

## **Flexibility of, and load distribution in, multi-bolt lap joints subject to in-plane axial loads**

**Associated software: ESDUpac A9812**

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## FLEXIBILITY OF, AND LOAD DISTRIBUTION IN, MULTI-BOLT LAP JOINTS SUBJECT TO IN-PLANE AXIAL LOADS

### CONTENTS

|                                                                                                               | Page      |
|---------------------------------------------------------------------------------------------------------------|-----------|
| <b>1. INTRODUCTION, THEORY AND APPLICATION</b>                                                                | <b>1</b>  |
| <b>1.1 INTRODUCTION</b>                                                                                       | <b>1</b>  |
| <b>1.2 NOTATION</b>                                                                                           | <b>1</b>  |
| <b>1.2.1 Suffixes</b>                                                                                         | <b>3</b>  |
| <b>1.3 ANALYSIS OF MULTI-BOLT SINGLE LAP JOINTS</b>                                                           | <b>4</b>  |
| <b>1.3.1 Analysis of Load Distribution in a Joint</b>                                                         | <b>5</b>  |
| <b>1.3.2 Joint Displacement, Flexibility and Stiffness</b>                                                    | <b>6</b>  |
| <b>1.3.3 Calculation of Bolt Flexibility</b>                                                                  | <b>7</b>  |
| <b>1.3.3.1 Basic relationships</b>                                                                            | <b>7</b>  |
| <b>1.3.3.2 Governing equation</b>                                                                             | <b>8</b>  |
| <b>1.3.3.3 Boundary conditions</b>                                                                            | <b>8</b>  |
| <b>1.4 APPLICATION OF THE THEORY FOR CALCULATING THE FLEXIBILITY OF A BOLT IN A SINGLE PLATE</b>              | <b>9</b>  |
| <b>1.4.1 Parameters Required to Calculate Joint Flexibilities</b>                                             | <b>9</b>  |
| <b>1.4.1.1 Effective stiffness, <math>k</math></b>                                                            | <b>10</b> |
| <b>1.4.1.2 Rotational restraint, <math>\lambda</math></b>                                                     | <b>11</b> |
| <b>1.4.1.3 Effective shear area, <math>A_{ps}</math></b>                                                      | <b>11</b> |
| <b>1.4.1.4 Effective areas of the plates to account for the presence of bolt holes</b>                        | <b>11</b> |
| <b>1.4.2 Influence of the Major Factors Governing the Flexibility of a Single Bolt</b>                        | <b>12</b> |
| <b>1.4.3 Application of Flexibility of a Bolt in a Single Plate to a Bolt in a Single Lap Joint</b>           | <b>13</b> |
| <b>1.4.4 Application of Flexibility of a Bolt in a Single Plate to a Bolt in a Symmetric Double Lap Joint</b> | <b>13</b> |
| <b>1.5 APPLICATION OF THE THEORY FOR CALCULATING JOINT FLEXIBILITY</b>                                        | <b>14</b> |
| <b>1.6 DERIVATION</b>                                                                                         | <b>14</b> |
| <b>2. ESDUpac A9812</b>                                                                                       | <b>23</b> |
| <b>2.1 INTRODUCTION TO THE PROGRAM</b>                                                                        | <b>23</b> |
| <b>2.2 PROGRAM SCOPE AND LIMITATIONS</b>                                                                      | <b>23</b> |
| <b>2.3 DATA INPUT REQUIREMENTS FOR THE FORTRAN PROGRAM</b>                                                    | <b>24</b> |
| <b>2.3.1 Units</b>                                                                                            | <b>24</b> |
| <b>2.3.2 Loading, Number of Bolts and Material Properties of the Plates</b>                                   | <b>24</b> |
| <b>2.3.3 Geometry of Plates Between Bolts</b>                                                                 | <b>25</b> |
| <b>2.3.4 Bolt Details</b>                                                                                     | <b>25</b> |
| <b>2.3.4.1 Bolt flexibilities known</b>                                                                       | <b>25</b> |
| <b>2.3.4.2 Bolt flexibilities to be calculated</b>                                                            | <b>25</b> |
| <b>2.3.5 Input checks</b>                                                                                     | <b>25</b> |

