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One-dimensional representation of steady, spatially non-uniform flow An equivalent mean-value set for compressible flow Part 2. Implementation for an ideal, thermally-perfect gas

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ONE-DIMENSIONAL REPRESENTATION OF STEADY, SPATIALLY NON-UNIFORM FLOW. AN EQUIVALENT MEAN-VALUE SET FOR COMPRESSIBLE FLOW. PART 2. IMPLEMENTATION FOR AN IDEAL, THERMALLY-PERFECT GAS.

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ONE-DIMENSIONAL REPRESENTATION OF STEADY, SPATIALLY NON-UNIFORM FLOW. An Equivalent Mean-value Set for Compressible Flow.

Part 2. Implementation for an ideal, thermally-perfect gas.

1. INTRODUCTION

Despite the fact that some degree of spatial non-uniformity is present in the flow in all real systems, design and performance analysis methods for ducted-flow systems often involve an explicit or implicit assumption that the flow is uniform. Consequently, it is often necessary to formulate a set of representative mean properties to describe the flow and to use those in the analysis methods.

This Data Item describes a mean-value set based on an equivalent-state, reference-mean-flow model derived from measured[†], local properties at a cross section of the flow in a duct. The flow considered is a steady, spatially non-uniform (profiled), compressible flow of an ideal, thermally-perfect gas. It is assumed to be axial and free from significant swirl. Measurement uncertainties on the raw measurements are assumed to be accounted for separately so that the definition starts with a “corrected” set of measurements. The model allows for the simultaneous variation of total and static temperatures and pressures and other flow properties across the section in the flow and is applicable to both subsonic and supersonic flow.

The Item is one of a group concerned with the definition of a mean-value set for the one-dimensional representation of conditions, at a cross section of a duct, of a steady, profiled flow and their use in the analysis of system performance. Other Items in the group include Reference 8, for ideal, calorically-perfect gas flow (this can be derived from the present Item by assuming constant specific heat capacities), Reference 10, presenting numerical examples of the application, and Reference 12, presenting a mean-value set for incompressible flow.

The layout of the Item is as follows.

- Section 2 gives the notation and some notes on specific terminology used in the Item.
- Section 3 describes the requirements of a mean-value set and the choice made in defining the present set.
- Section 4 derives the reference-mean flow that forms the basis of the mean-value set.
- Section 5 considers the relationships between properties of the measured, profiled flow and its corresponding reference-mean flow and derives profile factors that complete the mean-value set.
- Section 6 presents selected analytical examples illustrating applications of the reference-mean-flow model to ducted flows and continuous-flow, adiabatic compression and expansion processes.
- Appendix A summarises the equations for the mean-value set.
- Appendix B summarises the equations for uniform flow of an ideal, thermally-perfect gas[‡]. These equations apply also to local conditions in a profiled flow.
- Appendix C provides a glossary of terms as they are used in this Data Item.

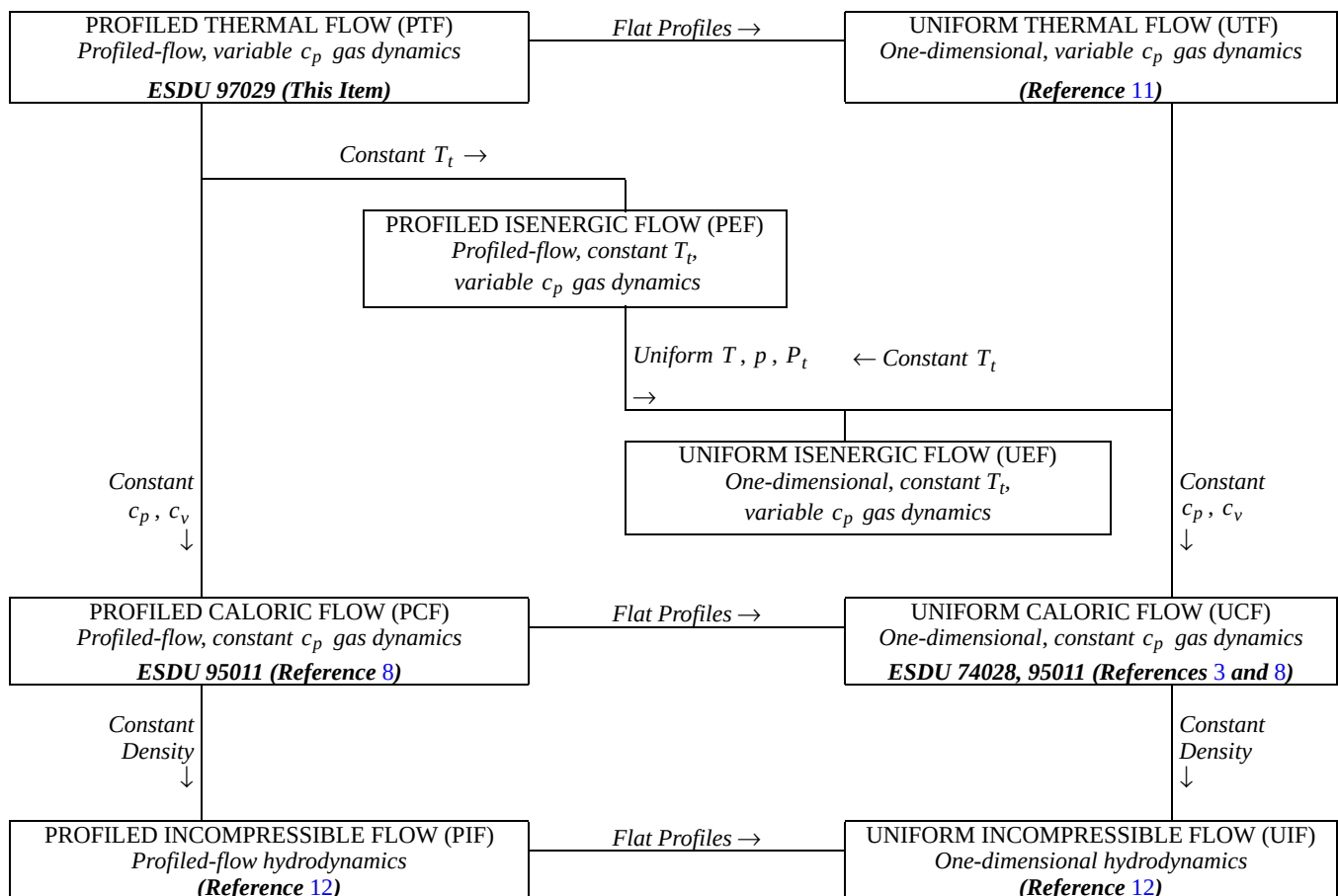
[†] The term “measured” may be extended to include local property values from computational methods (CFD).

