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Design and performance evaluation of heat exchangers: The effectiveness – N_{TU} method

Part 1: Introduction

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DESIGN AND PERFORMANCE EVALUATION OF HEAT EXCHANGERS: THE EFFECTIVENESS - N_{TU} METHOD

PART 1: INTRODUCTION

1. NOTATION AND UNITS

Two sets of self-consistent units are given.

		Units	
		SI	British
A	effective surface area of heat exchanger, see Section 4.1	m^2	ft^2
$a_0 - a_2$	regression coefficients in Equation (5.9)	—	—
B	coefficient in Equation (5.11)	—	—
$b_0 - b_4$	regression coefficients in Equation (5.10)	—	—
C	constant of integration	—	—
C^*	thermal capacity ratio, defined in Section 4.2.3	—	—
c_0	regression coefficient in Equation (5.10)	—	—
c_p	specific heat capacity	J/kg K	Btu/lb °F
$d_0 - d_4$	regression coefficients in Equation (5.12)	—	—
D	coefficient in Equation (5.10)	—	—
E	thermal effectiveness, defined in Section 4.2.1	—	—
E_l	upper limit of effectiveness, see Equation (5.13)	—	—
E_{tm}	approximation to limit of effectiveness for temperature meet, see Equation (5.14)	—	—
E_0	effectiveness for minimum number of thermal plates in plate-type exchanger	—	—
F_T	correction factor in LMTD method	—	—
G	coefficient in Equation (5.14)	—	—
$g_0 - g_2$	regression coefficients in Equation (5.14)	—	—

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H	enthalpy, $\dot{M}c_p T$	J	Btu
$\dot{M}$	mass flow rate	kg/s	lb/h
N	number of thermal plates in plate-type exchanger (see Section 5.4)	—	—
N_{ch}	number of channels for each stream in plate-type exchanger	—	—
N_p	number of stream passes in plate-type exchanger	—	—
N_{TU}	number of heat transfer units, defined in Section 4.2.2	—	—
P	thermal ratio (alternative symbol for effectiveness), or percentage of asymptotic value of effectiveness	—	—
$\dot{Q}$	rate of heat transfer between streams (heat load)	W	Btu/h
R	alternative definition of thermal capacity ratio, defined in Section 4.3	—	—
S	number of identical heat exchangers (shells) in train	—	—
T	stream temperature	K	°R
U	overall heat transfer coefficient, see Section 4.1	W/m ² K	Btu/h ft ² °F
ΔT	temperature difference between streams	K or °C	°F or °R
ΔT_c	temperature difference defined by Equation (C1.5)	K	°F or °R
ΔT_h	temperature difference defined by Equation (C1.4)	K	°F or °R
δ	factor on N_{TU} for F_T , see Equation (B1.6)	—	—
Γ	correction factor defined in Equation (5.8)	—	—

Subscripts

<i>asympt</i>	value as N_{TU} tends to infinity
<i>co</i>	co-current flow
<i>corr</i>	correction to asymptotic value
<i>cu</i>	counter-current flow
<i>in</i>	value of parameter at inlet to heat exchanger
<i>larger</i>	larger value of parameter in streams
<i>lm</i>	log mean value

<i>m</i>	mean value
<i>max</i>	maximum value
<i>min</i>	minimum value
<i>out</i>	value of parameter at outlet from heat exchanger
<i>smaller</i>	smaller value of parameter in streams
1	refers to stream number 1 or location 1
2	refers to stream number 2 or location 2

2. PURPOSE AND SCOPE

This is the first of a group of Data Items concerning the effectiveness (E) - number of heat transfer units (N_{TU}) method for the thermal analysis of two-stream heat exchangers. Aspects of the method presented in the Data Items in the group are as follows.

Data Item Number	Title	Description
98003	Part 1. Introduction	Description and overview of the $E-N_{TU}$ method.
98004	Part 2. Performance comparisons and selection of configurations presented in Part 3	Comparative performance and performance limitations of particular heat exchanger configurations.
98005	Part 3. Graphical and analytical data	Data required to use the method. The range of heat exchangers covered includes pure counter-current, pure co-current, crossflow, shell-and-tube and plate heat exchanger configurations.
98006	Part 4. Case studies	Case studies, incorporating all elements of the method presented.
98007	Part 5. Analytical techniques	Numerical and algebraic techniques for relating heat transfer effectiveness, E , to the number of heat transfer units, N_{TU} .