|            |                                                                 |           |
|------------|-----------------------------------------------------------------|-----------|
| <b>2.4</b> | <b>PROGRAM OUTPUT</b>                                           | <b>28</b> |
| 2.4.1      | Output of Input Data                                            | 28        |
| 2.4.2      | Interpretation of Successful Runs                               | 28        |
| 2.4.3      | Interpretation of Solutions With Warnings                       | 28        |
| 2.4.4      | Failure to Provide a Solution Owing to Errors in the Input Data | 29        |
| 2.4.5      | Failure to Provide a Solution With Valid Input Data             | 31        |
| <b>2.5</b> | <b>EXAMPLES</b>                                                 | <b>32</b> |
| 2.5.1      | Example 1, Single Lap, Single-bolt Joint                        | 32        |
| 2.5.1.1    | Input File for Joint with 6 mm Bolt                             | 33        |
| 2.5.1.2    | Output File for Joint with 6 mm Bolt                            | 33        |
| 2.5.2      | Example 2, Multi-bolt, Tapered, Double Lap Joint                | 35        |
| 2.5.2.1    | Input File for Example 2                                        | 37        |
| 2.5.2.2    | Output File for Example 2                                       | 37        |

## **APPENDIX A TEST FOR EFFECTIVE STIFFNESS OF PLATE SUPPORTING BOLT, $k$**

|            |                                                       |           |
|------------|-------------------------------------------------------|-----------|
| <b>A1.</b> | <b>INTRODUCTION</b>                                   | <b>41</b> |
| <b>A2.</b> | <b>THE TEST</b>                                       | <b>41</b> |
| <b>A3.</b> | <b>DETERMINATION OF <math>k</math> FROM TEST DATA</b> | <b>43</b> |

## FLEXIBILITY OF, AND LOAD DISTRIBUTION IN, MULTI-BOLT LAP JOINTS SUBJECT TO IN-PLANE AXIAL LOADS

### 1. INTRODUCTION, THEORY AND APPLICATION

#### 1.1 INTRODUCTION

This Data Item, together with ESDUpac A9812, provides a computer program for the analysis of multi-bolt, single row, lap joints. The Data Item is provided in two main sections. Section 1 describes the theoretical solution and its application to practical bolted joints. Section 2 describes the use of the Fortran program, its input, output and all messages that might be encountered. Example calculations are also provided. The program is available in the form of a Fortran source code and corresponding executable file. Both files can be found in ESDUscope (the CD version of the Series), on the internet version of the Series, and on disk in the Series Software Volume.

The analysis applies to multi-bolt, single row, lap joints loaded in either axial tension or compression. It requires the flexibilities of each of the bolts under shear loading. A theory for the calculation of these flexibilities is also described and may be used when they are not available. However, without test data to support the values of the flexibilities used, it is recommended that the calculation of the flexibilities of individual bolts via the program is only used for comparative purposes.

The analysis allows for joints in which the bolts are of different sizes and are not at a regular pitch and in which the thickness and/or width of the joined plates vary between bolt pitches. Although the analysis is performed in terms of a single lap joint, it may be applied to symmetric double lap joints. It may also be applied to joints having multiple rows of bolts provided that the rows are identical. Notes on the appropriate factors for the plate stiffness and bolt head restraints are included together with details of their derivation. Figures are provided that illustrate the influence of the major parameters on the bolt loads and joint flexibility.

#### 1.2 NOTATION

Both SI and British units are quoted but any coherent system of units may be used.

|           |                                                                                                                        |                  |                     |
|-----------|------------------------------------------------------------------------------------------------------------------------|------------------|---------------------|
| $A$       | cross-sectional area of plate ( $t \times w$ )                                                                         | m <sup>2</sup>   | in <sup>2</sup>     |
| $A_b$     | bolt cross-sectional area                                                                                              | m <sup>2</sup>   | in <sup>2</sup>     |
| $A_{ps}$  | effective area of plate over which shear acts (see Equation (1.13))                                                    | m <sup>2</sup>   | in <sup>2</sup>     |
| $D_b$     | bolt diameter                                                                                                          | m                | in                  |
| $D_{ref}$ | reference bolt diameter used when estimating $k$ (see Equation (1.21))                                                 | m                | in                  |
| $E$       | modulus of elasticity                                                                                                  | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $F_b$     | flexibility of single bolt/hole* joining two plates in single shear, $F_b = f_{bA} + f_{bC} = 1/S_b = \delta_{bT}/P_b$ | m/N              | in/lbf              |

