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Elastic buckling of flat isotropic stiffened panels and struts in compression

**Associated software: ESDUpacs
A9816
and B9816**

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ELASTIC BUCKLING OF FLAT ISOTROPIC STIFFENED PANELS AND STRUTS IN COMPRESSION

CONTENTS

1.	INTRODUCTION, THEORY AND APPLICATION	1
1.1	INTRODUCTION	1
1.1.1	General Limitations and Scope of the Method of ESDUpac A9816	2
1.2	NOTATION	3
1.2.1	Suffices	5
1.2.2	Sign Convention	6
1.3	OUTLINE OF THE STRIP METHOD AND MODEL DETAILS	6
1.3.1	Stringer Section Models	8
1.3.1.1	Predefined skin-stringer sections	8
1.3.1.2	General panel model	10
1.3.1.3	Special single-stringer section	10
1.3.1.4	Rolled-sheet and machined sections	10
1.3.2	Skin-stringer Joint Models	11
1.3.3	Analysis Options	12
1.4	RESULTS	14
1.4.1	Introduction	14
1.4.2	Comparison Cases from the Data Items of Table 1.1	14
1.4.3	Illustrative Examples	15
1.4.3.1	A corrugated sandwich panel	15
1.4.3.2	Buckling of an I-beam. The influence of shear deformation	17
1.4.3.3	Buckling of a sandwich column - using the adhesive layer to represent the sandwich core	19
1.4.3.4	Buckling of a lipped channel-section strut. On the minima in the $P-L$ curve	20
1.4.4	Aspects of the Finite Strip Models	23
1.4.4.1	The stringer corner bend radius. Local and overall buckling of a top-hat-section stiffened panel	23
1.4.4.2	The special single-stringer model	24
1.5	DERIVATION AND REFERENCES	26
1.5.1	Derivation	26
1.5.2	References	26
2.	ESDUpac A9816	27
2.1	INTRODUCTION TO THE PROGRAM	27
2.2	PROGRAM OPTIONS AND LIMITATIONS	27
2.3	RUNNING THE FORTRAN PROGRAM	28

2.3.1	Overview	28
2.3.1.1	File definitions	28
2.3.1.2	Naming of input and output files	29
2.3.1.2.1	ESDUpac A9816	29
2.3.1.2.2	ESDUpac B9816	29
2.3.1.3	Approaches to running the program	29
2.3.2	Panel Geometry Definition File	30
2.3.2.1	Units	31
2.3.2.2	Example panel geometry definition file	34
2.3.3	Material Data File	34
2.3.3.1	Example material data file	35
2.4	PROGRAM OUTPUT	35
2.4.1	Runtype and Output Specifiers	35
2.4.1.1	Runtype = 0, Minimal output, OP = 0	38
2.4.1.2		38
2.4.1.3		40
2.4.1.4		40
2.4.2	Strip Model Data File	41
2.5	General Notes on ESDUpac A9816	45
2.5.1	Introduction	45
2.5.2	Program Constants	45
2.5.3	Warnings and Error Messages	46
2.5.4	Data Flow in Program A9816	50

NOTES ON PREDEFINED SKIN-STRINGER SECTIONS

A1.	INTRODUCTION	51
A1.1	Single-stringer Models	51
A1.2	'Machined' versus 'Rolled-sheet' Stringer Sections	51
A1.3	Location of Z-section Stringers	53
A1.4	Node and Element Numbering Conventions for Sketches	53
A1.5	Illustration of Pitch and Edge Distances	53
A1.6	Example Input and Output Files for Type 1 Stringer	53
A1.6.1	Example input files	54
A1.6.1.1	User-prepared input files	54
A1.6.1.2	Program-generated input file, 020134ck.STO	55
A1.6.2	Example output files	56
A1.6.2.1	Input data check with extended output	56
A1.6.2.2	Output file for local buckling analysis with minimal output, 020134lb.PRT	58
A1.6.2.3	Output file for overall buckling analysis with extended output	60
A1.6.2.4	Output file for P - L curve analysis and minimal output	62