[‡] A more extensive consideration of uniform flow of thermally-perfect gases is presented in Reference 11.

The mean-value set provides a means of characterising a profiled flow at a particular cross section in the flow. In order to assess the performance of a system it will be necessary to make measurements at more than one cross section and derive corresponding mean-value sets at each measurement section.

The application of the reference-mean-flow model at the measurement sections in continuous flow systems leads to a correct representation of the entropy at each section and hence to a proper accounting of the reversible and actual work or heat transfer in the processes between the measurement sections. Thus a correct distribution of work and irreversibilities among the components of multi-process systems can be made, avoiding the attribution of artificial flow-mixing losses to system components upstream or downstream of a measurement section.

The reference-flow model, based on the conservation of entropy and mass flux between the profiled and reference-mean flows, provides a useful unification of the treatment of continuous, adiabatic flows as illustrated by the schema in Sketch 1.1. Calorically-perfect gas flows emerge as special cases when c_p is kept constant. Isenergetic flows emerge as special cases when the total temperature is constant everywhere. Uniform flows are special cases when all properties are uniform across the section (flat profiles). The flow model applies to incompressible flows, both profiled and uniform, when the density is made constant.



Sketch 1.1 Schema Illustrating Unification of Flow Descriptions

The present mean-value set provides a basis for the assessment and comparison of historical and alternative mean values and mean-value sets.

2. NOTATION AND TERMINOLOGY

2.1 Notation

		SI Unit
A	cross-sectional area of flow at duct section	m^2
A_{eff}	effective cross-sectional area of flow at duct section,	m^2
a	speed of sound in gas	m/s
B_t, b_t	sectional and specific availability (see Appendix C)	J, J/kg
C_d	discharge coefficient or mass-flow profile factor	—
C_{pr}, C'_{pr}	diffuser static-pressure recovery coefficients	—
C_v	thrust coefficient	—
c_p	specific heat capacity at constant pressure	J/kg K
c_v	specific heat capacity at constant volume	J/kg K
E_{an}, e_{an}	sectional and specific energy (see Appendix C)	J, J/kg
E_{ex}, e_{ex}	sectional and specific exergy (see Appendix C)	J, J/kg
E_i, e_i	sectional and specific internal energy	J, J/kg
E_k, e_k	sectional and specific kinetic energy	J, J/kg
E_t, e_t	sectional and specific total energy	J, J/kg
F_a, f_a	sectional and specific absolute stream force	N, N/kg
F_g	gross thrust	N
F_M, f_M	sectional and specific momentum force	N, N/kg
H, h	sectional and specific enthalpy	J, J/kg
I_E	kinetic-energy profile factor	—
I_M	momentum profile factor	—
I_p	static-pressure profile factor	—
J	profile factor relating profiled-flow thermodynamic property value to reference-mean value (see Section 5 and Appendix A)	—
K_{pt}, K'_{pt}, K_t	diffuser total-pressure loss coefficients	—

M	Mach number	—
m_w	molecular weight of gas	kg/kg-mol
$\dot{m}$	mass flow rate or mass flux through section	kg/s
n	exponent in polytropic compression or expansion	—
p	static pressure	Pa (N/m ²)
p_{res}	reservoir pressure	Pa (N/m ²)
p_t	total pressure	Pa (N/m ²)
pr	isentropic pressure ratio	—
Q_1, Q_2, Q_3	mass flow functions	—
R	gas constant, $R = R_u/m_w$	J/kg K
R_u	universal gas constant, 8314 J/kg-mol K	J/kg-mol K
S, s	sectional and specific entropy	J/K, J/kg K
T	static temperature	K
T_a	ambient or environmental temperature	K
T_{res}	reservoir temperature	K
T_t	total temperature	K
V	axial velocity	m/s
v	specific volume	m ³ /kg
vr	isentropic specific volume ratio	—
γ	ratio of specific heat capacities, c_p/c_v	—
η	process efficiency or effectiveness	—
θ	specific constant-volume entropy function	J/kg K
ρ	static density of gas	kg/m ³
ρ_{res}	reservoir density of gas	kg/m ³
ρ_t	total density of gas	kg/m ³
ρr	isentropic density ratio	—
ϕ	specific constant-pressure entropy function	J/kg K

Subscripts

Where appropriate, subscripts are used in combination.