\* For footnote see end of Notation Section

|           |                                                                                                                                      |                  |                     |
|-----------|--------------------------------------------------------------------------------------------------------------------------------------|------------------|---------------------|
| $F_j$     | flexibility of joint between reference points defined by $L_A$ and $L_C$                                                             | m/N              | in/lbf              |
| $f_b$     | flexibility of single bolt/hole* in single plate, $f_b = \delta_b/P_b$                                                               | m/N              | in/lbf              |
| $G$       | shear modulus                                                                                                                        | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $I_b$     | second moment of area of bolt cross-section                                                                                          | m <sup>4</sup>   | in <sup>4</sup>     |
| $k$       | effective stiffness, per unit thickness, of plate supporting bolt (see Sections 1.3.3.1 and 1.4.1.1 and the examples of Section 2.5) | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $L$       | length measured along joint                                                                                                          | m                | in                  |
| $L_A$     | length from arbitrary reference point to first bolt, see Sketch 1.1                                                                  | m                | in                  |
| $L_C$     | length from last bolt to arbitrary reference point, see Sketch 1.1                                                                   | m                | in                  |
| $l_{inf}$ | length of plate loaded in tension over which stress distribution is influenced by presence of a bolt (see Section 1.4.1.4)           | m                | in                  |
| $M_{bx}$  | internal moment in bolt at distance $x$ (see Section 1.3.3)                                                                          | N m              | lbf in              |
| $N$       | number of bolts along joint                                                                                                          |                  |                     |
| $P_b$     | shear load on bolt                                                                                                                   | N                | lbf                 |
| $P_j$     | total shear load on joint, tension positive, compression negative                                                                    | N                | lbf                 |
| $Q_{bx}$  | shear in bolt at distance $x$ (see Section 1.3.3)                                                                                    | N                | lbf                 |
| $q_{px}$  | reaction in plate supporting bolt per unit length at distance $x$ (see Equation (1.13))                                              | N/m              | lbf/in              |
| $S_b$     | stiffness of bolt/hole* at joint of two plates in single shear, $S_b = 1/F_b$                                                        | N/m              | lbf/in              |
| $S_j$     | joint stiffness between arbitrary reference points defined by $L_A$ and $L_C$ , $S_j = 1/F_j$                                        | N/m              | lbf/in              |
| $t$       | thickness of plate                                                                                                                   | m                | in                  |
| $w$       | width of plate                                                                                                                       | m                | in                  |
| $w_{eff}$ | effective value of $w$ for plate loaded in tension (see Section 1.4.1.4)                                                             | m                | in                  |
| $\beta$   | rotation of bolt axis due to shear, $\beta = Q_{bx}/\mu G_b A_b$                                                                     | rad              | rad                 |
| $x$       | longitudinal bolt co-ordinate, see Sketch 1.3                                                                                        | m                | in                  |
| $y$       | bolt deflection, see Sketch 1.3                                                                                                      | m                | in                  |

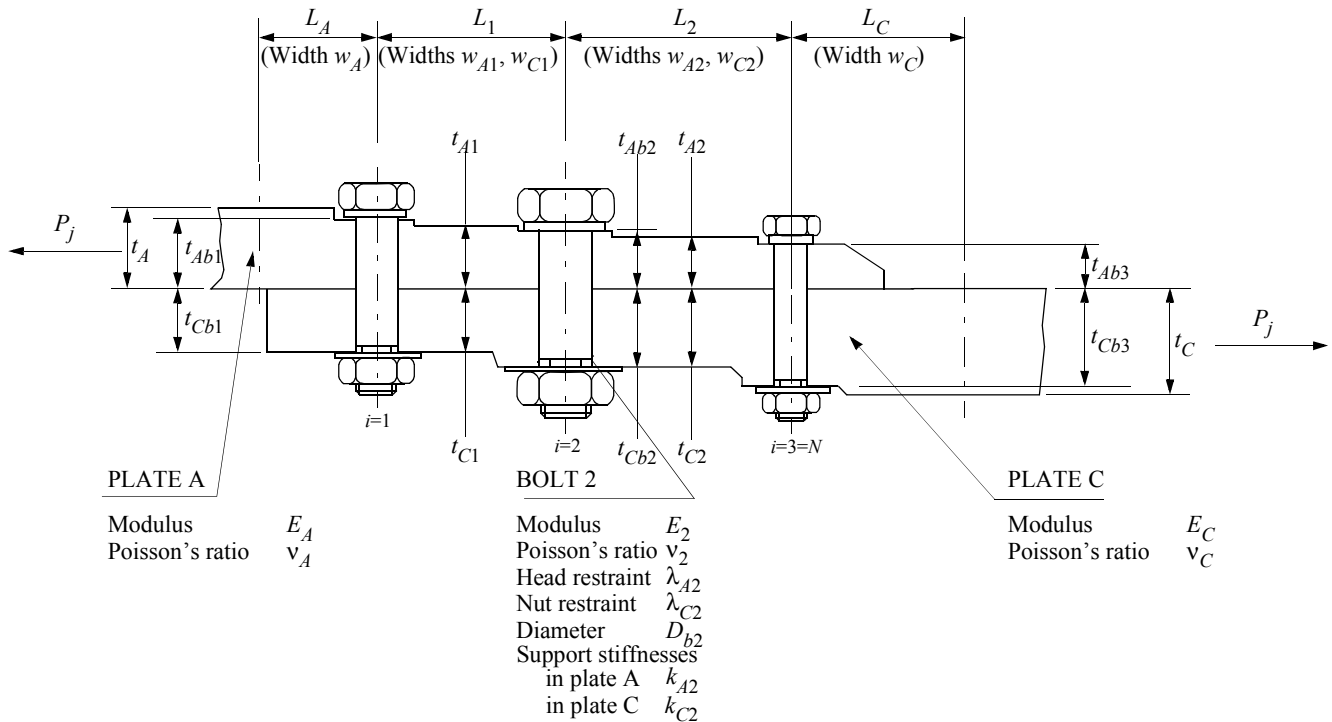
\* For footnote see end of Notation Section

