

AERODYNAMIC PRINCIPLES OF WINGLETS

1. INTRODUCTION

Winglets are small wing-like lifting surfaces, fitted at the tips of some wings, usually with the objective of decreasing the trailing-vortex drag and thereby increasing the aerodynamic efficiency of the basic wing. Winglets may comprise both upper and lower elements or upper elements only. Those comprising upper elements only are the more common.

This Item deals with the aerodynamic effects of winglets at subsonic freestream speeds, mainly so far as they affect the aerodynamic efficiency of a configuration, defined either as L/D or ML/D . Fitting winglets can provide improvements in aerodynamic efficiency for a range of lift and Mach number conditions by decreasing the trailing-vortex drag by amounts that more than compensate for any increases in the profile and wave drag contributions. In addition to their effects on trailing-vortex drag, some other beneficial effects may arise from winglets, for example on the vortex wake characteristics (important for aircraft used for agricultural spraying), see Refs 1 and 2, for the directional stability of canard configurations (Ref. 3), and, arguably, on the appearance of the aircraft.

Winglets may be designed to be retrofitted to an existing aircraft wing, or included in a new version of a pre-existing design, or included as a feature to be considered at the design stage. The design constraints applicable differ in the different situations. Section 3 provides a brief description of winglets and a discussion of their associated benefits and disadvantages. As indicated in Sections 3, 6 and 7, deciding on whether or not winglets are worthwhile in any particular case may be complicated, since a number of factors, in addition to the aerodynamic efficiency, must be considered.

In Section 4, an overview of the various methods of aerodynamic analysis appropriate for treating the effects of wing/winglets is presented, with emphasis given to a simple approach suited to an initial assessment.

This simple approach is applied in Section 5. In designing winglets there is an important influence of the required strength (and associated weight) of the wing/winglet structure, and in Section 5 this influence is taken into account in a simple manner by consideration of the aerodynamic bending moment about the wing centre-line. Graphs showing the minimum trailing-vortex drag of wings with upper winglets are presented for winglets having various values of cant angle and relative length, with constraints imposed on the values of lift and centre-line bending moment. In each case the trailing-vortex drag is expressed as a ratio to the minimum trailing-vortex drag of the basic wing (without winglets) supporting the same lift. Deductions are made relevant to situations where winglets are included at the initial design stage and also where they are considered as modifications to a pre-existing design. Comparisons of winglets and wing tip-extensions are made for both situations. Consideration is also given to a number of other factors, including the effect of changes in the wing loading distribution, a brief treatment of winglets with upper and lower elements, and winglets designed for aircraft with constrained span.

In Section 6, the detailed design aspects of winglets and the aerodynamic interactions between a wing and winglet are described. Two major studies relating to the application of winglets to large subsonic transport aircraft, one by Boeing and the other by Douglas, are examined and discussed in Section 7. In Section 8, a summary of conclusions is given.

2. NOTATION AND DEFINITIONS

2.1 Notation

A	aspect ratio of wing (excluding any winglets or tip-extensions), $= 4s^2/S$
b	span of wing (excluding any winglets or tip-extensions), $= 2s$
b_a	span of wing (including any winglets or tip-extensions), $= 2s_a$
b_s	structural span, $= 2s_s$
C_D	drag coefficient (based on S)
C_{DTV}	trailing-vortex drag coefficient (based on S)
C_L	lift coefficient (based on S)
C_{Ldes}	value of C_L corresponding to design conditions
C_{LL}	local lift coefficient
C_{Lcr}	value of C_L corresponding to cruise
C_{Lstr}	value of C_L corresponding to limiting manoeuvre or gust case for design of wing structure
C_{NL}	local normal force coefficient
C_p	pressure coefficient
c	local wing chord, $= c(\eta)$
$\bar{c}$	mean wing chord (excluding any winglets or tip-extensions), $= S/b$
c_t	tip chord (of wing less any winglet)
c_{wr}	root chord of winglet
c_{wt}	tip chord of winglet
c_0	centre-line chord (of gross wing)
D	aerodynamic drag
D_{oa}	minimum trailing-vortex drag of wing (having no winglets) of span b_a , for conditions of given lift, L , and kinetic-pressure, q (see Section 5.1)
D_{ob}	minimum trailing-vortex drag of wing (having no winglets) of span b , for conditions of given lift, L , and kinetic-pressure, q (see Section 5.1)
D_{TV}	trailing-vortex drag
e	efficiency factor associated with trailing-vortex drag, $e = L^2/(4\pi qs^2 D_{TV})$

h_w	height of winglet, $= l_w \cos \phi$ (see Sketch 3.1)
L	aerodynamic lift of gross wing including any winglets or tip-extensions
l_w	front-view length of winglet or wing tip-extension (see Sketch 3.1)
M	flight Mach number (or wind-tunnel test Mach number)
M_{des}	value of M corresponding to design conditions
M_r	bending moment about wing centre-line due to aerodynamic loading on one (gross) wing panel, including any winglet(s) or tip-extension fitted to that panel
q	freestream kinetic-pressure, $= \frac{1}{2} \rho V^2$
S	reference wing area, <i>i.e.</i> gross planform area of wing less any winglets or tip-extensions
s	semi-span of wing (less any winglets or tip-extensions)
s_a	semi-span of wing (including any winglet or tip-extension), $= s + l_w \sin \phi$
s_s	half structural-span, $= s + l_w$
t	distance (in front-view) from point on winglet to wing/winglet junction (see Sketch 3.1)
V	freestream speed
y	distance from point on wing to plane of symmetry of wing
y_b	effective moment arm, $= 2M_r/L$
η	non-dimensional location of point on winglet or wing, $= 1 + (t/s)$ for point on winglet, $= y/s$ for point on wing
η_b	effective non-dimensional moment arm, $= y_b/s$
η_{bmin}	value of η_b corresponding to minimum value of D_{TV}/D_{ob}
η_{str}	value of η_b corresponding to C_{Lstr}
ϕ	winglet cant-angle <i>i.e.</i> angle between mean plane of winglet and wing plane of symmetry, taken positive in direction shown in Sketch 3.1
Λ_m	sweepback angle of wing half-chord line
λ	wing taper-ratio, c_t/c_0
ρ	freestream density

Subscripts

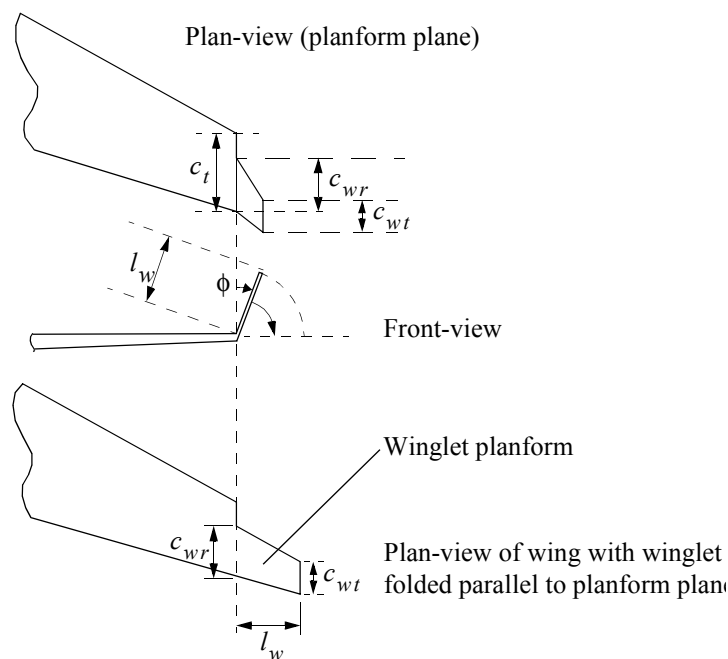
l	denotes lower winglet
TV	denotes trailing-vortex
w	refers to case where winglets are present
wt	refers to case where wing tip-extensions are present
wo	refers to case where no winglets or wing tip-extensions are present