A2.	PREDEFINED SECTIONS	66
A2.1	SIMPLE INTEGRAL BLADE STRINGER, TYPE 1	66
	A2.1.1 Example panel geometry definition file	67
A2.2	BEADED INTEGRAL BLADE SECTION, TYPE 2	68
	A2.2.1 Example panel geometry definition file	69
A2.3	BEADED T-SECTION, TYPE 3	70
	A2.3.1 Example panel geometry definition file	71
A2.4	I-SECTION, TYPE 4	72
	A2.4.1 Example panel geometry definition file	73
A2.5	LIPPED J-SECTION, TYPE 5	74
	A2.5.1 Example panel geometry definition file	75
A2.6	MACHINED LIPPED Z-SECTION, TYPE 6	76
	A2.6.1 Example panel geometry definition file	77
A2.7	MACHINED UNLIPPED Z-SECTION, TYPE 7	78
	A2.7.1 Example panel geometry definition file	79
A2.8	ROLLED-SHEET LIPPED Z-SECTION, TYPE 8	80
	A2.8.1 Example panel geometry definition file	81
A2.9	MACHINED TOP-HAT SECTION, TYPE 9	82
	A2.9.1 Example panel geometry definition file	83
A2.10	ROLLED-SHEET TOP-HAT SECTION, TYPE 10	84
	A2.10.1 Example panel geometry definition file	85
 APPENDIX B STRINGER-SKIN JOINT MODELS		
B1.	NOTATION	87
B2.	INTRODUCTION	87
	B2.1 Adhesive Layer Strain-displacement Relationships	90
 APPENDIX C EXAMPLE PROBLEM		
C1.	EXAMPLE OF MODIFYING THE STRIP MODEL DATA FILE TO SPECIFIC USER REQUIREMENTS	93

ELASTIC BUCKLING OF FLAT ISOTROPIC STIFFENED PANELS AND STRUTS IN COMPRESSION

1. INTRODUCTION, THEORY AND APPLICATION

1.1 INTRODUCTION

This Data Item accompanies ESDUpac A9816 which provides a Fortran computer program that calculates initial elastic compressive buckling loads for stiffened rectangular panels and struts made of isotropic materials. The method on which the program is based is the finite strip method (FSM), which is particularly well-suited to the analysis of prismatic structures that show periodic deformation patterns under loading. A brief outline of the finite strip method is presented in Section 1.3.

The purpose of the program is to automate and extend the range of applicability of the methods employed in various Data Items in the Aerospace Structures Series on the subject of buckling of elastic structures. Table 1.1 summarises the relevant ESDU Data Items, for which the program provides an alternative computational method for estimating the buckling loads of panels and struts. These Items are mainly examples of applications of the general theory presented in Data Items Nos 02.01.28* and 94005†. Although the program is based on FEM-like discretisation procedures, it possesses significant advantages in speed compared to a full-scale finite element approach.

In the interest of quick and effective use, and in order to ease the input preparation process, the program is equipped with features that allow the user to specify a panel geometry using predefined skin-stringer sections. This aspect is elaborated more fully in Section 1.3.1.1. The option to analyse models of arbitrary cross-section, for which the user prepares the input, is also available. In addition, and extending further the capabilities and scope of the methods involved, adhesive layer and rivet line elements are provided to enable the joint between stringer and skin to be modelled in a more realistic way. Section 1.3.2 gives more detailed information on these joint models. Section 1.3.3 briefly explains the different choices available to the user for selecting an appropriate type of analysis. Table 1.3 in that section presents a compact overview of the available panel/strut models, types of boundary or support conditions provided, applicability of the joint models and a summary of the run types.

The computational capabilities of the program are demonstrated in Section 1.4. Firstly, in Section 1.4.2, direct comparisons of results produced by the program with those given by the Data Items listed in Table 1.1 are given. Secondly, Section 1.4.3 contains selected examples that illustrate the scope of the program and highlight its capabilities. Thirdly, typical features that are inherent to the finite strip formulation used by the program are outlined in Section 1.4.4. This section has been included to offer the user a broader insight into what is possible and what to expect when using program A9816.