|               |                                                                                                                     |         |            |
|---------------|---------------------------------------------------------------------------------------------------------------------|---------|------------|
| $\delta$      | in-plane longitudinal extension                                                                                     | m       | in         |
| $\delta_b$    | total deformation of bolt/hole* in a single plate as shown in Sketch 1.3                                            | m       | in         |
| $\delta_{bT}$ | total deformation of bolt/hole* at a joint of two plates in single shear, $\delta_{bT} = \delta_{bA} + \delta_{bC}$ | m       | in         |
| $\delta_j$    | total in-plane extension of joint between arbitrary reference points defined by $L_A$ and $L_C$                     | m       | in         |
| $\eta$        | function, see Equation (1.16)                                                                                       |         |            |
| $\lambda$     | rotational stiffness provided by bolt head or nut                                                                   | N m/rad | lbf in/rad |
| $\mu$         | shape factor for circular beam, $\mu = \frac{6(1 + \nu_b)}{(7 + 6\nu_b)}$                                           |         |            |
| $\nu$         | Poisson's ratio of material                                                                                         |         |            |
| $\psi_b$      | rotation of bolt axis due to bending                                                                                | rad     | rad        |

\* The term 'bolt/hole', used throughout this Item, refers to the bolt plus the material surrounding it which loads and/or supports it.

## 1.2.1 Suffixes

|          |                                                                                                             |
|----------|-------------------------------------------------------------------------------------------------------------|
| $A$      | relates to plate A of a single lap joint, see Sketch 1.1                                                    |
| $b$      | relates to bolt material, or load or plate dimension at a bolt                                              |
| $C$      | relates to plate C of a single lap joint, see Sketch 1.1                                                    |
| $eff$    | denotes effective value                                                                                     |
| $i$      | identifies $i$ th bolt or pitch along joint (pitch $i$ lies between bolts $i$ and $i + 1$ , see Sketch 1.1) |
| $m$      | bolt number (see Sketch 1.2 and Equations (1.2), (1.3) and (1.5) to (1.7))                                  |
| $max$    | denotes maximum value                                                                                       |
| $p$      | denotes plate (see Section 1.3.3)                                                                           |
| $\alpha$ | 1, 2, 3 or 4, see Equation (1.16)                                                                           |



**Sketch 1.1 Multi-bolt single lap joint showing notation**  
(Equilibrium moment omitted for clarity)

### 1.3 ANALYSIS OF MULTI-BOLT SINGLE LAP JOINTS

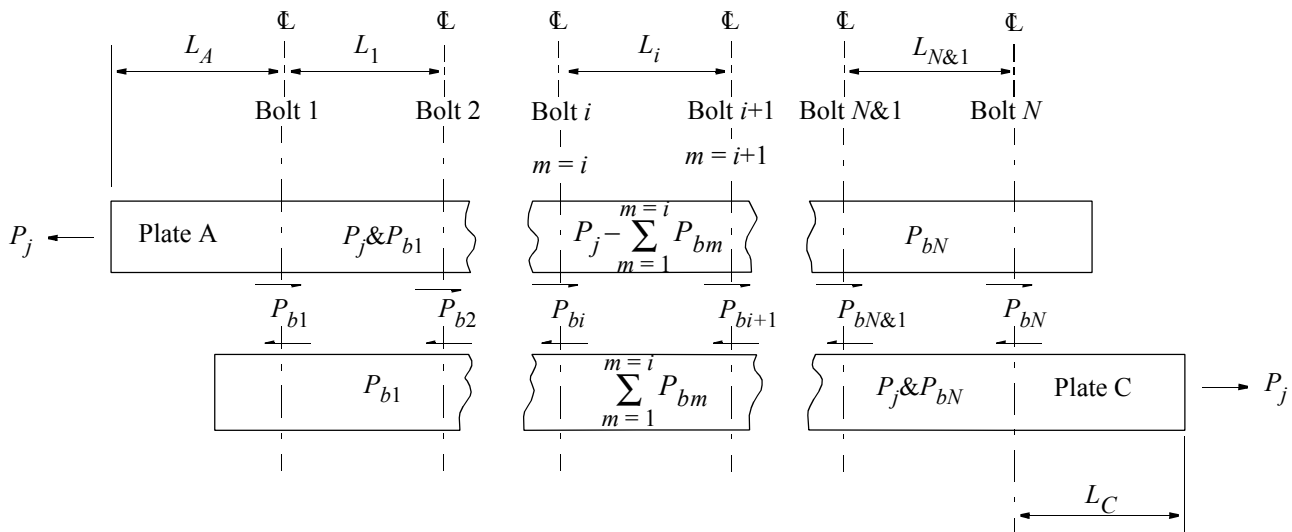
The analysis is based upon Derivation 4 of Section 1.6 in which the elastic behaviour of multi-bolt joints of the type illustrated in Sketch 1.1 is analysed to determine the load transferred at each bolt and the overall joint flexibility using the following assumptions.