A	denotes integration over whole duct area at section
a	denotes value at ambient or environmental conditions
aw	denotes area-weighted-mean value (see Appendix C)
CM	denotes a continuity-mean value
hs, ht	denote values related to static and total enthalpy
i	denotes a value applicable to isentropic process (except as applied to E_i, e_i denoting internal energy)
$ideal$	applicable to ideal, nozzle expansion process
lm	denotes logarithmic-mean value (see Appendix C)
mw	denotes mass-flow-weighted-mean value (see Appendix C)
$\dot{m}$	denotes integration over whole of mass flow at section
p, p_t	denote values applicable to static and total pressures
res	denotes reservoir-state value
rms	denotes root-mean-square value
s	denotes static-state value
T, T_t	denote values applicable to static and total temperatures
t	denotes total-state value
θ_s, θ_t	denote values related to static and total constant-volume entropy function
ρ, ρ_t	denote values applicable to static and total densities
0	denotes value for a reference datum condition
1 to 6	denote values at successive sections along duct

Superscripts

$'$	Section 6.1.1, denotes an alternative parameter definition, Sections 6.2 to 6.4, denotes a value related to temperature-mixed flow condition
$\star$	denotes a value at sonic conditions ($M = 1$)
$\#$	denotes a value of pr related to exit temperature condition

Dressings

- denotes mean value of type defined by subscript
- ~ denotes reference-mean-flow value

2.2 Terminology

For convenience, the following abbreviations have been used in this Item:

- UCF refers to uniform flow of an ideal, calorically-perfect gas,
- UTF refers to uniform flow of an ideal, thermally-perfect gas,
- PCF refers to profiled flow of an ideal, calorically-perfect gas,
- PTF refers to profiled flow of an ideal, thermally-perfect gas.

The following operators are defined, relating to specific heat capacities:

- Tcp – True c_p operator, $c_p = \text{Tcp}\{T\}$, is interpreted as “ c_p is determined at the appropriate static temperature from an empirical $c_p \sim T$ relationship for the gas”,
- Tcv – True c_v operator, $c_v = \text{Tcv}\{T\}$, is interpreted as “ c_v is determined at the appropriate static temperature from an empirical $c_v \sim T$ relationship for the gas”.

The following further operators are defined, relating to other properties:

- Tse – True specific internal energy operator,
- Tsh – True specific enthalpy operator,
- Ts ϕ – True specific constant-pressure entropy function operator,
- Ts θ – True specific constant-volume entropy function operator,
- Tpr – True isentropic pressure ratio operator.

For example, $e_i = \text{Tse}\{T\}$ is interpreted as “the value of specific internal energy, e_i , is determined at the appropriate static temperature, T , from an empirical *specific-internal-energy* $\sim$ *temperature* relationship”.

The same empirical relationship is used[†] for the definition of both static-state and total-state values of a particular property, *e.g.* the static- and total-enthalpy definitions, $h_s = \text{Tsh}\{T\}$ and $h_t = \text{Tsh}\{T_t\}$, must both use the same *specific-enthalpy* $\sim$ *temperature* relationship.

Corresponding inverse operations are defined for each of the operators, using the same empirical relationship as for the forward operation. For example, the inverse of the operation $\phi_s = \text{Tsf}\{T\}$ is $T = \text{Tsf}^{-1}\{\phi_s\}$, interpreted as “the value of static temperature, T , is obtained from the value of specific static constant-pressure entropy function, ϕ_s , by use of an empirical, *specific* $\phi \sim$ *temperature* relationship”.

These operators are implemented through relationships in the form of tabulated data, graphical data or equations, as available.

[†] See Section B2 of Appendix B.

3. THE MEAN-VALUE SET

In order to account properly for the irreversibilities and heat, work and momentum changes involved in irreversible continuous-flow processes it is necessary, in any mean-value description, to represent the totality of the mass, entropy, enthalpy, momentum and other fluxes of the flow.

However, it is not possible to represent all of these fluxes for a non-uniform flow solely in terms of the properties of a consistent, uniform “reference flow”. The reference-flow properties have to be complemented by other factors in the definitions of one or more of the fluxes to represent properly the non-uniform flow at a section. A *mean-value set* therefore must include both the properties of some hypothetical uniform flow and a number of corresponding profile factors.

The “*reference-mean flow*” at a flow section is defined in Section 4 of this Item on the basis of the direct equivalence of the sectional mass flow and sectional entropy between the non-uniform flow and the reference-mean flow. The reference-mean static and total state properties are linked through isentropic relationships. This ensures that the concept of thermodynamic reversibility and correct accounting for irreversibilities can be extended to profiled flow of a thermally-perfect gas (PTF) and enables a thermodynamically-rigorous, uniform, reference-mean flow to be achieved.

The reference-mean-flow condition of the gas is dependent only on the profiles of flow properties at a section in the non-uniform flow and is independent of the process paths on either side of the section. It therefore meets the formal requirement of a thermodynamic state. Thus the proposed mean-value set enables the use of the temperature ~ entropy and Mollier diagrams for the gas to be extended to the analysis of non-uniform flow.

The conditions imposed on the reference-mean flow in Section 4.1 define its specific total entropy, $\bar{s}_t$. In order to specify its total state fully, one other thermodynamic property must be defined[†]. In the proposed methodology the reference-mean total pressure is chosen as discussed in Section 4. With the requirement of an isentropic relationship between total and static states, all other properties then follow from the conventional uniform flow relationships.

The *profile factors* required to complete the mean-value set are derived in Section 5. Some of these relate values of the sectional extensive properties, such as enthalpy and momentum, for the profiled flow directly to the corresponding reference-mean-flow values. Others relate mean intensive properties for the profiled flow to reference-mean-flow values, allowing the conventional uniform-flow relationships for specific extensive properties to be extended to profiled flow.

The mean-value set for the profiled, compressible flow of a calorically-perfect gas (PCF), given in ESDU 95011^[8], is a special case of the set for PTF and can be derived from it by setting c_p constant. The *definition* of the reference-mean flow for PTF is identical to that for PCF. The development differs solely as a consequence of the fact that the specific heat capacities are functions of temperature for a thermally-perfect gas whereas they are constant for a calorically-perfect gas.

[†] If more than two properties of the thermally-perfect gas are defined independently, the state of the reference-mean flow will be over-specified. In that case it will not be possible to write the governing equations of the reference-mean flow without the introduction of profile factors in addition to its basic properties.