Section 2 of the Data Item may be considered as a Program User's Manual. It consists of descriptions of the input and output files, interwoven with appropriate examples to explain or elucidate detailed aspects of the finite strip input/output models.

The introduction of predefined skin-stringer sections requires the listing of the typical section parameters, such as widths, thicknesses, *et cetera*. Information on these parameters is brought together in Appendix A of this Data Item. The introduction to that appendix explains the reasons behind the values assigned to some of these parameters. Among others, the differences between milled or extruded and rolled-sheet stringer section models are elaborated there. The sub-sections of the appendix present the geometric and finite strip models for all the predefined skin-stringer sections.

* Data Item No. Struct 02.01.28 "Information on the use of Item No. 73007".

† Data Item No. 94005 "Buckling of flat rectangular orthotropic plates".

Since the skin-stringer attachment models used for the adhesive layer and rivet line connections may be new to some users, Appendix B provides background material on which most of the assumptions adopted for those models are based.

1.1.1 General Limitations and Scope of the Method of ESDUpac A9816

ESDUpac A9816 has been written to compute the buckling loads of models that are presumed to behave within the realm of classical thin plate theory. See Derivation 1 for the pertinent assumptions that are basic to that theory.

It should be noted, however, that the restriction to classical thin plate theory holds for the formulation of the equations of equilibrium of a strip. This means that the actual panel may well show deformation patterns that can analytically only be described by more comprehensive, or higher order, theories. For example, the influence of transverse shear deformation of the total panel cross-section on the buckling load is included, despite the fact that the strip element formulation does not account for through-the-thickness shearing within each strip. However, since in-plane shear deformations are represented in the behaviour of the strip as a consequence of the utilisation of the two-dimensional thin plate theory, the influence of this deformation shows up, via the in-plane shear strains arising in webs and uprights, as a reduction of the buckling load of a built-up panel. The same effect is described in Chapter 2, Article 2.17, of Derivation 2, where it is introduced as the “effect of shearing forces” on the critical load. The example in Section 1.4.3.2 is presented to show this effect for an I-section beam.

Because of the inherent freedom, on account of the finite strip formulation, for the buckling modes to develop across the panel, it has been possible to extend the scope of the computable modes to include the combined torsional-flexural mode. No particular fixed points of rotation of the section, or parts of the section, are presumed to exist, other than those enforced by the user through imposition of boundary conditions. A comprehensive account of local, flexural and torsional-flexural buckling may be found in Section 3 of Data Item No. 72012*.

Hence, the present version of ESDUpac A9816 is able to compute the elastic buckling stresses and associated modes for local, flexural and torsional or torsional-flexural buckling. It is emphasised, however, that the method presented has, as yet, no capabilities to analyse the response of a panel loaded under axial compression into the post-buckling range. See Data Item No. 78021† for more information on that subject.

In addition, no attempt has been made to include in the model of a rivet line connection the possibility of analysing inter-rivet buckling phenomena. Data for such cases are provided in Data Items Nos Struct 02.01.08‡ and Struct 02.01.09**.

The program provides the **elastic** buckling load for panels and struts. This means that solutions must still be checked to establish whether any stress levels occur in the plastic range of the selected materials. The plasticity reduction factors, as utilised in the various Items, still apply. The user is referred to Section 3 of Data Item No. 83044†† for information on how to select a reduction factor η , where η is the ratio of the plastic to the elastic buckling stress of the panel. It is also emphasised that care should be taken when selecting a particular form for η because the expressions given for η in that Item apply strictly to specially supported plates that do not typically occur in built-up panel sections. In addition, particular attention should be paid to cases where the skin and stringer materials have different mechanical properties.

* Data Item No. 72012 “Information on the use of Data Items on the buckling of plates and compression panels manufactured from isotropic materials”.

† Data Item No. 78021 “Guide to Items on the strength and stability of struts”.

‡ Data Item No. Struct 02.01.08 “Buckling in compression of sheet between rivets”.