- All components of the joint behave elastically.
- The flexibilities of the assemblies of individual bolts in the plate are available. (If the required flexibilities are not available, Section 1.3.3 provides a theoretical method for estimating them.)
- The transfer of load by friction is ignored and load transfer takes place only by shear through the bolts.
- The increase in flexibility of the plates under tensile loading attributable to the presence of the bolt holes can be accounted for by use of reduced plate areas. Under compressive loading the bolt holes are ignored, see Section 1.4.1.4.
- The overall moment arising from the offset loading in a single lap joint is not considered.
- The loading can be tensile or compressive. Under compressive loading no account is taken of the possibility of buckling.
- The bolts are assumed to be perfect fits in both plates and therefore to have perfectly matched pitches in the plates.

Mismatching pitches and/or lack of bolt fit in the plates can significantly influence the load distribution among the bolts.

### 1.3.1 Analysis of Load Distribution in a Joint

Joints of the type illustrated in Sketch 1.1 may be idealised as illustrated in Sketch 1.2. Note that the dimensions  $L_A$  and  $L_C$  define the ends of the joint and can be arbitrarily chosen. Over each pitch the end load is constant at the values shown within each plate.



### Sketch 1.2 Idealised single lap joint

Each pitch between the bolts, as shown in Sketch 1.2, is considered in turn and the effects summed, from  $i = 1$  to  $N - 1$ . For compatibility the extensions over pitch  $i$  are equal so that

$$\delta_{iA} = \delta_{iC} . \quad (1.1)$$

The pitch extensions are made up of two components: (a) plate extensions and (b) bolt/hole deformations.

Note that throughout the following equations ‘effective’ areas are used. These are used to take account of the increased flexibility of the plates in tension owing to the presence of the bolt holes, see Section 1.4.1.4.

(a) Plate extensions only

For plate A,

$$\delta_{iAp} = \frac{L_{iA}}{A_{iAeff} E_{iA}} \left[ P_j - \sum_{m=1}^{m=i} P_{bm} \right] \quad (1.2)$$

and for plate C,

$$\delta_{iCp} = \frac{L_{iC}}{A_{iCeff} E_{iC}} \left[ \sum_{m=1}^{m=i} P_{bm} \right]. \quad (1.3)$$

(b) Bolt/hole extensions

Since the bolt/hole flexibility,  $F_b$ , relates to both plates A and C, the effective extension of pitch  $i$  owing to bolt/hole flexibility is the difference between the displacements of the bolts at each end of the pitch,  $i$ , in both plates A and C. Thus,

$$\delta_{bTi} = P_{b(i+1)} F_{b(i+1)} - P_{bi} F_{bi}. \quad (1.4)$$

Combining Equations (1.1) to (1.4) to give the compatibility equation for each pitch gives  $N - 1$  equations as follows.

$$\left( \frac{L_{iA}}{A_{iAeff} E_{iA}} \right) \left[ P_j - \sum_{m=1}^{m=i} P_{bm} \right] - \left( \frac{L_{iC}}{A_{iCeff} E_{iC}} \right) \left[ \sum_{m=1}^{m=i} P_{bm} \right] + P_{b(i+1)} F_{b(i+1)} - P_{bi} F_{bi} = 0. \quad (1.5)$$

For overall load equilibrium the sum of the bolt loads must equal the applied load, or

$$P_j = \sum_{m=1}^{m=N} P_{bm}. \quad (1.6)$$

Equations (1.5) and (1.6) now represent  $N$  simultaneous equations which may be solved to yield the values of the bolt loads.