4. THE REFERENCE-MEAN FLOW

The mean-value set is based upon a thermodynamically consistent, equivalent-state reference-mean flow, constrained by the conditions set out in Section 4.1. The principal intensive properties of the reference-mean flow are derived in Section 4.2 and the remainder of Section 4 develops its other properties, extending uniform-flow concepts to non-uniform (profiled) flow.

The properties of the reference-mean flow are distinguished by a tilde ($\sim$) dressing, other mean property values are distinguished by a bar ($\bar{}$) dressing and an appropriate subscript.

4.1 Definition

The reference-mean flow is an equivalent uniform flow of the same gas as in the measured, profiled flow and has the following basis:-

- (i) its properties are uniform across the section,
- (ii) its mass flow is equal to the sectional mass flow of the measured, non-uniform flow,
- (iii) its total-state properties are equal to those that obtain in a hypothetical reservoir that defines a quiescent end state into which the whole of the section flow is decelerated at constant sectional entropy and in which the ideal gas equation of state applies,
- (iv) its static-state properties are related isentropically to its total state properties,

These conditions are identical to those adopted for profiled flow of a calorically-perfect gas in ESDU 95011^[8].

4.2 Entropy Considerations and Reference-mean Intensive Properties

4.2.1 Reservoir conditions

In order to implement condition (iii) of Section 4.1, two independent state properties must be specified to define the reservoir state. In this Item, following custom and practice for UTF and the approach adopted in ESDU 95011 for PCF, pressure and temperature are chosen[†]. The choice of reservoir pressure as one property with the condition that specific entropy is conserved dictates that the reservoir temperature is defined from the constant-pressure entropy function. This choice is consistent with the fact that specific heat capacity variations in UTF and PTF occur only with temperature and with the important practical consideration that pressure is a directly-measured quantity.

The reservoir condition requires that the sectional total entropy of the profiled flow, S_t , is conserved in the reservoir, so that

$$S_{res} = S_t \quad (4.1)$$

and, since the mass flux is conserved in the reservoir, that the specific entropy in the reservoir is equal to

[†] In principle, density and temperature could be chosen as the definitive reservoir state properties and hence as the reference-mean total-state properties. Entropy conservation would then require that the reservoir temperature be defined from the constant-volume entropy function. This would lead to a different reference-mean total pressure from that of the present model. In PCF, with c_p constant, the two approaches lead to a common set of reference-mean intensive properties that are all logarithmic-mean values.

the mass-flow-weighted-mean total entropy of the profiled flow, that is

$$s_{res} = \bar{s}_{t,mw} . \quad (4.2)$$

The local[†], specific total entropy in a streamtube in the profiled flow is given, with an appropriate datum[‡], by

$$s_t = \int_{T_t} \frac{c_p}{T} dT - \int_{p_t} \frac{R}{p} dp = \phi_t - R \ln p_t , \quad (4.3)$$

where ϕ_t is the local total-state value of the constant-pressure entropy function and p_t is the local total pressure.

The mass-flow-weighted-mean, total-state entropy of the profiled flow is then

$$\bar{s}_{t,mw} = \frac{1}{\dot{m}} \int_{\dot{m}} s_t d\dot{m} = \frac{1}{\dot{m}} \int_{\dot{m}} [\phi_t - R \ln p_t] d\dot{m} . \quad (4.4)$$

Introducing the mass-flow-weighted-mean of the constant-pressure entropy function, $\bar{\phi}_{t,mw}$, given by

$$\bar{\phi}_{t,mw} = \frac{1}{\dot{m}} \int_{\dot{m}} \phi_t d\dot{m} = \frac{1}{\dot{m}} \int_{\dot{m}} \int_{T_t} \frac{c_p}{T} dT d\dot{m} , \quad (4.5)$$

and the logarithmic-mean total pressure, $\bar{p}_{t,lm}$, given by

$$\ln \bar{p}_{t,lm} = \frac{1}{\dot{m}} \int_{\dot{m}} \ln p_t d\dot{m} , \quad (4.6)$$

Equation (4.4) can be written as

$$\bar{s}_{t,mw} = \bar{\phi}_{t,mw} - R \ln \bar{p}_{t,lm} . \quad (4.7)$$

The reservoir specific entropy can be written as

$$s_{res} = \int_{T_{res}} \frac{c_p}{T} dT - \int_{p_{res}} \frac{R}{p} dp = \phi_{res} - R \ln p_{res} . \quad (4.8)$$

Substituting Equations (4.7) and (4.8) into Equation (4.2) then gives

$$\phi_{res} - R \ln p_{res} = \bar{\phi}_{t,mw} - R \ln \bar{p}_{t,lm} . \quad (4.9)$$

In order to maintain direct equality of the separate contributions of the pressure and temperature terms in Equation (4.9), the reservoir pressure is specified as equal to the logarithmic-mean of the values across the

[†] Relationships between properties in uniform flow of a thermally-perfect gas, which apply locally in the profiled flow, are summarised in Appendix B.

[‡] The requirements of the datum condition are discussed in Reference 11.

profile, that is,

$$p_{res} = \bar{p}_{t, lm} . \quad (4.10)$$

Equation (4.9) then requires that

$$\phi_{res} = \bar{\phi}_{t, mw} \quad (4.11)$$

and reservoir temperature is defined, using the $Ts\phi$ operator, as

$$T_{res} = Ts\phi^{-1}\{\phi_{res}\} = Ts\phi^{-1}\{\bar{\phi}_{t, mw}\} . \quad (4.12)$$

In the reservoir, the equation of state for the gas applies, so that

$$p_{res} = \rho_{res} RT_{res} \quad (4.13)$$

and hence the reservoir density is defined as

$$\rho_{res} = \frac{p_{res}}{RT_{res}} . \quad (4.14)$$

All other properties of the reservoir state can be defined in terms of the uniform flow relationships in Appendix B.