This Data Item describes the $E - N_{TU}$ method and its application to the design, performance evaluation and performance limitations of specified heat exchange equipment. The main features of the method are listed below.

General features of method	• Applicable to all heat exchangers, from simple pipework to trains of shell-and-tube exchangers. • Allows direct performance comparison of exchangers and exchanger configurations. • Indicates suitable exchanger configurations for a given thermal specification (that is, for known stream temperatures, flow rates and specific heat capacities). • Allows assessment of total possible heat exchange.
In exchanger design	• Allows rapid selection of a range of suitable heat exchanger configurations (and elimination of others) for a given thermal specification. • Indicates minimum terminal temperature difference between the streams, ΔT_{min}, and subsequent potential for process integration.
In exchanger performance evaluation	• Allows rapid assessment of the suitability of an existing exchanger for a new thermal specification. • Indicates directly any requirement for temporary additional flow control on clean exchangers, prior to expected in-service fouling being attained. • Indicates heat loading in the exchanger (see Section 5.1.1)

3. APPLICATION OF THE $E - N_{TU}$ METHOD

3.1 General Assumptions

The method, in common with **all** similar methods used in two-stream heat exchanger design, is based on the following assumptions:

- heat exchange with the surroundings is negligible,
- constant specific heat capacities are assumed,
- the overall heat transfer coefficient between the streams is constant throughout the heat exchanger,
- mixing occurs in each stream so that fluid temperature is uniform over any cross section at right angles to the direction of flow,
- where a heat exchanger consists of multiple parallel paths, such as tubes in a row or channels within adjacent plates, the flow rates and heat transfer areas in each path are identical.

Any further assumptions relating to specific geometries are discussed in detail where data for these geometries are presented.

3.2 Stream Assumptions

A number of additional assumptions apply depending on the phases of the streams, and are summarised in the table below. Although the method can be used most reliably where heat is exchanged between two single-phase fluid streams, it can be used as a first approximation in a number of other applications. For example, it is applicable in the design of air compressor inter- and after-coolers where the air side is predominantly single phase, but where account must be taken of small amounts of condensation of water vapour.

Phase of cold stream	Phase of hot stream	Additional assumptions and notes.
Single (all gas or all liquid)	Single (all gas or all liquid)	Variations in specific heat capacity with temperature are assumed to be small. See Section 3.3 if this is not the case.
Isothermal boiling of pure substance	Single (all gas or all liquid)	The method is applicable where the effective specific heat capacity, c_p , in the stream with the phase change is infinite (that is, the case of isothermal or near-isothermal boiling or condensing). Under such circumstances, where the thermal capacity ratio, C^* , is zero, the heat transfer relationships are identical for most heat exchangers.
Single (all gas or all liquid)	Isothermal condensation of pure substance	
Boiling of mixture	Single (all gas or all liquid)	The method is applicable when there is a linear relationship between heat release and temperature change (that is, when the effective specific heat capacity, c_p , is a function of sensible and latent heat), and the overall heat transfer coefficient, U , can be assumed to remain constant throughout the exchanger. The latter condition will not hold if there is a significant change of phase.
Single (all gas or all liquid)	Condensation of mixture	

3.3 Validity of the Method

The applicability of the $E - N_{TU}$ data in the design and performance evaluation of heat exchangers can be assessed from the list of assumptions in Sections 3.1 and 3.2. It must be recognised that, even when the method is applied to single-phase fluids, the assumptions are unlikely to be met completely. Analyses of designs of heat exchangers where small variations of overall heat transfer coefficient or specific heat capacity occur show that the $E - N_{TU}$ method can be applied using average values of the varying parameters without introducing significant errors. For larger variations it is possible to use an incremental approach, whereby the exchanger model is broken down into a number of sections over which local constant values of overall heat transfer coefficient or specific heat capacity are assumed. The N_{TU} for the whole exchanger is given by the sum of the N_{TU} values of all sections.

Although the data presented only cover two-stream heat exchangers, the $E - N_{TU}$ method can be applied to exchangers with three or more streams; consideration of such cases is outside the scope of this group of Data Items.