2.2 Definitions

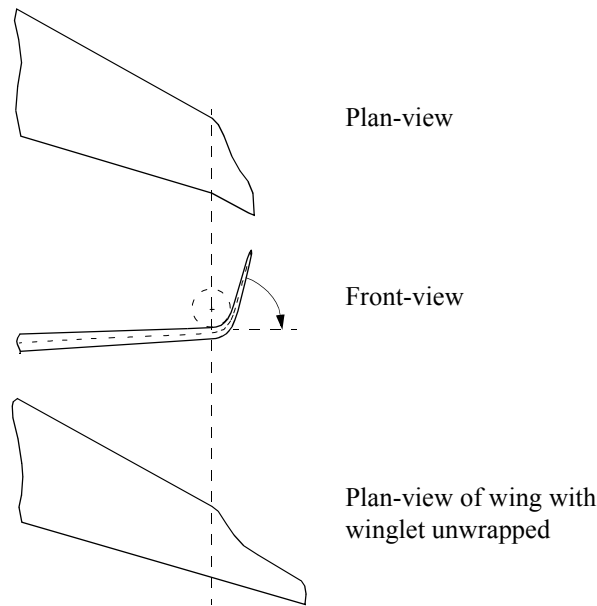
(a) Winglet planform and geometry

The planform of a winglet is the outline figure obtained by projecting the winglet on to its mean plane. It may alternatively be thought of as the extension to the wing planform that would be formed were the winglet to be rotated to lie in the aircraft planform plane, see Sketch 2.1. Winglet geometry may be considered to be defined by planform, cant angle, fore-and-aft location in relation to the wing, toe-in angle, section shapes and fairing or blend shapes. The toe-in angle may vary along the length of the winglet; it may then be referred to in terms of the toe-in angle at the winglet root, together with a winglet twist distribution.

The wing and winglet junction may form a sharp angle (see the front view in Sketch 2.1) or, in the case of a 'blended winglet', a smooth curve – see the front view in Sketch 2.2. A blended winglet may be envisaged as being partially wrapped around an imaginary cylinder – see Sketch 2.2, and may be described in terms of its planform when unwrapped to lie in the aircraft planform plane, and the mean curve of the junction blend, see Sketch 2.2.



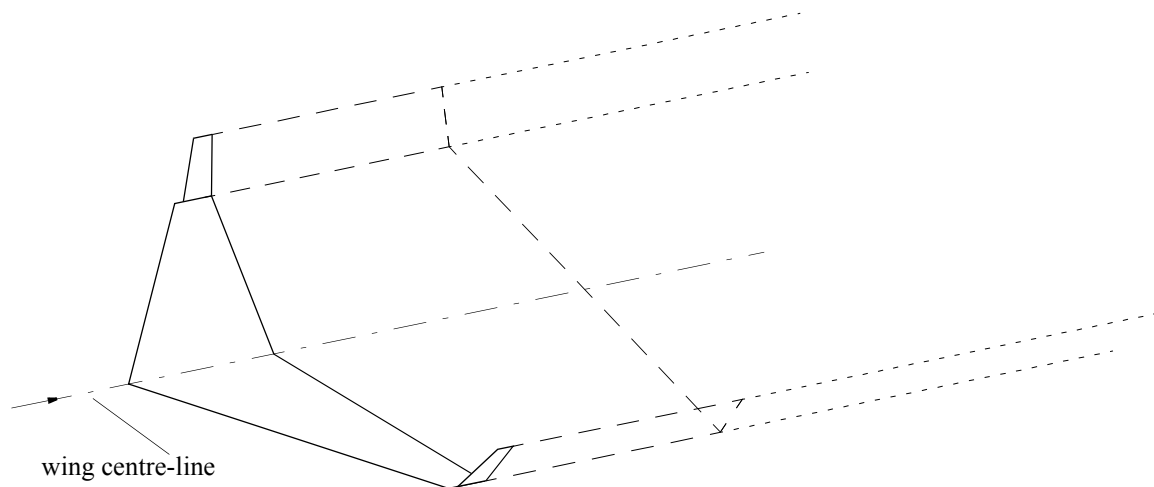
Sketch 2.1 Illustration of winglet planform



Sketch 2.2 Illustration of blended winglet

(b) Trefftz plane

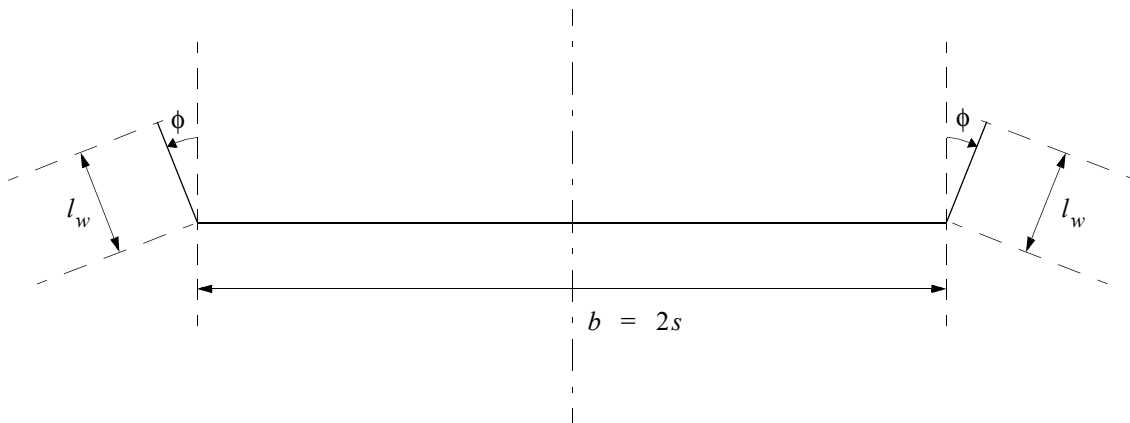
The wake model generally used in lifting-surface theory is a thin sheet of vorticity that leaves the trailing edge in the freestream direction, and develops free of deformation and dissipation, *i.e.* it is considered to be a rigid surface, see Sketch 2.3. In this Item, the trailing-vortex drag is evaluated by integration of the flow properties of such a wake in the Trefftz plane, that is a (or any) hypothetical plane normal to the freestream direction that is sufficiently far downstream of the wing for the flow there to be effectively two-dimensional.



Sketch 2.3 Idealised form of wake used in lifting-surface theory

(c) Trace (of the lifting surface)

The intersection of the idealised wake, as described above, with the Trefftz plane, is sometimes referred to as the trace of the wing or lifting surface(s). To the appropriate degree of approximation, the trace of most wings is simply a straight line (of length equal to the wing span) unless an unusually high dihedral or anhedral angle is utilised. The form of the trace for a wing with upper winglets is shown in Sketch 2.4, for which case the trace length is $2(s + l_w)$.



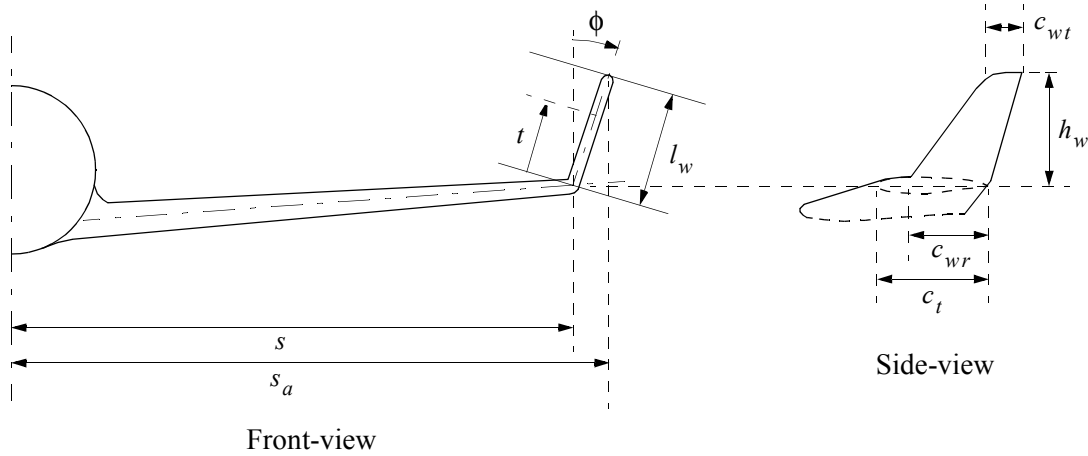
Sketch 2.4 Form of trace for wing with upper winglets