4.2.2 Reference-mean-flow total state

As, by condition (iii) of Section 4.1, the reference-mean total state properties are defined to be equal to the reservoir properties, the reference-mean total pressure is

$$\tilde{p}_t = p_{res} = \bar{p}_{t, lm} , \quad (4.15)$$

the reference-mean total temperature is

$$\tilde{T}_t = T_{res} = Ts\phi^{-1}\{\bar{\phi}_{t, mw}\} \quad (4.16)$$

and the reference-mean total density is

$$\tilde{\rho}_t = \rho_{res} = \frac{p_{res}}{RT_{res}} = \frac{\tilde{p}_t}{R\tilde{T}_t} . \quad (4.17)$$

The reference-mean specific total entropy can be expressed as

$$\tilde{s}_t = \tilde{\phi}_t - R \ln \tilde{p}_t , \quad (4.18)$$

where $\tilde{\phi}_t$ is the reference-mean value of the total-state, constant-pressure entropy function, defined by

$$\tilde{\phi}_t = \int_{\tilde{T}_t} \frac{c_p}{T} dT \quad (4.19)$$

and equal to the reservoir value and hence to the mass-flow-weighted-mean value, so that,

$$\tilde{\phi}_t = \phi_{res} = \bar{\phi}_{t, mw} . \quad (4.20)$$

The entropy can be expressed also in terms of the reference-mean constant-volume entropy function and reference-mean density, relative to the same datum as for Equation (4.18), as

$$\tilde{s}_t = \tilde{\theta}_t - R \ln \tilde{\rho}_t - R \ln R , \quad (4.21)$$

where $\tilde{\theta}_t$ is given by

$$\tilde{\theta}_t = \text{Ts}\theta\{\tilde{T}_t\} = \int_{\tilde{T}_t} \frac{c_v}{T} dT . \quad (4.22)$$

The central difference of PTF from PCF lies in the difference in the $\phi \sim T$ relationship, leading to a difference in the evaluation of $\tilde{T}_t$. For PCF, with c_p constant, Equation (4.5) can be reduced as follows,

$$\bar{\phi}_{t, mw} = \frac{1}{\dot{m}} \int_{\dot{m}} \int_{T_t} \frac{c_p}{T} dT d\dot{m} \Rightarrow \bar{\phi}_{t, mw} = \frac{c_p}{\dot{m}} \int_{\dot{m}} \int_{T_t} \frac{1}{T} dT d\dot{m} = c_p \int_{\dot{m}} \ln T_t d\dot{m} , \quad (4.23)$$

so that in PCF, the reference-mean total temperature, defined as in PTF by $\tilde{T}_t = \text{Ts}\phi^{-1}\{\bar{\phi}_{t, mw}\}$, can be evaluated as the logarithmic-mean value, given by

$$\ln \bar{T}_{t, lm} = \frac{1}{\dot{m}} \int_{\dot{m}} \ln T_t d\dot{m} . \quad (4.24)$$

From the equation of state, the reference-mean total density is then the logarithmic-mean value, given by

$$\ln \bar{\rho}_{t, lm} = \frac{1}{\dot{m}} \int_{\dot{m}} \ln \rho_t d\dot{m} . \quad (4.25)$$

Thus, the reference-mean total pressure for PTF is the logarithmic-mean value, $\bar{p}_{t, lm}$, as it is for PCF (ESDU 95011^[8]), and the reference-mean total temperature and total density for PTF reduce to the logarithmic-mean values, $\bar{T}_{t, lm}$ and $\bar{\rho}_{t, lm}$, which are the reference-mean values for PCF, if c_p is made constant.

For uniform flow, UTF, Equation (4.5) reduces to the conventional definition

$$\bar{\phi}_{t, mw} = \frac{1}{\dot{m}} \int_{\dot{m}} \int_{T_t} \frac{c_p}{T} dT d\dot{m} \Rightarrow \phi_t = \int_{T_t} \frac{c_p}{T} dT \quad (4.26)$$

and the reference-mean properties are then the same as those of the uniform flow.

4.2.3 Reference-mean-flow static state

Condition (iv) of Section 4.1 requires conservation of entropy between static and total states as well as between the reference-mean flow, the reservoir and the profiled flow. The reference-mean specific static entropy is equal therefore to the mass-flow-weighted-mean specific static entropy of the profiled flow and to the corresponding specific total entropies so that,

$$\tilde{s}_s = \bar{s}_{s,mw} = \tilde{s}_t = \bar{s}_{t,mw} = s_{res}. \quad (4.27)$$

Following a development similar to that in Section 4.2.1 (Equations (4.3) to (4.9)); the mass-flow-weighted-mean value of the specific static entropy can be written as

$$\bar{s}_{s,mw} = \frac{1}{\dot{m}} \int \dot{m}_s s_s d\dot{m} = \frac{1}{\dot{m}} \int \dot{m} [\phi_s - R \ln p] d\dot{m}, \quad (4.28)$$

which, with the mass-flow-weighted mean of the constant-pressure entropy function defined by

$$\bar{\phi}_{s,mw} = \frac{1}{\dot{m}} \int \dot{m} \phi_s d\dot{m} = \frac{1}{\dot{m}} \int \dot{m} \int_T \frac{c_p}{T} dT d\dot{m}, \quad (4.29)$$

can be written as

$$\bar{s}_{s,mw} = \bar{\phi}_{s,mw} - R \ln \bar{p}_{lm}, \quad (4.30)$$

and the reference-mean static entropy can be written as

$$\tilde{s}_s = \tilde{\phi}_s - R \ln \tilde{p}, \quad (4.31)$$

where
$$\tilde{\phi}_s = \int_T \frac{c_p}{T} dT, \quad (4.32)$$

leading to

$$\tilde{\phi}_s - R \ln \tilde{p} = \bar{\phi}_{s,mw} - R \ln \bar{p}_{lm}. \quad (4.33)$$