### 1.3.2 Joint Displacement, Flexibility and Stiffness

The joint extension is found by adding (a) the plate A (pitch) extensions, (b) the extension due to the flexibility of the end bolt/hole,  $N$ , and (c) the free plate extensions in plates A and C at each end over the lengths  $L_A$  and  $L_C$ .

$$\delta_j = \left[ \sum_{i=1}^{i=N-1} \frac{L_{iA}}{A_{iAeff} E_{iA}} \left[ P_j - \sum_{m=1}^{m=i} P_{bm} \right] \right] + P_{bN} F_{bN} + \frac{P_j L_A}{A_{Aeff} E_A} + \frac{P_j L_C}{A_{Ceff} E_C}. \quad (1.7)$$

Joint flexibility is then

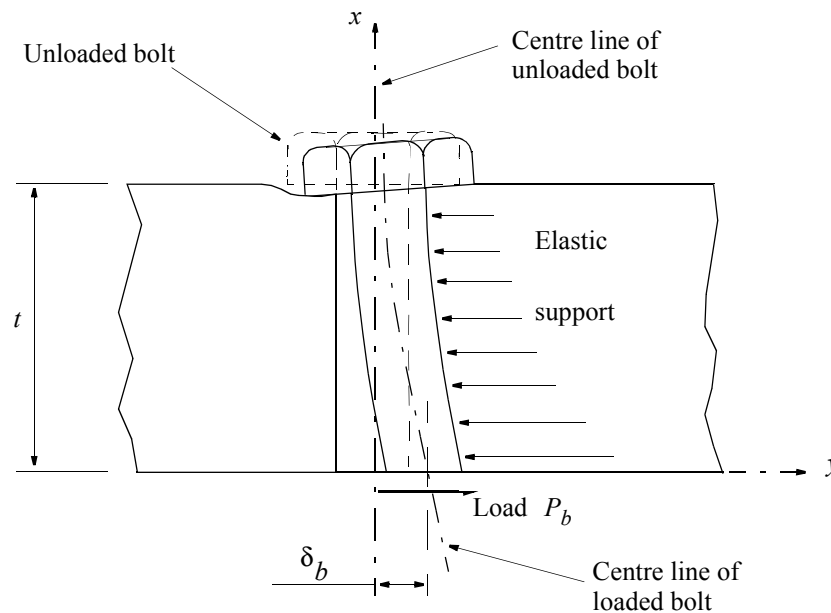
$$F_j = \frac{\delta_j}{P_j} \quad (1.8)$$

and joint stiffness is

$$S_j = \frac{1}{F_j} \quad (1.9)$$

### 1.3.3 Calculation of Bolt Flexibility

The calculation of bolt flexibility is based on the theoretical analysis of Derivation 7 of Section 1.6. The theory, summarised below, considers elastic behaviour of an individual bolt in a single plate subjected to a shear load as shown in Sketch 1.3 and so leads to the calculation of  $f_b$ . The bolt is assumed to act as a beam on an elastic foundation. Bending and shear of the bolt are considered together with the through-the-thickness shear stiffness of the plate and the effect of bolt head rotational restraint. The plate width and length are both assumed to be sufficiently large for plate boundary effects not to influence the solution significantly.



**Sketch 1.3 Theoretical bolt and plate configuration showing reference axes**

#### 1.3.3.1 Basic relationships

At any point,  $x$ , along the bolt the following relationships apply.

Rotation:

$$\frac{dy}{dx} = \beta + \psi_b \quad (1.10)$$

Beam equilibrium:

$$\frac{dM_{bx}}{dx} + Q_{bx} = 0 \quad (1.11)$$

and

$$\frac{dQ_{bx}}{dx} + q_{px} = 0. \quad (1.12)$$

The plate reaction,  $q_{px}$ , may be simply represented by the product of the foundation stiffness and the bolt deflection,  $ky$ . However, this only accounts for the direct stiffness of the foundation acting as if it were a set of independent springs acting on the bolt. The plate material shear stiffness couples adjacent regions of the foundation. This can be accounted for by taking the slope  $dy/dx$ , together with the material through-the-thickness shear stiffness,  $G_p$ , and assuming action over an effective area,  $A_{ps}$ , to give an effective shear force in the plate. The rate of change of this shear force gives an additional plate reaction so that, in total

$$q_{px} = -ky + G_p A_{ps} \frac{dy}{dx}. \quad (1.13)$$

Note that in Equation (1.13)  $k$  and  $A_{ps}$  are specific to the bolt under consideration. Hence, changing bolt diameter changes both  $k$  and  $A_{ps}$ .