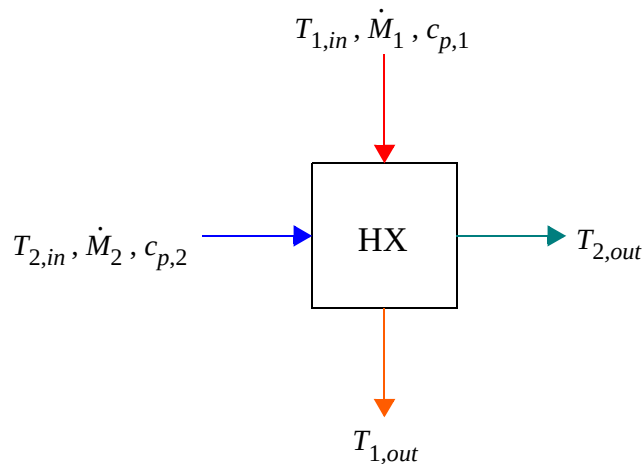
Because of the likely deviation from the idealised assumptions of conditions in real heat exchangers, there is little merit in seeking greater algebraic precision in the relationships between E and N_{TU} than is provided by the graphical and algebraic presentations.

4. DESCRIPTION OF THE $E - N_{TU}$ METHOD

The method of analysing the transfer of heat in heat exchangers presented in these Items was first published in the 1930s and subsequently developed by, amongst others, Ten Broeck¹, Kern², Jakob³, Hausen⁴, Taborek⁵, Kays and London⁶ and in unpublished work by ESDU. Its applicability was until recent years restricted by the large computational calculations necessary to generate the design charts for a comprehensive range of heat exchanger types and configurations.

Data for a wide range of exchanger configurations are given in Part 3 of this group of Data Items, in graphical, analytical and empirical (where appropriate) forms.

4.1 Model of Two-stream Heat Exchanger



Sketch 4.1 Simplified representation of a heat exchanger

The method starts from the consideration of a generalised heat exchanger in which heat is transferred from a hot stream (stream 1) to a cold stream (stream 2).

In passing through the heat exchanger, the hot stream cools from $T_{1,in}$ to $T_{1,out}$ while the cold stream is heated from $T_{2,in}$ to $T_{2,out}$. By a simple heat balance the rate of heat transfer (which is commonly referred to as the “thermal duty” or “heat load” of the exchanger) between the two streams is

$$\dot{Q} = (\dot{M}c_p)_1(T_{1,in} - T_{1,out}) = (\dot{M}c_p)_2(T_{2,out} - T_{2,in}), \quad (4.1)$$

where $(\dot{M}c_p)_1$ and $(\dot{M}c_p)_2$ are the average thermal capacity rates in each stream. Alternatively, the rate of heat transfer may be expressed in terms of the larger and smaller stream properties as

$$\dot{Q} = (\dot{M}c_p)_{\text{smaller}} |(T_{in} - T_{out})|_{\text{larger}} \quad (4.2)$$

$$\text{or} \quad \dot{Q} = (\dot{M}c_p)_{\text{larger}} |(T_{in} - T_{out})|_{\text{smaller}}. \quad (4.3)$$

In addition the rate of heat transfer may also be expressed by

$$\dot{Q} = UA\Delta T_m, \quad (4.4)$$

where U is the overall heat transfer coefficient between the streams, A is the effective heat transfer surface area of the heat exchanger and ΔT_m is the effective mean temperature difference between the streams.

Care must be taken when evaluating U and A to ensure that they are compatible. For example, for a shell-and-tube exchanger, if U is calculated on the basis of the outside tube surface then the area term in Equation (4.4) will be that of the outside surface of the tubes.

4.2 Definitions of Non-dimensional Parameters

The $E - N_{TU}$ method allows a range of heat exchangers capable of achieving a required duty to be selected or the performance of specified exchangers to be assessed. Three non-dimensional parameters are used to achieve this:

- thermal effectiveness, E (see Section 4.2.1),
- number of heat transfer units, N_{TU} (see Section 4.2.2), and
- thermal capacity ratio, C^* (see Section 4.2.3).