Since, locally, p_t and p are related isentropically then the mean values $\bar{p}_{t,lm}$ and $\bar{p}_{lm}$ are also related isentropically. Condition (iv) is satisfied with respect to pressures, therefore, if

$$\tilde{p} = \bar{p}_{lm}. \quad (4.34)$$

Equation (4.33) then requires that

$$\tilde{\phi}_s = \bar{\phi}_{s,mw} \quad (4.35)$$

and the reference-mean static temperature is given by

$$\tilde{T} = \text{TS}\phi^{-1}\{\tilde{\phi}_s\} = \text{TS}\phi^{-1}\{\bar{\phi}_{s,mw}\}. \quad (4.36)$$

As for the total-state values, the specific static entropy of the reference-mean flow can be expressed also in terms of the constant-volume specific-entropy function and the density as

$$\tilde{s}_s = \tilde{\theta}_s - R \ln \tilde{\rho} - R \ln R, \quad (4.37)$$

where
$$\tilde{\theta}_s = \int_{\tilde{T}} \frac{c_v}{T} dT. \quad (4.38)$$

Equating the total and static specific entropies of the reference-mean flow, Equations (4.18) and (4.31) lead to

$$\tilde{\phi}_t - \tilde{\phi}_s = R \ln \frac{\tilde{p}_t}{\tilde{\rho}} \quad (4.39)$$

and Equations (4.21) and (4.37) to

$$\tilde{\theta}_t - \tilde{\theta}_s = R \ln \frac{\tilde{p}_t}{\tilde{\rho}}. \quad (4.40)$$

From the definitions of θ and ϕ ,

$$\tilde{\phi}_s - \tilde{\theta}_s = R \ln \tilde{T}_t \quad \text{and} \quad \tilde{\phi}_s - \tilde{\theta}_s = R \ln \tilde{T}. \quad (4.41)$$

Combining Equations (4.39), (4.40) and (4.41) then shows that

$$R \ln \frac{\tilde{p}_t}{\tilde{\rho}} - R \ln \frac{\tilde{\rho}_t}{\tilde{\rho}} = R \ln \frac{\tilde{T}_t}{\tilde{T}} \quad (4.42)$$

or
$$\frac{\tilde{p}_t}{\tilde{\rho}} = \frac{\tilde{\rho}_t R \tilde{T}_t}{\tilde{\rho} R \tilde{T}}. \quad (4.43)$$

Since the thermal equation of state applies to the total-state properties, $\tilde{p}_t = \tilde{\rho}_t R \tilde{T}_t$, Equation (4.43) confirms that it applies also to the static-state properties, $\tilde{p} = \tilde{\rho} R \tilde{T}$. The static density can be derived therefore as

$$\tilde{\rho} = \frac{\tilde{p}}{R \tilde{T}} = \frac{\bar{p}_{lm}}{R \tilde{T}}. \quad (4.44)$$

As for the total-state properties, the reference-mean static pressure for PTF is the logarithmic-mean value, $\bar{p}_{lm}$, as it is for PCF. The reference-mean static temperature and static density are not the same as for PCF, but can be shown to reduce, if c_p is made constant, to the logarithmic-mean values, defined by

$$\ln \bar{T}_{lm} = \frac{1}{\dot{m}} \int \ln T d\dot{m} \quad \text{and} \quad \ln \bar{\rho}_{lm} = \frac{1}{\dot{m}} \int \ln \rho d\dot{m}, \quad (4.45)$$

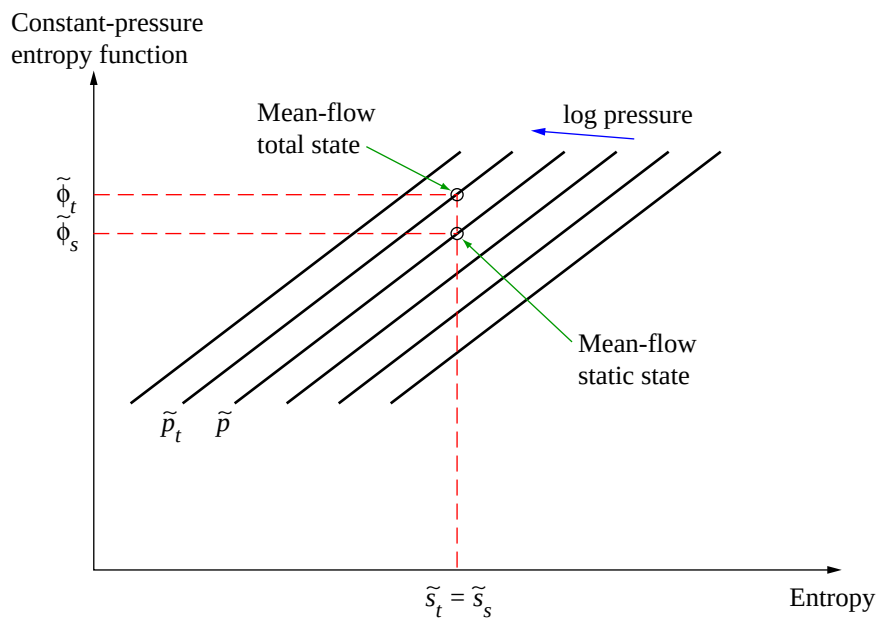
which are the PCF values.