3. WINGLET FEATURES AND BENEFITS

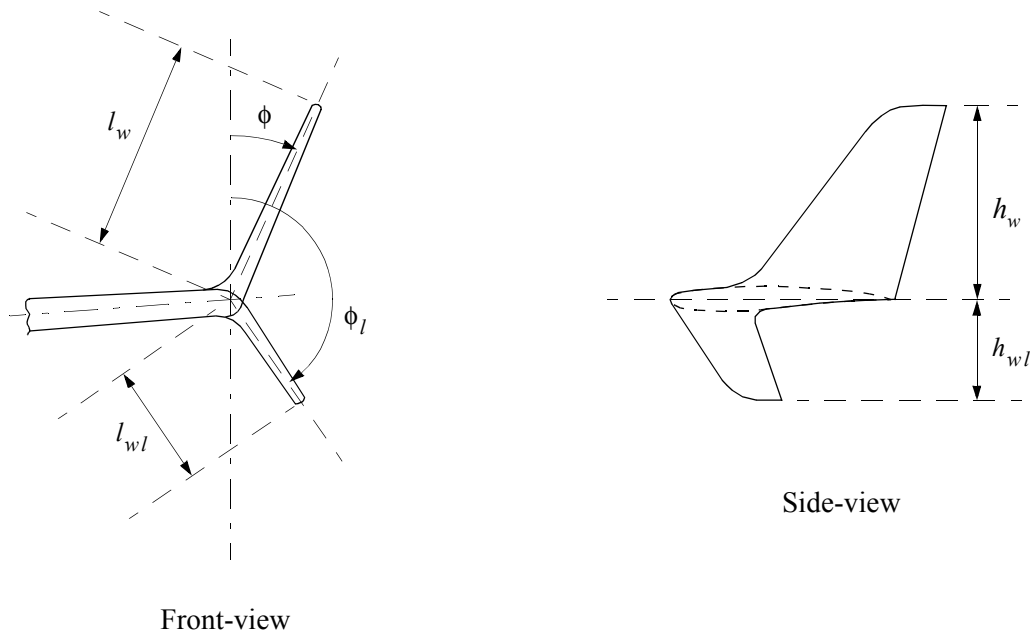
3.1 Winglet Features

Winglets are wing-like surfaces fitted at the tips of some wings. In their simplest form they are like wing-tip extensions, except that they often have a smaller chord than that of the main wing at the tip, and they are turned upwards from the plane of the main wing. In this case the wing and winglet trailing edges generally are aligned at the wing/winglet junction. Generally, such upper winglets are not vertical but are canted outwards at an angle between 15 and 45 deg, see Sketch 3.1. In the case of some upper winglets, there is a blend region between the wing and winglet, which then form a smooth shape with no distinct junction between them. Some winglets are more complex, and consist of both upward and downward turned elements, see Sketch 3.2, with the downward turned elements being smaller and placed forward of the upward ones. Some examples of applications of upper winglets are shown in Sketch 3.3, and some examples of applications of combined upper and lower winglets are shown in Sketch 3.4.

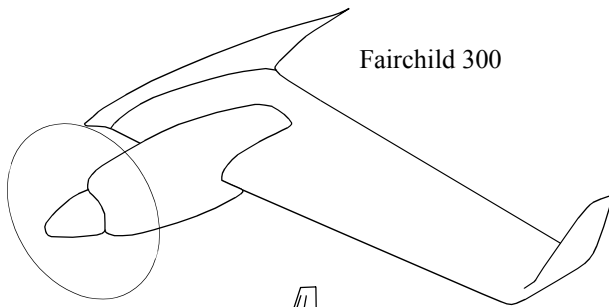
Most aircraft types fitted with winglets operate in the high-subsonic speed range, although there have been applications to lower speed aircraft. No application to aircraft having supersonic flight capability has been noted, and it is probable that such application has not been found appropriate. No consideration of supersonic flight is presented in this Item.



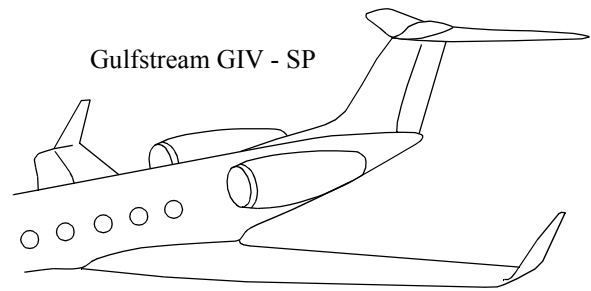
Sketch 3.1 Wing with Upper Winglets



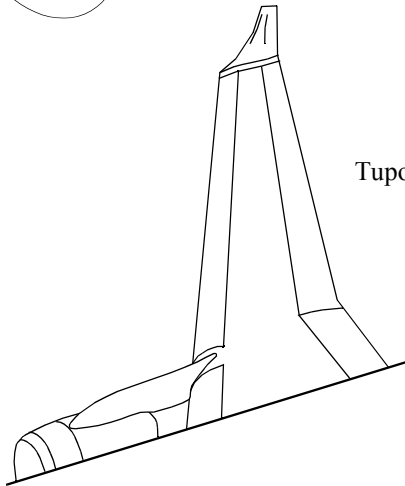
Sketch 3.2 Wing with Upper and Lower Winglets



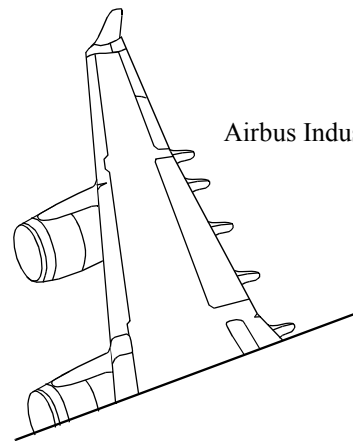
Fairchild 300



Gulfstream GIV - SP

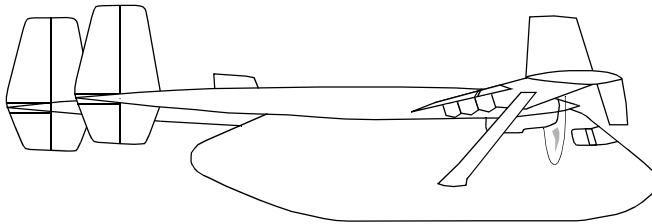


Tupolev Tu 204

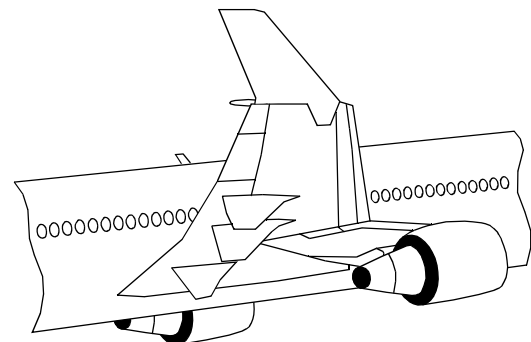


Airbus Industrie A340-300

Sketch 3.3 Examples of Application of Upper Winglets



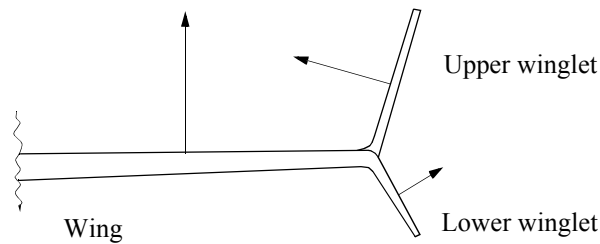
IAI 202 Arava



McDonnell Douglas MD-11

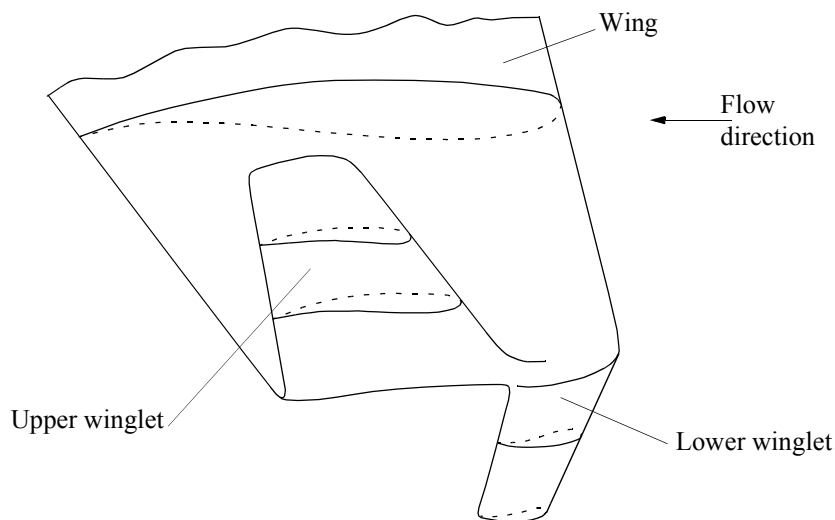
Sketch 3.4 Examples of Application of Upper and Lower Winglets