Linear small displacement assumption:

$$M_{bx} = E_b I_b \frac{d^2 \psi_b}{dx^2}. \quad (1.14)$$

### 1.3.3.2 Governing equation

From Equations (1.10) to (1.14) the following relationship is obtained.

$$\left[ 1 + \frac{G_p A_{ps}}{\mu G_b A_b} \right] \frac{d^4 y}{dx^4} - \left[ \frac{k}{\mu G_b A_b} + \frac{G_p A_{ps}}{E_b I_b} \right] \frac{d^2 y}{dx^2} + \frac{k}{E_b I_b} y = 0. \quad (1.15)$$

The general solution of Equation (1.15) is given by,

$$y = A_\alpha e^{\eta_\alpha x} \quad (1.16)$$

where  $\alpha$  is 1, 2, 3 or 4 and  $\eta$  is a function of the coefficients in Equation (1.15).

### 1.3.3.3 Boundary conditions

The four boundary conditions are as follows.

At  $x = 0$ ,

$$Q_{bx} = P_b - G_p A_{ps} \frac{dy}{dx} \quad (1.17)$$

and a point of contraflexure is assumed in the bolt so that

$$M_{bx} = 0 \quad (1.18)$$

This latter condition is strictly only valid in single lap joints in which the plates are of the same thickness and stiffness; however, this is still a reasonable assumption in most practical structural joints.

At  $x = t$ ,

$$Q_{bx} = -G_p A_{ps} \frac{dy}{dx} \quad (1.19)$$

and

$$M_{bx} = -\lambda \psi_b \quad (1.20)$$

Equation (1.19) relates the shear load at the bolt head to the bolt slope,  $dy/dx$ , and the plate shear stiffness. Since, from Equation (1.10), the slope  $dy/dx$  includes a component due to bolt bending,  $\psi_b$ , Equation (1.19) is only strictly valid when  $\psi_b$  is negligible, that is when the bolt head restraint,  $\lambda$ , is large. This will be the case in most practical structural joints, see Section 1.4.1.2.

By using Equations (1.10) to (1.14) and (1.17) to (1.20) equations may be developed to solve for the coefficients  $A_\alpha$  of Equation (1.16). The coefficients are all that is required to calculate the required deflection at  $x = 0$  via Equation (1.16) since all other terms are functions of  $e^x$  and are therefore unity at  $x = 0$ .

## 1.4 APPLICATION OF THE THEORY FOR CALCULATING THE FLEXIBILITY OF A BOLT IN A SINGLE PLATE

### 1.4.1 Parameters Required to Calculate Joint Flexibilities

The theory described in Section 1.3.3 for calculating the flexibility of a bolt in a single plate requires knowledge of the following.

- The bolt diameter, the plate thickness and the bolt and plate material properties.
- The effective areas of the plates (reduced from their nominal areas by the presence of the bolt holes).
- The rotational restraint,  $\lambda$ , provided by the nut or bolt head.
- The effective area of plate,  $A_{ps}$ , over which the bolt shear is taken to act.
- The effective stiffness,  $k$ , of the plate as an elastic foundation supporting the bolt.

The tests reported in Derivation 2 on single bolt joints show that the measured flexibility of nominally identical joints usually varies by a factor in excess of 1.3 and can vary by a factor of up to 2.

The values listed in (a) above are readily available. The effective areas of (b) may be estimated from the dimensions and properties of the plates using the method of Section 1.4.1.4 and that method is incorporated in the program.

Figure 1 shows that, for two different joint geometries, as  $\lambda$  is increased, variations in bolt flexibility as a result of errors in estimating the value of  $\lambda$  become insignificant, see Section 1.4.1.2.

The value of  $A_{ps}$  is only approximately known, see Section 1.4.1.3. However, its influence on the joint flexibility is small. This is illustrated by Figure 2 where, for two different joint geometries, the change in  $f_b$  for a range of values of  $A_{ps}$  is shown to be relatively small.

The remaining factor,  $k$ , is a major influence on the flexibility of a joint and so it is essential that a good estimate of  $k$  is available if an accurate value of joint flexibility is to be obtained.

### 1.4.1.1 Effective stiffness, $k$

While the best estimate of  $k$  may be obtained by experiment, the practical difficulties inherent in measuring a particular parameter when testing a representative joint should not be underestimated. Appendix A describes a simple test on a specially manufactured double lap joint having relatively rigid outer plates. This design eliminates all overall joint eccentricity effects, minimises bolt bending effects and allows measurement of the extension that arises principally from the deformation of the material around the inner plate bolt hole. The relatively small plate extensions are calculated and subtracted from the measured extension and the resulting value is used to estimate  $k$ .