4.2.4 The $\phi \sim s$ and $T \sim s$ diagrams

Sketch 4.1 shows the general form of the $\phi \sim s$ diagram for a thermally-perfect gas. The reference-mean-flow total and static states can be expressed in functional form as

$$\tilde{\phi}_t = \phi\{\tilde{s}_t, \tilde{p}_t\} \text{ and } \tilde{\phi}_s = \phi\{\tilde{s}_s, \tilde{p}\}, \quad (4.46)$$

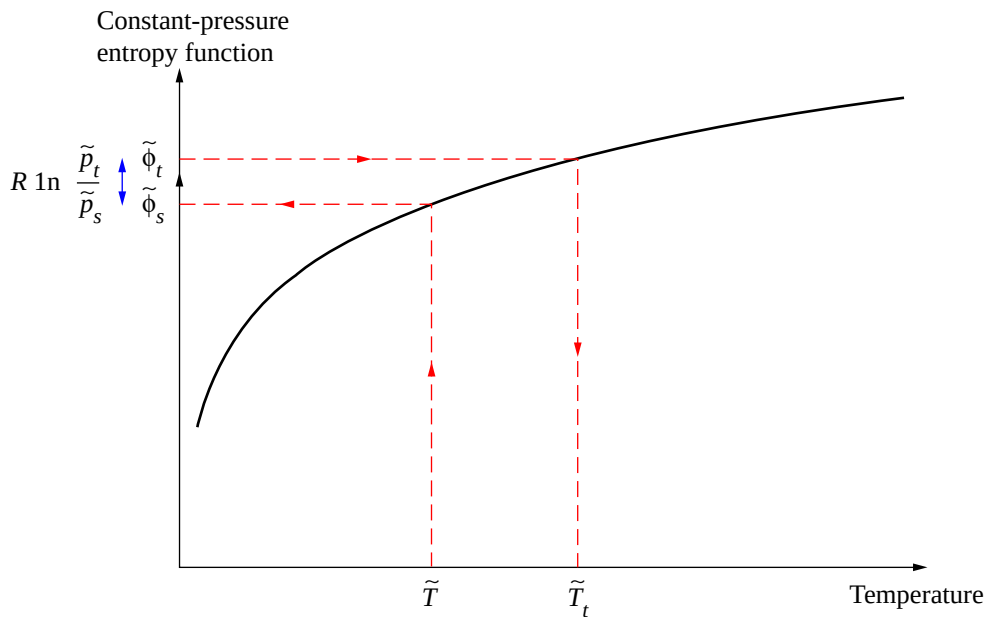
where these are explicit functions. These states can be represented on the $\phi \sim s$ diagram as shown in the sketch, extending the application of the diagram to profiled flow.



Sketch 4.1 The $\phi \sim s$ diagram for a thermally-perfect gas with the reference-mean states for a profiled flow

The isentropic relationship between the reference-mean total and static pressures and the corresponding entropy functions (Equation (4.39)) is also shown on the sketch.

Sketch 4.2 shows the $\phi \sim T$ relationship representing the $Ts\phi$ operator. By use of the inverse operator, $Ts\phi^{-1}$, the reference-mean-flow temperatures can be obtained from the constant-pressure entropy functions and the total and static states can then be represented on a $T \sim s$ diagram.



Sketch 4.2 The $\phi \sim T$ relationship for a thermally-perfect gas with the Ts_ϕ and Ts_ϕ^{-1} operations for the reference-mean flow

4.2.5 Isentropic pressure, density and specific-volume ratios

The pressure ratios for *isentropic* processes between a datum pressure, p_0 , and the reference-mean static and total pressures $\tilde{p}$ and $\tilde{p}_t$, are functions only of the corresponding isentropic temperature difference and can be expressed in terms of the constant-pressure entropy functions as

$$p\tilde{r}_s = \frac{\tilde{p}}{p_0} = \exp\left(\frac{\tilde{\phi}_s - \phi_0}{R}\right) \quad (4.47)$$

and

$$p\tilde{r}_t = \frac{\tilde{p}_t}{p_0} = \exp\left(\frac{\tilde{\phi}_t - \phi_0}{R}\right), \quad (4.48)$$

where ϕ_0 is the value of ϕ at the pressure p_0 and at the same entropy as $\tilde{\phi}_s$ and $\tilde{\phi}_t$.

Similarly, the density ratios for isentropic processes between a datum density, ρ_0 , and the reference-mean static and total densities, $\tilde{\rho}$ and $\tilde{\rho}_t$, are functions only of the corresponding isentropic temperature difference and can be expressed in terms of the constant-volume entropy functions as

$$\rho\tilde{r}_s = \frac{\tilde{\rho}}{\rho_0} = \exp\left(\frac{\tilde{\theta}_s - \theta_0}{R}\right) \quad (4.49)$$

and

$$\rho\tilde{r}_t = \frac{\tilde{\rho}_t}{\rho_0} = \exp\left(\frac{\tilde{\theta}_t - \theta_0}{R}\right), \quad (4.50)$$

where θ_0 is the value of θ at the density ρ_0 and at the same entropy as $\tilde{\theta}_s$ and $\tilde{\theta}_t$.

The isentropic specific-volume ratios are the reciprocals of the isentropic density ratios and are given by

$$\tilde{v}r_s = \frac{\tilde{v}}{v_0} = \frac{1}{\tilde{\rho}r_s} = \exp\left(-\frac{\tilde{\theta}_s - \theta_0}{R}\right) \quad (4.51)$$

and

$$\tilde{v}r_t = \frac{\tilde{v}_t}{v_0} = \frac{1}{\tilde{\rho}r_t} = \exp\left(-\frac{\tilde{\theta}_t - \theta_0}{R}\right). \quad (4.52)$$

4.2.6 Total-to-static pressure and density ratios

Since the relationship between total and static pressures in the reference flow is, by definition, isentropic, the following important relationships can be written,

$$\frac{\tilde{p}_t}{\tilde{p}} = \frac{\tilde{p}r_t}{\tilde{p}r_s} = \exp\left(\frac{\tilde{\Phi}_t - \tilde{\Phi}_s}{R}\right) \quad (4.53)$$

and

$$\frac{\tilde{\rho}_t}{\tilde{\rho}} = \frac{\tilde{\rho}r_t}{\tilde{\rho}r_s} = \exp\left(\frac{\tilde{\theta}_t - \tilde{\theta}_s}{R}\right). \quad (4.54)$$

Thus in PTF, as in UTF, the total-to-static pressure and density relationships are not related explicitly to the total-to-static temperature ratio.