At their design conditions the lift (or normal force) generated on any upper winglet has a component directed inboard, whilst any lower winglet has a component directed outboard (see Sketch 3.5). The sections utilised for winglets would be selected to give their highest efficiency for such conditions. As a result the winglet sections would generally be cambered in the manner shown in Sketch 3.6. It is important that whether or not winglets are utilised, there is an acceptable range of flight conditions that are free of buffet, and this consideration influences the choice of section shapes and twist distributions. Winglet sections often have a lower thickness/chord ratio than even those used on the outboard area of the wing, partly because it is possible to design them to have an adequate strength with thin sections without being excessively heavy, and partly because it is easier to ensure that thin sections do not generate undesirable interference flows in the area of the wing/winglet junctions.



Front-view

Sketch 3.5 Direction of normal force contributions from wing and winglets at design conditions



Sketch 3.6 Form of camber generally used for winglets

3.2 Benefits Attributable to Winglets

The principal attribute of winglets is their ability to decrease the trailing-vortex drag (see Section 5) at given conditions of freestream kinetic-pressure, Mach number and lift, in comparison with that of the basic wing without winglets. (Such conditions are generally referred to simply as 'given lift', it being taken for granted that the freestream kinetic-pressure and Mach number are held fixed.) Consider, as an example, vertical upper winglets of length equal to 15 per cent of the basic wing semispan. Such winglets could be expected to decrease the trailing-vortex drag at given lift by about 15 per cent (see Section 5.2), the actual amount depending mainly on the spanwise distribution of lift of the basic wing, see Section 5.3.2. The decrease of 15 per cent is appropriate for cases where the basic spanwise wing loading distribution is near the elliptical form (which is the form yielding minimum trailing-vortex drag for given lift for a wing without winglets, see Section 5.1). Suppose such winglets do achieve a reduction in the cruise condition trailing-vortex drag by this amount, and that at this condition the trailing-vortex drag constitutes about half the total aircraft drag (which is fairly typical of transport and business aircraft). Then, if the winglets added nothing to the profile drag coefficient, they would yield an improvement in the cruise lift/drag ratio of just over 7 per cent. Inevitably, winglets would add to the profile drag coefficient, and the improvement in the maximum lift/drag ratio resulting from winglets of this size fitted to wings with a nearly elliptical form of basic spanwise wing

loading would be likely to be in the region of 4 to 5 per cent. The improvement in the maximum lift/drag ratio tends to be lower when the basic wing is relatively highly loaded inboard, see Section 5.3.2. At conditions of relatively high C_L flight, such as in the climb or 'holding', the trailing-vortex drag is a higher proportion of the total drag than in the cruise, and so the percentage improvements in lift/drag ratio due to winglets would then generally be greater than in the cruise. Such improvements would also generally be further increased by the tendency of the spanwise loading distribution on many wings to shift outboard with increasing C_L (see Section 5.3.2). (This outboard shift occurs on wings with twist 'washout'. Such washout is applied to wings of swept-tapered planform in order to achieve the desired spanwise loading distribution at cruise conditions. Counteracting the outboard shift is the effect of aeroelastic washout, which tends to increase with increasing lift, see Section 6 of Ref. 16.)

There are a number of factors that must be counted against such improvements, the principal one being the increased structure weight that is entailed when winglets are fitted. Part of this weight increase is due to the jointed nature of a wing/winglet structure. Other possible disadvantages are related to trim, stability, control effectiveness, flutter and the initial and recurrent costs. The decision on whether to proceed with the consideration of winglets may thus often be finely balanced. A simple analysis of winglets considered as an *ab initio* design feature is given in Appendix A, and from this analysis it would appear unlikely that winglets would offer worthwhile benefits unless the aircraft span were limited due to ground operations. Where there is no such span limitation, it would appear better to achieve any desired improvement in cruise L/D by increasing the wing span, rather than by adding winglets. This may be understood by considering the fact that similar decreases in trailing-vortex drag to those provided by 15 per cent semispan winglets could be provided by an 8 per cent increase in wing span, and, so long as it was well designed, such an increased span wing would result in lower additional profile and wave drag increments than those of the wing with winglets. It would also avoid the element of weight increase associated with the jointed nature of a wing/winglet structure. Heyson *et al.* (Ref. 4) drew the opposite conclusion, favouring winglets over wing tip-extensions. However, the conclusions of Ref. 4 resulted from several factors, including significant errors in some of the lifting-surface theory results utilised, an assumption that the tip-extensions would not be optimised, and the discounting of profile drag changes. The analysis of Asai (Ref. 5) correctly indicated the superiority of the planar wing, *i.e.* the wing with tip-extensions, over the wing with winglets, for *ab initio* designs.

When considering winglets to be added to a pre-existing design of wing, somewhat different criteria apply. In particular, if the wing structure had been designed so as to cater for the critical loads on the basic wing, with only a very small degree of latitude for any increase in these loads, it might well be possible to achieve a greater improvement in L/D by adding nearly vertical winglets than by even well-designed extensions to the wing tips. This would occur as a result of the wing strength limitations permitting a much greater length for near-vertical winglets than for wing tip-extensions. It was this situation that was considered by Whitcomb (Ref. 6) in the first and seminal published report on winglets, where it appeared that the improvement in wing lift-drag ratio given by winglets was more than twice as great as that given by the small wing tip-extension allowed by wing strength limitations – see also Section 5.3. The tip-extension tested had a length which was 38 per cent of the upper winglet length, and increased the wing-root bending moment by about the same amount as the upper winglets. It may be noted, however, that although great care was taken by Whitcomb in the winglet design, comparable care was not taken in the design of the tip-extension. It is also important to take into account the influence of the spanwise loading distribution of the basic wing in the cruise. The benefits from winglets are potentially larger if the basic wing has relatively high loading over the outboard regions, such as with 'first-generation' designs of turbojet transport aircraft, see Section 5.3.2. Comprehensive reports dealing with a number of issues that must be addressed when winglets are being considered as a modification of an existing wing design are given in Refs 7 and 30, see also Section 7.

4. METHODS OF AERODYNAMIC ANALYSIS APPROPRIATE TO WING/WINGLET CONFIGURATIONS

4.1 Early Work on Optimally-loaded Configurations

Optimally loaded wings with tip-fins (*i.e.* those giving minimum trailing-vortex drag) are considered in a number of early papers, including those of Mangler in 1937 (Ref. 8), Falkner and Darwin in 1945 (Ref. 9), Weber in 1956 (Ref. 10), and Cone in 1962 (Ref. 11). The optimal loading sought for such geometries is the equivalent of the elliptic loading that is optimum for planar wings. Falkner and Darwin presented very accurate calculations for the optimal loading of a small number of examples of wings with tip fins, and noted an unwarranted approximation in the work of Mangler (Ref. 8). (It later transpired that this approximation introduced negligible errors, see Ref. 28.) They observed that in lifting-line theory the problem of determining the optimal loading on a nonplanar configuration is equivalent to the calculation in complex-variable analysis of a conformal transformation of the wake trace (see Section 2.2), and used elliptic functions and theta functions to express this transformation. Some of the results of Ref. 9 are presented in Figure 1. As noted in Section 5, the same spanwise loading distributions are optimal in both lifting-surface and lifting-line theories. Cone (Ref. 11) appreciated this fact and considered a wide variety of forms of trace, including those corresponding to a wing with vertical upper winglets (the term winglets was introduced about a decade after the publication of Cone's work), but he rejected the use of conformal transformations for such traces because of its extreme difficulty. He referred to a translation of Mangler's work (Ref. 12), but not to the work of Falkner and Darwin (Ref. 9) or Weber (Ref. 10). Cone presented a number of results obtained from use of an analogy of the velocity potential for the Trefftz-plane flow with an electrical potential. (The analogy method requires voltmeter measurements to be made at various points on a sheet of uniformly conducting material in which a cut having the shape of the trace has been made.) For the example of an optimally loaded wing with upper tip-fins of height equal to $0.15s$, the value for the efficiency factor e presented from this method was 1.22. The correct value for e in this case is 1.166 (corresponding to $D_{TV}/D_{ob} = 0.858$, see Section 5.2), as may be found from the work of Falkner and Darwin (Ref. 9). Lundry and Lissaman (Ref. 28) subsequently commented on and explained the source of the errors in Ref. 11. Advances in computing power have for a considerable time made it possible to determine the optimal loadings and the corresponding values of efficiency factor to 3 significant figures without recourse to the complex analysis of Ref. 9.