Their relationship may best be illustrated by the analysis of a simple counter-current heat exchanger, described in detail in Appendix A. In principle, expressions can be derived for any geometry and flow arrangement in a heat exchanger in the general form

$$E = \text{function}(N_{TU}, C^*). \quad (4.5)$$

4.2.1 Thermal Effectiveness - E

The thermal effectiveness is the ratio of the actual rate of heat transfer in a heat exchanger to the thermodynamic maximum rate between the two streams, and is defined by

$$E = \frac{\text{actual rate of heat transfer}}{\text{maximum possible rate of heat transfer}} = \frac{\dot{Q}}{\dot{Q}_{max}}, \quad (4.6)$$

where E is the proportionality factor between the actual rate of heat transfer and the maximum rate of heat transfer possible. The thermal effectiveness describes the performance of a given heat exchanger compared with the performance of an infinitely large counter-current flow heat exchanger supplied with the same streams at the same process conditions. The maximum amount of heat that can be transferred per unit time in the counter-current flow heat exchanger is limited by the stream with the smaller thermal capacity rate and the theoretical maximum temperature change that could occur in either stream, that is $(T_{1,in} - T_{2,in})$, where $T_{1,in}$ is the inlet temperature of the hot stream and $T_{2,in}$ is the inlet temperature of the cold stream:

$$\dot{Q}_{max} = (\dot{M}c_p)_{\text{smaller}}(T_{1,in} - T_{2,in}). \quad (4.7)$$

By substitution of Equation (4.7) into Equation (4.6), this maximum possible rate of heat transfer can be re-expressed by introducing the proportionality factor E as follows.

$$\dot{Q} = E\dot{Q}_{max} = E(\dot{M}c_p)_{\text{smaller}}|T_{1,in} - T_{2,in}|. \quad (4.8)$$

Substitution of Equation (4.2) or (4.3) into Equation (4.8) relates E to the larger or smaller temperature difference in the heat exchanger, as follows

$$E = \frac{|(T_{in} - T_{out})_{larger}|}{(T_{1,in} - T_{2,in})} \times \frac{(\dot{M}c_p)_{smaller}}{(\dot{M}c_p)_{smaller}} \quad (4.9)$$

$$E = \frac{|(T_{in} - T_{out})_{smaller}|}{(T_{1,in} - T_{2,in})} \times \frac{(\dot{M}c_p)_{larger}}{(\dot{M}c_p)_{smaller}}, \quad (4.10)$$

and Equation (4.9) is a common definition of thermal effectiveness

$$E = \frac{|(T_{in} - T_{out})_{larger}|}{(T_{1,in} - T_{2,in})}. \quad (4.11)$$

The “ineffectiveness” of heat transfer between the two streams, expressed as $1 - E$, is

$$\begin{aligned} 1 - E &= \frac{\text{maximum possible rate of heat transfer} - \text{actual rate of heat transfer}}{\text{maximum possible rate of heat transfer}} \\ &= \frac{|(T_{1,in} - T_{2,in})| - |(T_{in} - T_{out})_{larger}|}{|(T_{1,in} - T_{2,in})|} \\ &= \frac{\Delta T_{smaller}}{|(T_{1,in} - T_{2,in})|}, \end{aligned} \quad (4.12)$$

where $\Delta T_{smaller}$ is the modulus of the smaller of the temperature differences $|T_{1,in} - T_{2,out}|$ or $|T_{2,in} - T_{1,out}|$.

The use of “ineffectiveness” and $\Delta T_{smaller}$ in process integration (where it is referred to as ΔT_{min}) is discussed in Appendix C.

Thermal effectiveness, like the thermal capacity ratio, is dependent upon the applied process conditions, and not upon the properties of the heat exchanger. It therefore does not have a unique value for a particular heat exchanger type.

It should be noted that the subscripts ‘smaller’ and ‘larger’ refer to the magnitude of the terms and not to one or other of the streams. Therefore $(\dot{M}c_p)_{smaller}$ and $|(T_{in} - T_{out})_{larger}|$ relate to the same stream.

4.2.2 Number of transfer units - N_{TU}

The number of heat transfer units, N_{TU} , is a non-dimensional combination of the physical size of a heat exchanger (as indicated by the area for heat transfer, A), the overall heat transfer coefficient between the two streams, U , and the smaller of the two thermal capacity rates, $(\dot{M}c_p)_{smaller}$:

$$N_{TU} = \frac{UA}{(\dot{M}c_p)_{smaller}}. \quad (4.13)$$

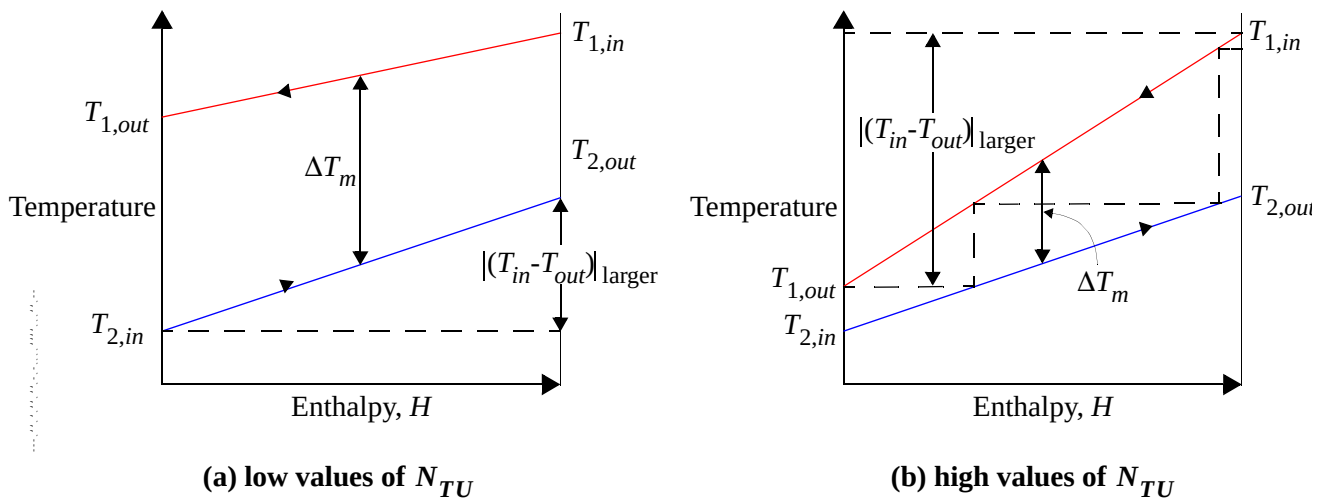
The number of heat transfer units can alternatively be described as the heat exchanging capacity between the two streams, as shown in Equation (4.15) and given by the substitution of Equations (4.2) and (4.4) in Equation (4.13):

$$N_{TU} = \frac{UA}{(\dot{M}c_p)_{\text{smaller}}} \times \frac{(\dot{M}c_p)_{\text{smaller}} |(T_{in} - T_{out})_{\text{larger}}|}{UA \Delta T_m}, \quad (4.14)$$

so

$$N_{TU} = \frac{|(T_{in} - T_{out})_{\text{larger}}|}{\Delta T_m}. \quad (4.15)$$

From Equations (4.13) and (4.15), it can be seen that N_{TU} can take any positive value. Experience has shown that heat exchangers commonly have N_{TU} values between 0.5 and 4. This does not mean that a heat exchanger design for which the N_{TU} value lies outside this range is not satisfactory. In some cases, for example with compact exchangers, the N_{TU} value can be significantly higher and will still produce a feasible design. Guidance on design N_{TU} ranges is given in Section 5.



Sketch 4.2 Plots of temperature versus enthalpy in heat exchangers

Sketch 4.2 shows two plots of temperature versus enthalpy for two-stream heat exchangers. For truly counter-current configurations ΔT_m will be ΔT_{lm} (that is, the LMTD), see Equation (B1.2). In Sketch 4.2a the mean temperature difference is large compared with the change in temperature of either stream. This indicates a “soft” duty as the effectiveness is low (see Section 5.1.1) and the two outlet temperatures are far from the conditions of an overall temperature meet (see Section 5.1.3). In Sketch 4.2b the change in temperature of either stream (or both as shown) is large in comparison with the mean temperature difference. The condition of an overall temperature meet occurs (see Section 5.1.3). In this case the thermal effectiveness is large, indicating a higher heat duty and a higher value of N_{TU} : this is a “hard” duty (see Section 5.1.1).

Values of N_{TU} can be estimated using the “stepping off” method which is a graphical means of evaluating Equations (4.13) and (4.15). The dashed lines on Sketch 4.2b illustrate this method, whereby each full step is one heat transfer unit, and incomplete steps are a portion of an N_{TU} . For Sketch 4.2b the approximate value of N_{TU} is 2.2 whereas, for Sketch 4.2a, it is less than unity. This technique, which is approximate except for cases where $C^* = 1$, is unlikely to be used in detailed calculations, but can give a quick assessment of the severity of a process demand; that is, whether the duty is “hard” or “soft”.

It should be noted again that the subscripts ‘smaller’ and ‘larger’ refer to the magnitude of the terms and not to one or other of the streams. Therefore $(\dot{M}c_p)_{\text{smaller}}$ and $|(T_{in} - T_{out})_{\text{larger}}|$ relate to the same stream.