In contrast to PCF, for PTF the ratios $\tilde{T}_t/\tilde{T}$, $\tilde{p}_t/\tilde{p}$ and $\tilde{\rho}_t/\tilde{\rho}$ are related implicitly to the reference-mean Mach number, $\tilde{M}$. Since γ is a function of temperature for a thermally-perfect gas, relationships involving the explicit use of γ as an exponent are unhelpful in analysing UTF and PTF.

4.3 Enthalpy

The reference-mean specific static and total enthalpies are defined by

$$\tilde{h}_s = \int_{\tilde{T}} c_p dT \quad \text{and} \quad \tilde{h}_t = \int_{\tilde{T}_t} c_p dT, \quad (4.55)$$

that is, by

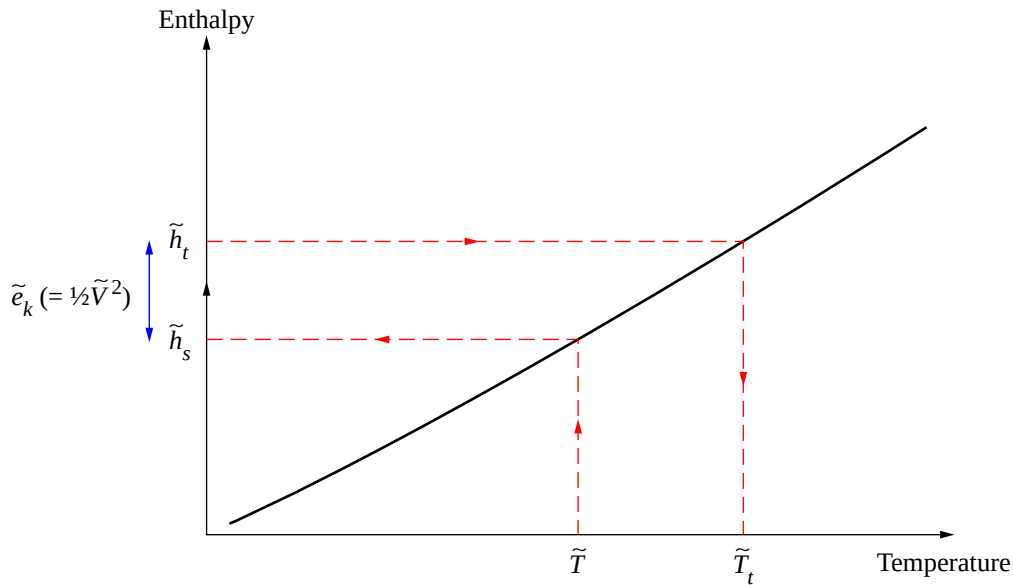
$$\tilde{h}_s = \text{Tsh}\{\tilde{T}\} \quad \text{and} \quad \tilde{h}_t = \text{Tsh}\{\tilde{T}_t\}. \quad (4.56)$$

The Tsh operation and its inverse are shown schematically in Sketch 4.3.

The corresponding reference-mean sectional values are given by

$$\tilde{H}_s = \dot{m}\tilde{h}_s \quad \text{and} \quad \tilde{H}_t = \dot{m}\tilde{h}_t. \quad (4.57)$$

For a calorically-perfect gas, with c_p constant, the curve in Sketch 4.3 becomes a straight line so that the reference-mean enthalpies for PCF are given by $\tilde{h}_s = c_p \tilde{T}$ and $\tilde{h}_t = c_p \tilde{T}_t$.



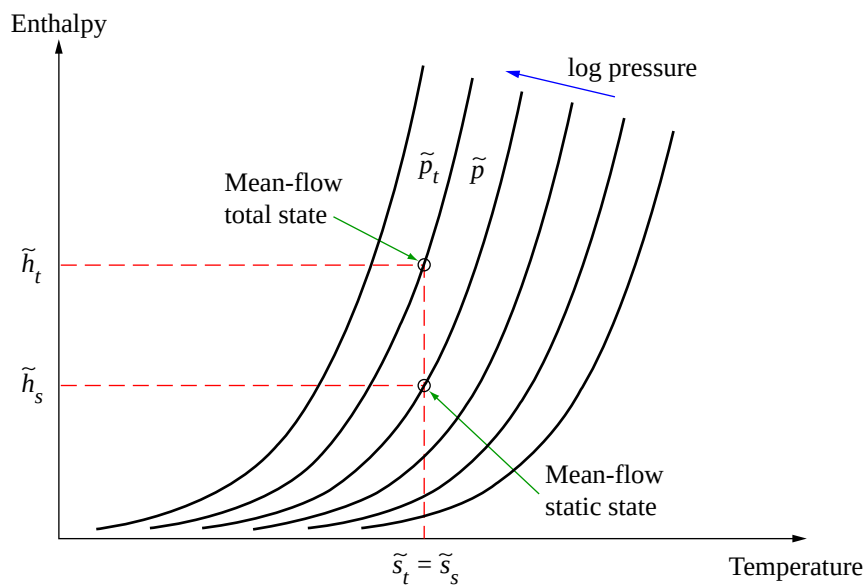
Sketch 4.3 The $h \sim T$ relationship for a thermally-perfect gas with the Tsh and Tsh⁻¹ operations for the reference-mean flow

4.3.1 The Mollier diagram

The reference-mean-flow states can be expressed also in terms of reference-mean enthalpy and entropy as

$$\tilde{h}_t = h\{\tilde{s}_t, \tilde{p}_t\} \text{ and } \tilde{h}_s = h\{\tilde{s}_s, \tilde{p}\}, \quad (4.58)$$

enabling the states to be shown on an enthalpy ~ entropy or Mollier diagram for the gas, as in Sketch 4.4.



Sketch 4.4 The Mollier diagram for a thermally-perfect gas with the reference-mean-flow states for a profiled flow

The isentropic relationship between the reference-mean total and static enthalpies is also shown on the Sketch.