Despite the overestimate of the improvement in e due to tip-fins obtained by Cone (Ref. 11), he did very clearly state that careful design of the fins would be needed in order to realise the full aerodynamic advantages of the devices, observing that test results with end plates (*i.e.* relatively large chord, uncambered tip fins, usually extending above and below the wing) were not indicative of the results possible with carefully designed fins.

4.2 Introduction of Winglets

Winglets were introduced and first tested in a wind tunnel by Whitcomb in 1974. At that time the methods available for their analysis and design were relatively simple, particularly so far as the important three-dimensional effects are concerned – for which reliable implementations of lifting-surface methods suited to nonplanar configurations were only just coming into use. Indeed Whitcomb, in the first report on winglets, (Ref. 6) looked forward to the use of lifting-surface methods such as those of Lamar (Ref. 13) and Goldhammer (Ref. 14) to assist the aerodynamic design and lead to more nearly optimum winglet configurations than he had obtained. Whitcomb (Ref. 6) presented data for upper winglets and for combined upper and lower winglets. The body of earlier work which had indicated that a nonplanar lifting surface should have less trailing-vortex drag than a planar wing was acknowledged by Whitcomb (Ref. 6), but he presented no numerical results based on it. Such results could have thrown some light on the analysis of test data and on the relative advantages of optimally designed winglets and tip-extensions, see Section 5.3.1.

4.3 Computational Fluid Dynamics and Other Approaches

Since the initial introduction and exploration of winglets, much development of CFD (computational fluid dynamics) methods has taken place, and many such methods are routinely used in aerodynamic design. In order to be of maximum value in engineering analysis and design, the capabilities and limitations of the various CFD methods available need to be well understood. In principle, the most accurate methods for practical application are those that are based on the time-averaged Navier-Stokes equations, use advanced transition prediction and turbulence models, and employ closely spaced grid nodes. Even so, such methods cannot be relied upon fully to provide reliable indications of features such as boundary-layer transition and shock/boundary layer interactions, and the present need to utilise wind tunnel test data in the design process (even if only as a means of 'calibrating' the CFD results) is likely to continue. A number of less advanced methods (some of which include allowance for viscous boundary-layers and wakes) also remain valuable at various stages in the design process. These include (a) methods that provide solutions of the Euler equations, (b) methods that solve the velocity potential equation rather than the Euler equations, (c) methods that solve the small-perturbation potential equation rather than the full potential equation, (d) panel methods, and (e) methods such as lifting-surface theory that are based on the linearised potential equation. A useful introduction to the capabilities and limitations of these methods is given by Boppe (Ref. 15).

CFD methods for three-dimensional configurations are expensive in terms of their initial implementation and their use. It remains a very demanding requirement to predict the aerodynamic drag of a configuration to an accuracy of 1 per cent, but even this degree of accuracy may not be sufficient to assess the relative merits of alternative wing/winglet configurations. However, CFD methods which may be inadequate for providing absolute drag data may be used in an incremental way, and relatively simple methods may provide results for pressure fields and shock waves that, once interpreted, can be valuable in indicating undesirable flow features that might lead to drag increments or flow separation. Examples of this nature derived from the application of solutions of the Euler equation (coupled with a three-dimensional boundary-layer calculation) to wing-body-winglet flows are presented by Vassberg *et al.* (Ref. 34), and Yu *et al.* (Ref. 35). In Ref. 34 the application of a panel method is also described, whilst in Ref. 35 remarks concerning a velocity potential solution method are included. The task of aerodynamic design also has to be considered, and this can be expected to involve, or be equivalent to, a considerable number of analysis calculations. It appears probable that the capability to perform high quality detailed design of either a wing or a wing/winglet configuration will continue to require a number of specialist skills and experience, including the critical utilisation of wind tunnel test data. Guidance in the case of the aerodynamic design of a wing is given in Ref. 16.

4.4 Methods for Initial Assessment of Winglets

For an initial assessment of the merits of including winglets in the design process for a new project, or for an assessment of the potential improvement that could be achieved by fitting winglets to an existing wing, the use of advanced CFD methods may be impractical, and for these tasks it may well be preferable to use as simple a method of aerodynamic analysis as possible, so long as it does not produce misleading indications. Such a method is provided, so far as the trailing-vortex drag element is concerned, by the vortex-lattice theory, as long as the aerodynamic loading distributions involved are not significantly affected by features such as shock waves and boundary-layer separation. Vortex-lattice theory, in common with other implementations of lifting-surface theory, does not, of course, provide any indication of the important contributions from a winglet to the profile drag and the wave drag elements of a configuration. However, so far as the contribution to the profile drag is concerned, a simple assessment may be adequate, such as one based on the extra skin friction that can be expected from the additional wetted area introduced by the winglets, and which could take into account the relatively low chord Reynolds numbers of the winglet sections. It is likely that such an assessment would tend to underestimate the actual profile drag increment, by taking no account of the interference between the wing and winglet flow fields. The wave drag contribution of any winglet depends on the strength and form of any shock waves in the region of the outer

wing and winglet, and these in turn depend on the particular section geometries employed and the local lift coefficients. Thus the wave drag contributions are related to details of the design, and do not depend simply on the winglet planform, location and twist parameters. It was argued by Whitcomb (Ref. 6), that a successful winglet design would locate the winglet and specify its section profiles and twist parameters in such a way that only negligible additional wave drag would be incurred (relative to the basic wing), at least at and near the design Mach number and lift conditions. Guidelines for achieving this were presented by Whitcomb (Ref. 6), see also Section 6.

In this Item, assessments of the likely aerodynamic effects of winglets operating near their design conditions are obtained on the following basis.

- (a) Values of the trailing-vortex drag are determined by a lifting-surface analysis using vortex-lattice theory.
- (b) The effects of a winglet on the profile drag are (implicitly) related to the additional surface area.
- (c) The winglets are assumed to contribute negligible additional wave drag.
- (d) It is assumed that there is no mutual interference between the winglets and other airframe components except the wing, and so the fuselage, tail, pylons and nacelles are not represented. The configuration is represented simply by the gross wing, together with any winglets. This simplification is common in studies of winglets effects, see, for example, Refs 4, 5, 9 and 31.

In order to assess the influence of winglets on wing structure-weight (for *ab initio* designs) the close correlation between wing structure-weight and wing root bending-moment found by Heyson *et al.* (Ref. 4) may be utilised. Since the effects of the fuselage are considered to be negligible so far as winglet assessment is concerned, the wing root bending-moment in this correlation is replaced, in this Item, by the aerodynamic moment about the wing centre-line, M_r , generated on one wing panel (*i.e.* port or starboard), together with any winglet attached to that panel.

A number of comparisons between predictions of winglet effects made on this basis and observed effects show generally good agreement provided the wing/winglet configurations are not operating with shock waves or flow separations producing significant drag increments, see Refs 17 and 18, and Section 5.3.1, and this justifies the approach adopted.

4.5 Remarks Concerning Drag Comparisons

In studying reports relating to winglets or tip-extensions, it is often found that conclusions are drawn concerning the magnitude of the drag coefficient decreases (usually at specific values of lift coefficient), and sometimes conclusions are drawn concerning corresponding decreases in trailing-vortex drag coefficient. There are two matters in particular that need to be considered when assessing the accuracy of such drag changes, as follows.

