τ can be regarded as linked to both thickness and angle of attack. The form of τ is arbitrary in the sense that it is used to characterise geometric extent in the z -direction. Chordwise variation of τ can be explicitly defined as a function of the x -coordinate or implicitly defined as it is in the example of Section 5.

For $M < 1$ the solution of Equation (3.1) (in the elliptic form quoted) and Equation (3.2) can be represented as

$$\frac{C_p}{B} = F_1 \left[\frac{\tau}{B(1-M^2)^{1/2}} \right] \quad (3.3)$$

which is the general form for the equation connecting pressure coefficient, a thickness parameter and freestream Mach number in subsonic *two-dimensional, inviscid* and *linearised* flow.

For $M > 1$ the solution of Equation (3.1) (in hyperbolic form, *i.e.* the term in brackets is expressed as $-(1/(M^2 - 1))$) and Equation (3.2) can be represented as

$$\frac{C_p}{B} = F_2 \left[\frac{\tau}{B(M^2 - 1)^{1/2}} \right]. \quad (3.4)$$

Another representation of the solutions of Equations (3.1) and (3.2), which can apply both to subsonic and supersonic freestream conditions, is

$$\frac{C_p}{B} = F_3 \left[\frac{\tau^2}{B^2(1-M^2)} \right]. \quad (3.5)$$

Equations (3.3), (3.4) and (3.5) are the general forms of the classical similarity rules for subsonic and supersonic linearised flow. Because Equation (3.1) is homogeneous in the potential function, the value of B may be chosen at will – so permitting particular forms of the similarity rules to be derived. See Section 4 regarding the application of the similarity rules.

Note that, because of the influence of the thickness parameter and angle of attack, it is not possible to quote ranges of values of Mach number where the results for linearised flow may be used. For advice of this nature, the user is referred to the separate Data Items giving illustrative examples.