It should be noted that the value of  $k$  used in Equation (1.13) depends on bolt diameter. For example, doubling the bolt diameter may be expected to double approximately the value of  $k$ .

Derivation 2 gives details of some 300 tests and covers the types of joint listed in Table 1.1. The larger groups of nominally identical tests have been analysed in order to determine an approximate value for  $k$ . For this it was assumed that  $\lambda$  was effectively infinite and that  $A_{ps} = 0.1 D_b^2$  (see Section 1.4.1.3);  $k$  was then varied until, by trial, the average value of  $F_j$  for each set of tests was obtained. This indicated that  $k$  is approximately related to the modulus of the plates and the bolt diameter. The values of  $k$  obtained were then related to an arbitrarily chosen reference bolt diameter,  $D_{ref}$ , of 6.35 mm (0.25 in). The mean value of  $k$  was then found to be given by

$$k = 0.32 E_p \frac{D_b}{D_{ref}} \quad (1.21)$$

This analysis indicated that the factor of 0.32 in Equation (1.21) has a standard deviation of about 0.1 and consequently a coefficient of variation in the region of 0.3. The analysis assumes that all variability between the test results is attributable to  $k$ , which is clearly not the case. However, in the absence of specific information on  $k$  for the joint under design, application of Equation (1.21) should lead to reasonable estimates of  $F_j$ .

**TABLE 1.1**

| Joint type | Maximum and minimum bolt diameters |                    | Bolt head type  | Bolt material  | Maximum and minimum plate thicknesses                    |                        | Plate material  |
|------------|------------------------------------|--------------------|-----------------|----------------|----------------------------------------------------------|------------------------|-----------------|
|            | mm                                 | (in)               |                 |                | mm                                                       | (in)                   |                 |
| Double lap | 6.35<br>(only)                     | (0.25)             | Countersunk     | Titanium alloy | 3.1<br>(only) outer plates<br>6.5<br>(only) inner plates | (0.124)<br><br>(0.256) | Aluminium alloy |
| Single lap | 4.0<br>12.7                        | (0.156)<br>(0.50)  | Countersunk     | Titanium alloy | 2.0<br>12.7                                              | (0.080)<br>(0.50)      | Titanium alloy  |
| Single lap | 4.0<br>4.8                         | (0.156)<br>(0.188) | Protruding head | Titanium alloy | 1.0<br>4.8                                               | (0.040)<br>(0.188)     | Titanium alloy  |
| Single lap | 4.8<br>9.5                         | (0.188)<br>(0.375) | Protruding head | Steel          | 0.8<br>4.8                                               | (0.031)<br>(0.188)     | Aluminium alloy |

### 1.4.1.2 Rotational restraint, $\lambda$

In the majority of structural joints the value of  $\lambda$  is sufficiently high to be taken as infinite with minimal loss of accuracy, see Section 1.4.2. The effect of decreasing the rotational restraint,  $\lambda$ , is to increase joint flexibility.

### 1.4.1.3 Effective shear area, $A_{ps}$

In Derivation 6 it has been found by experiment that a suitable value for the effective shear area of the plate,  $A_{ps}$ , is  $0.1D_b^2$ . This is the default value used by the program of ESDUpac A9812. A facility for using other values for  $A_{ps}$  is provided in the program.

Figure 2 shows the effect on bolt/hole flexibility of varying  $A_{ps}$  from 1.0 to 10.0 mm<sup>2</sup> for the 6.35 mm bolt of Figure 1 in 10 mm and 25 mm thick plates. Also shown on the figure is the suggested value of  $A_{ps} = 0.1D_b^2$ .

### 1.4.1.4 Effective areas of the plates to account for the presence of bolt holes

Throughout the description of the theory in Section 1.3 the plate areas used were termed effective values. The effective values are designed to account for the presence of the bolt holes in the flexibility calculation. Under a tensile load the presence of the bolt hole increases the flexibility of the plates. For plates loaded in compression the bolt is capable of passing the load directly through its diameter and, to account for this, the plate flexibility is calculated as if the plate contains no bolt holes.

For joints in tension the increased flexibility of the plates arising from the presence of the bolt holes has been assessed using finite element analyses and the experimental work of Derivation 5. The finite element analyses took no account of the presence of the bolt in the hole. However, the experimental work of Derivation 5 measured the flexibility of plates containing a bolt and good agreement between that work and the finite element analyses was observed. This has led to the determination of an effective width for the plates that may be used to calculate the area for the flexibility calculation. The analysis of the ends of