- (a) Drag measurements in a wind tunnel are subject to a large number of influences that make it very difficult to determine model drag to better than one or two per cent. One often significant source of error is the wall-induced upwash, see, for example, Ref. 19, which, in a solid-liner wind tunnel, for a model with a wing of moderate aspect ratio and a span to tunnel width ratio of about 0.6 (which is typical), can result in corrections to the raw balance drag of the order of half the trailing-vortex drag itself (see Ref. 17). With such large corrections, any uncertainty in the wall-induced upwash is potentially important, and it is clearly desirable to assess the effects of possible changes to this upwash resulting from the application of winglets. However, for small winglets (up to about 0.15s in length), it would appear that differences in drag attributable to

changes in wall-induced upwash in solid-liner wind tunnels are likely to be negligible. Nevertheless, unless the wall-induced upwash is accurately estimated, although the drag changes might be accurate, the drag values themselves would be erroneous, and this would make drag changes expressed as percentage changes misleading.

- (b) Measured drag coefficient data corresponding to a fixed test Mach number are often plotted against measured lift coefficient. These curves are termed drag polars. Typical drag polars for a wing-fuselage model and for the same model with winglets fitted are shown in Sketch 4.1. The data points in the range $0 \leq C_L \leq 0.5$ are also shown to a different scale in Sketch 4.2. The overall effects of winglets in increasing C_D at relatively low values of C_L , and decreasing C_D at relatively high values of C_L , are evident. In this case the cross-over, where there is no change in C_D , occurs (for positive C_L) at around $C_L = 0.3$. Over a range of moderate C_L values, drag polars are often nearly parabolic, but for most models the parabola does not have its minimum value for C_D at $C_L = 0$. The reason for such an offset is generally almost entirely attributable to the behaviour of the profile drag (and possibly the wave drag) of the wing, and the way these drag components vary with lift. Even though at fixed Mach number the trailing-vortex drag coefficient of a cambered and twisted wing behaves according to subsonic lifting-surface theory as,

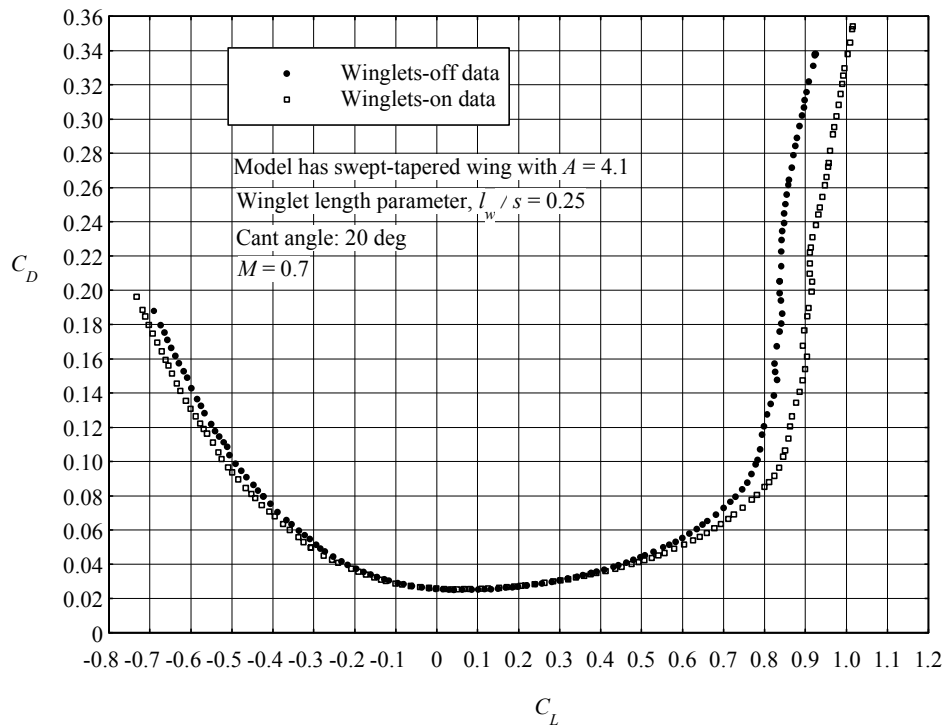
$$C_{DTV} = \bar{a} + \bar{b}(C_L - C_{L0})^2, \quad (4.1)$$

where $\bar{a}$, $\bar{b}$ and C_{L0} are constants, the magnitudes of C_{L0} that arise from the theory are so small (typically much less than 0.02), that they can be neglected, and Equation (4.1) can be replaced by

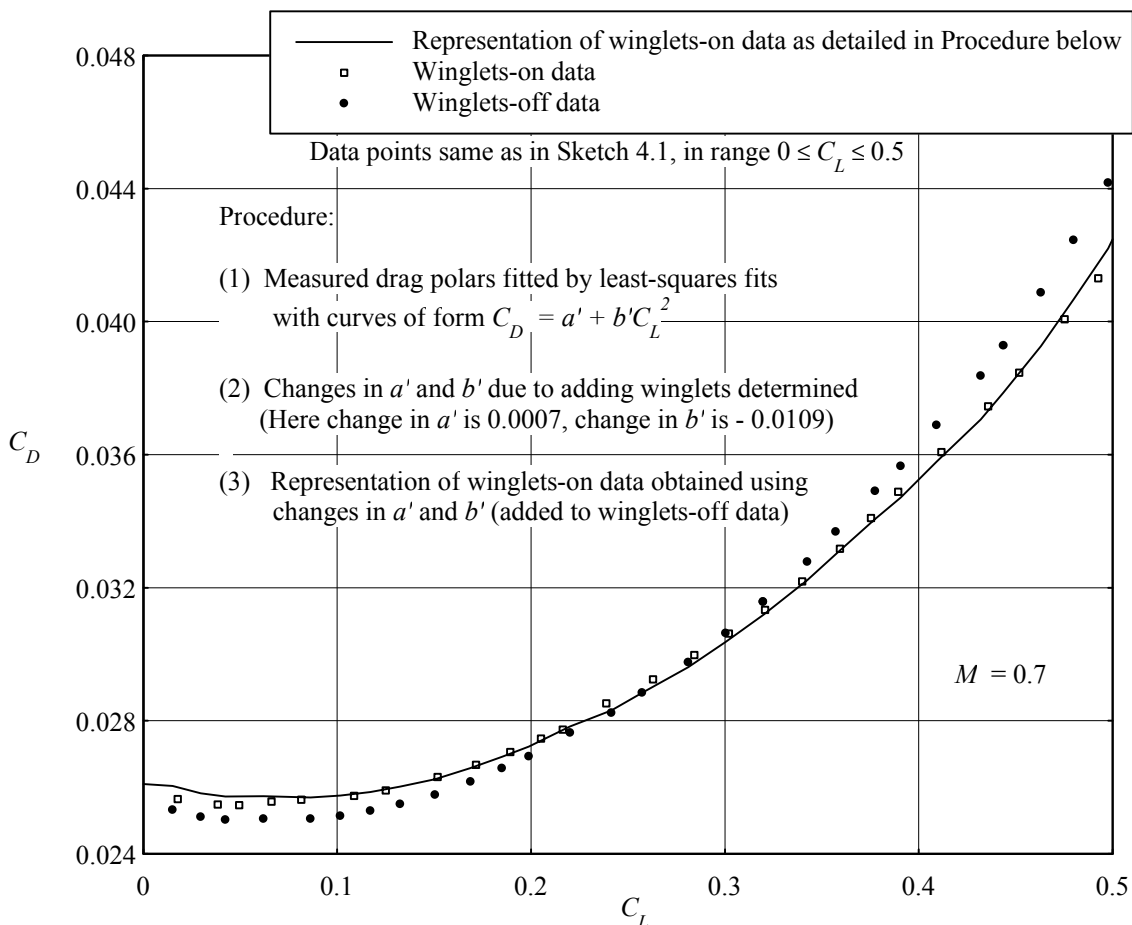
$$C_{DTV} = a' + b'C_L^2 \quad (4.2)$$

with negligible error, with $a' \cong \bar{a}$ and $b' \cong \bar{b}$. (For two examples which demonstrate this, see Ref. 37, Sections 7.1 and 7.2.)

Thus when analysing drag polars to deduce the effects of winglets it is suggested that the sets of winglets-on and the winglets-off drag coefficient data each be fitted over specific ranges of C_L by equations of the form of Equation (4.2) (with C_D replacing C_{DTV}), and the changes to the fitted a' and b' coefficients associated with adding winglets then determined, or, alternatively, the differences in C_D (at fixed C_L) may be so fitted. This latter procedure is not so simple in practice, since the values of C_L are unlikely to be equal in the two sets of tests, and interpolation will be required. A careful analysis of a particular series of tests with and without winglets over a range of Mach numbers up to 0.85 (Ref. 17) showed that the two procedures gave essentially the same results in terms of changes in the a' and b' coefficients associated with adding winglets, even though the individual drag polars were not well fitted by curves of the form of Equation (4.2) except over quite limited ranges of C_L . An example of the former procedure is shown in Sketch 4.2 in which the data points fitted are those in the range $0 \leq C_L \leq 0.5$. It is seen that for this example the drag coefficient changes due to fitting winglets are well represented by the deduced changes in the a' and b' coefficients. Such good representation does not generally extend to lift coefficients where significant regions of separated flow occur. (Note that there are slight irregularities in the curve shown by a solid line in Sketch 4.2, these being the result of scatter in the winglets-off data.) In attached flow conditions, it can be expected that the change in the value of the a' coefficient will be mainly due to the effect of the winglets on the profile and wave drag components, whilst the change in the value of the b' coefficient will be mainly due to the change in trailing-vortex drag. However, profile drag and wave drag changes due to winglets can produce significant changes in the value of b' , and trailing-vortex drag changes can slightly alter the value of a' .



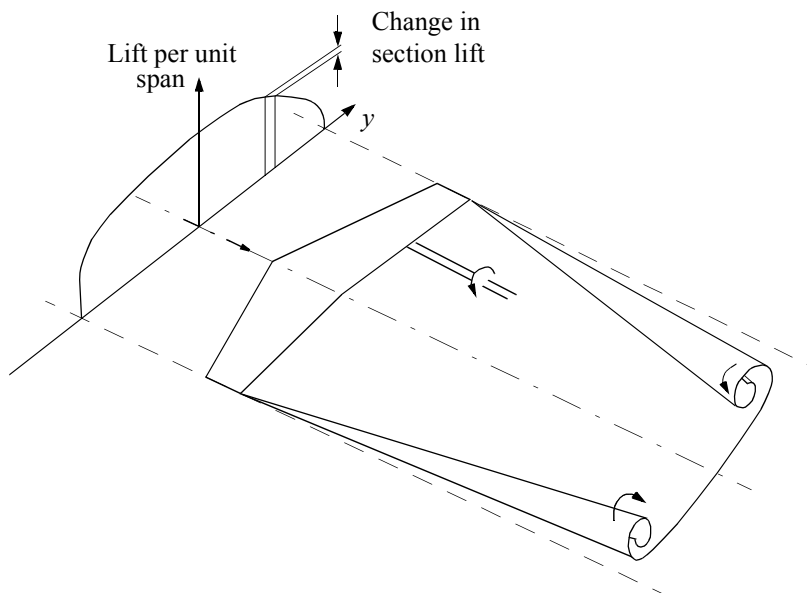
Sketch 4.1 Example of drag polars determined from wind-tunnel measurements on (a) wing-fuselage model and (b) wing/winglet-fuselage model



Sketch 4.2 Example of drag polar analysis indicating effects of winglets

5. EFFECTS OF WINGLETS ON TRAILING-VORTEX DRAG

To explain how winglets can decrease the trailing-vortex drag, this drag component may be considered as being a manifestation of the work done in producing and sustaining the vortex motion in the wake of the wing. For most lifting wings, particularly in cruise flight conditions, this motion comprises two counter-rotating vortices of equal strength. These vortices form as a result of the process of rolling-up (and slight diffusion) of the vortex-sheet as it extends downstream from the wing trailing-edge. (Such a pair of counter-rotating vortices generally remains intact for a considerable distance downstream, but distortions produced by the naturally occurring disturbances in the atmosphere eventually become greatly amplified, the vortices break up, usually into a number of ring-shaped vortices, and these then fairly rapidly dissipate, see, for example, Refs 20, 21 and 22.) At any spanwise position, the strength of an element of the vortex sheet at the trailing-edge (see Sketch 5.1) is related to the change of section lift over the span of the element. The strength (circulation) of a rolled-up wake vortex is equal to the sum of the vortex elements that have rolled up to form it, and for the example shown (and generally if the loading decreases monotonically with distance from the centre line) it is equal to the circulation around the centre-line section. If this circulation is kept constant and the span of the wing is increased, then the wing lift is increased whilst the vortex drag remains almost unchanged. Alternatively, if the wing lift is kept constant as the span is increased, the strength of the vortices decreases, and the trailing-vortex drag decreases. It is seen to be beneficial, in terms of trailing-vortex drag, to disperse the given lift of the wing laterally as much as possible, *i.e.* to have as high a trace length as possible (see Section 2.2(c)). The best result for a trace of given length is obtained for a straight trace with an elliptic distribution of lift per unit span. However, for given span of trace (rather than a given length), the straight trace is not the best, and traces corresponding to a wing with winglets, for example, are better, see Section 5.5.



Sketch 5.1 Schematic of vortex-sheet roll-up downstream of wing

Although the idea of achieving a low trailing-vortex drag by dispersing the lift over a large length of trace is qualitatively sound, the argument above has to be used with caution, since the drag associated with the rolled-up vortex wake is not related solely to the strengths of the trailing vortices that comprise it but also to the (viscous) core sizes of these vortices. The smaller a vortex core size the greater is the kinetic energy associated with it, and the greater the corresponding vortex drag. Rather than pursue the idea of the rolled-up wake as a way of determining the trailing-vortex drag, it is far better to utilise Munk's theory (Ref. 23), which obtains this drag under the assumption of a rigid wake (see Section 2.2). Adopting the assumption

of a rigid wake has been shown to give remarkably accurate results for the trailing-vortex drag, see Ref. 24. This accuracy is partly explained by the fact that the velocities induced by the vortex wake in the neighbourhood of the wing are dominated by the near-wake properties. Munk's theory indicates that the trailing-vortex drag of a lifting surface (either planar or non-planar) depends on the shape of the trace and the spanwise distribution of normal force (*i.e.* the chordwise integral of pressure difference) over the lifting surface, but that the chordwise variations of the pressure difference across the lifting-surface have no influence on the trailing-vortex drag. (It also indicates that the trailing-vortex drag is independent of the Mach number of the freestream, except insofar as the spanwise distribution of loading may be affected by changes in this Mach number.) This being so, it is possible to determine the trailing-vortex drag by assuming the aerodynamic load of the lifting surface to be concentrated over an arc (*i.e.* a surface of zero chord) lying in a plane normal to the freestream velocity. This is the approximation adopted in lifting-line theory (*e.g.* Ref. 25). In general the lifting-line theory is only appropriate for use with high-aspect ratio unswept wing configurations, but, for any configuration, provided the distribution of spanwise loading is determined by some more accurate means (such as lifting-surface theory), the lifting-line theory may then be utilised to give the trailing-vortex drag. Similarly, for any configuration, the lifting-line theory may equally well be used to yield the optimum spanwise distributions of loading corresponding to various constraints, such as those of given conditions of lift and non-dimensional moment arm, η_b .

5.1 Wing/winglet Cases for Application of Vortex-lattice Theory

In Sections 5.2 to 5.5 the vortex-lattice theory is used, together with the lifting-line theory evaluation of the trailing-vortex drag, to find this drag component for various wing and wing/winglet configurations. For a specified configuration of lifting surface, the vortex-lattice theory (*e.g.* Refs 13, 14, 26, and 27) is first used to determine an approximation to the distribution of aerodynamic loading over the wing and winglet surfaces. From this the lateral distribution of lift (or, more generally, normal force) may be determined, simply by summing the loads along each chord. Knowledge of this distribution, and the shape of the trace, is sufficient to determine the trailing-vortex drag. The use of a number of different vortex lattice layouts to represent the lifting surface, of various degrees of refinement, enables the accuracy of the numerical scheme to be estimated. Two programs to implement this vortex-lattice procedure, one for planar lifting surfaces, and one for wing/winglet configurations, were developed and utilised to obtain a number of results.

In addition to the determination of the trailing-vortex drag for specified configurations, it is of interest to determine the shapes of the lifting surface needed to achieve various optimum conditions. For example, it is of interest to find the variation of wing twist needed to give the spanwise loading corresponding to minimum trailing-vortex drag at given conditions of lift for a wing of specified planform. (Section camber also influences the loading distribution, but the choice of camber shapes is more likely to be related to the requirement to achieve high values of section efficiency at the design lift and Mach number conditions, than as a means of controlling the spanwise loading.)

Also, it is of interest to find the optimum linear variation of twist across the span, and to determine how much the restriction to a linear variation of twist adds to the trailing-vortex drag. Further optimum conditions that take account of the wing root-bending moment (replaced in this Item by M_r , see Section 4.4) are also of interest, since this moment correlates closely with the wing weight for an *ab initio* wing design (Ref. 4). Design studies show that it is worth trading a small increase in cruise trailing-vortex drag against a decrease in wing weight, and when this is done the wing loading distribution is weighted more heavily inboard than for the case of minimum trailing-vortex drag. It has to be noted that the wing root-bending moment that correlates with the wing weight corresponds to that at a considerably higher lift coefficient than applies to the cruise, C_{Lcr} , namely that corresponding to the limiting design gust or manoeuvre condition, denoted here by C_{Lstr} . (It is assumed that the flight Mach number in the cruise is the same as that of the limiting gust or manoeuvre.) In the study of Heyson *et al.* (Ref. 4), the values selected for C_{Lcr} and C_{Lstr} were 0.4 and 1.0 respectively. Both for a wing and a wing/winglet configuration, the loading distribution for this limiting gust or manoeuvre case can then be found as the sum of two loading distributions; these are

- (a) the loading distribution corresponding to the cruise conditions,
- and
- (b) the loading distribution produced by an increment in overall incidence that generates a lift coefficient increment of $C_{Lstr} - C_{Lcr}$.

In the present Item, since only a comparative initial analysis is made, the effects of structural flexibility on these loadings are neglected. Then, according to lifting-surface theory, the loading (b) is related solely to the wing/winglet planform and flight Mach number. The camber and twist distributions employed affect only loading (a). It can thus be seen that the wing and winglet twist can only be used to reduce the moment M_r corresponding to (a), whilst the (larger) moment corresponding to (b) is unaffected. As a result, although not negligible, there is only a relatively limited scope for using twist as a means of controlling the limiting moment M_r . In broad terms, a decrease in M_r of 2.5 per cent at C_{Lcr} can be expected to result in a decrease in M_r of only about 1 per cent at C_{Lstr} . In practice, the occurrence of boundary-layer separation may result in significant differences between predicted and actual loading distributions for C_{Lstr} , but such differences would be expected to have only a relatively small effect on the comparisons between various wing and wing/winglet performance parameters.

Further comparisons of interest arise when considering the addition of a winglet to a wing of specified configuration. For a given winglet planform and position relative to the wing, it is possible to determine the winglet twist so as to minimise the trailing-vortex drag of the total configuration, either for conditions of fixed lift, or for conditions of fixed lift and moment M_r (the latter being appropriate to C_{Lstr}).

The foregoing types of calculation of optimum conditions provide valuable indications of the magnitudes of the effects on the trailing-vortex drag of influences such as wing and winglet twist, and moment M_r , and as such can inform design choices in actual cases. A number of examples follow. In these examples results are given for the trailing-vortex drag together with the associated spanwise load distributions, corresponding to minimum trailing-vortex drag conditions, for a variety of wing and wing/winglet cases. In order to simplify the presentation of the example results, when a particular planform and Mach number have to be considered, the planform and design Mach number studied by Whitcomb (Ref. 6) are adopted, see Figure 2.

In the examples presented in Sections 5.2 to 5.4, the trailing-vortex drag of each wing/winglet combination of interest is given in relation to the minimum trailing-vortex drag, D_{ob} , produced by an isolated wing of the same span, b , as that of the *basic wing*, and generating the same lift, L , as the wing/winglet combination, that is:

$$D_{ob} = L^2 / \pi q b^2 = L^2 / 4 \pi q s^2.$$

(This isolated wing minimum is achieved when the spanwise load distribution is elliptic.)

The moment, M_r , at cruise conditions is expressed in terms of the non-dimensional moment arm, η_b . When the limiting design moment is being referred to, the corresponding non-dimensional moment arm is denoted by η_{str} .

In the examples in Section 5.5, the trailing-vortex drag of a wing/winglet combination is given in relation to the minimum trailing-vortex drag, D_{oa} , produced by an isolated (planar) wing having the same *overall span*, b_a , as that of the wing/winglet combination, and generating the same lift, L , as the wing/winglet

combination, that is:

$$D_{oa} = L^2/\pi q b_a^2 = L^2/4\pi q s_a^2.$$

(This isolated wing minimum is again achieved when the spanwise load distribution is elliptic.)

Relating the trailing-vortex drag to D_{oa} is a suitable basis for comparison of trailing-vortex drag values in those cases where the overall span is constrained, as it is for the examples considered in Section 5.5.

5.2 Wing/winglets having Minimum Trailing-Vortex Drag with Lift and Bending Moment Constraints – *ab initio* Designs

These considerations relate to overall optimal loadings, and are therefore only relevant to *ab initio* designs.

Considering a wing with an upper winglet, when the configuration is optimally loaded, the parameter of interest is D_{TV}/D_{ob} , and this is affected by η_b , l_w/s and ϕ . Constraints of fixed lift and non-dimensional moment arm, η_b , are considered, and these are the same, in effect, as constraints of fixed lift and moment, M_r , since $\eta_b = 2M_r/Ls$. The span of the wing, $b(=2s)$, excluding any winglets, is considered fixed. Graphs of D_{TV}/D_{ob} against η_b are precisely parabolic, according to the linear theory. Such graphs for $l_w/s = 0.05, 0.1, 0.15$ and 0.2 are given in Figures 3 to 6, for cant angles ranging from -30 deg to 90 deg (90 deg is the tip extension case). A negative cant angle is included for completeness, but should not be taken as an indication of any practical utility, although it could be employed as a means of obtaining low values of η_b . It should also be noted that results for cant angles greater than 90 deg may be obtained from the figures since *results for cant angles of ϕ and $180 - \phi$ are identical*. For civil transport aircraft, unless combined upper and lower winglets are used, a cant angle greater than 90 degrees (implying lower winglets) would generally be avoided, for reasons of ground operations. In the figures, the range of η_b is restricted to exclude high values of D_{TV}/D_{ob} , since these would not be of interest. Results for D_{TV}/D_{ob} and η_b , for those cases where M_r is not subject to a constraint, are given in Figure 7.

Examples of optimal spanwise loading distributions, $C_{NL}c/C_L\bar{c}$ against η , are given in Figures 8 to 10 for $l_w/s = 0.15$, for $\phi = 0$ and 90 deg for cases where lift and moment, M_r , or just lift, are subject to constraint.

It is clear from Figure 7, for example, that if no regard is taken of M_r , then for any selected value of l_w/s the best cant angle to choose is 90 deg, since this yields the lowest value of D_{TV}/D_{ob} . (This fact is not always appreciated, as may be seen from Ref. 29, where $\phi = 0$ is stated as giving a lower value than $\phi = 90$ deg.) It is also seen from Figure 8 that the winglet loading values are only required to be slightly higher for $\phi = 90$ deg than for $\phi = 0$ deg. There is thus little advantage accruing to the low cant angle cases on the basis that they could use lower local chords as they would be required to produce lower loadings, an argument that has been suggested.

The fact that at the wing/winglet junction the winglet root chord, c_{wr} , is generally chosen to be lower than the wing tip chord, c_t , has been implied in Sections 2.2 and 3.1, and is alluded to in Section 6.2, where it is linked to the rapid variations in the optimal normal loading distributions near $\eta = 1$ which occur for a range of values of ϕ as shown in Figures 8 to 10. When $\phi = 90$ deg, the loading distributions show no such rapid variations near $\eta = 1$, and on this basis the planform of a wing with a well-designed tip-extension would be expected to have a smooth chord length variation in the vicinity of the junction of the basic wing and the tip-extension.

If a constraint on M_r is taken into account, then Figures 3 to 6 again seem to indicate that the best choice for ϕ is 90 deg, since over the range of η_b where D_{TV}/D_{ob} is below 1, for any fixed value of η_b (and thus for fixed M_r) the lowest value of D_{TV}/D_{ob} is obtained with $\phi = 90$ deg. However, as has been noted in