** Data Item No. Struct 02.01.09 “Inter-rivet buckling curves for specific materials”.

†† Data Item No. 83044 “Plasticity correction factors for plate buckling”.

The plate strip element matrix is derived for flat elements. A buckling analysis for a model of curved panel or shell-type geometry, where, amongst other things, significant membrane action could be expected as a result of its overall curvature, could be carried out, by approximating the curved geometry with flat strip elements, but is not recommended.

TABLE 1.1 Relevant Data Items on Buckling

<i>Data Item No.</i>	<i>Volume No.</i>	<i>Title</i>
Struct 02.01.34	8	Estimation of the initial buckling stress under uniaxial compression of a skin with integral unflanged stiffeners.
Struct 02.01.35	8	Estimation of the initial buckling stress under uniaxial compression of a skin with Z-section stringers.
Struct 02.01.36	8	Estimation of the initial buckling stress under uniaxial compression of a skin with top hat section stringers.
Struct 02.01.37	8	Estimation of the initial buckling stress under uniaxial compression of a corrugated sandwich panel.
70003	8	Local buckling of compression panels with unflanged integral stiffeners.
71014	8	Local buckling of compression panels with flanged stringers.
76023	11	Buckling of struts. Lipped and unlipped channel sections.
77030	11	Buckling of struts. Lipped and unlipped Z sections.
78020	11	Local buckling and crippling of I-, Z- and channel-section struts.
78021	11	Guide to Items on the strength and stability of struts.

1.2 NOTATION

Both SI and British units are quoted but any coherent system of units may be used. Additional notation is defined where required.

A	cross-sectional area	m^2	in^2
A_w	shear area	m^2	in^2
e_1, e_2	edge widths, see Sketch A1.2	m	in
E	Young's modulus	N/m^2	lbf/in^2
D	diameter	m	in
G	shear modulus	N/m^2	lbf/in^2

h	height	m	in
i	counter or index		
I	second moment of area		
ID_{type}	predefined skin-stringer section identifier		
I_{y0}	second moment of area of element about axis through its centroid and parallel to model y-axis (see Sketches A2.1 to A2.10 for location of y-axis)	m^4	in^4
I_{yy}	second moment of area of element about axis joining its node points	m^4	in^4
I_{zz}	second moment of area of element about axis through its centroid and perpendicular to axis joining its node points	m^4	in^4
k	boundary condition identifier		
L	buckling half-wavelength	m	in
L_0	starting length, see Section 1.3.3 and Table 2.2	m	in
L_{inc}	incremental length	m	in
L_p	panel length	m	in
MC_i	material property code		
n	correction factor		
N_{st}	number of stringers		
OP	output specifier		
p	pitch	m	in
P	load	N	lbf
R	stringer corner inside bend radius	m	in
Runtype	run type specifier		
S	shear stiffness	N/m	lbf/in
t	thickness	m	in
u, v, w	displacements in x, y- and z-directions, respectively, in buckling mode	m	in
W_i	dimension	m	in
W_{snw}	width of sandwich column	m	in

x, y, z	co-ordinates	m	in
z_{avg}	distance of centroid of element from y-axis	m	in
α	angle of inclination of web of machined top-hat section, see Sketch A2.9	rad	rad
γ_{xy}	shear strain		
ϵ	strain ('Eps' in program output)		
η	plasticity reduction factor, see Section 1.1.1		
θ	angle of orientation of element about x-axis, measured anticlockwise from y-axis ('Theta' in program output)	rad	rad
ν	Poisson's ratio ('Nu' in program output)		
σ	stress ('Sigma' in program output)	N/m ²	lbf/in ²
τ_{xy}	shear stress	N/m ²	lbf/in ²
ϕ	rotational displacement about x-axis in buckling mode ('Phi' in program output)	rad	rad

1.2.1 Suffices

1, 2, ...	indices
<i>adh</i>	adhesive
<i>c</i>	core of sandwich
<i>chord</i>	chord of sector of stringer corner
<i>cr</i>	critical (or buckling)
<i>E</i>	Euler
<i>EL</i>	element
<i>f</i>	flat (in this Data Item, a flat refers to any of the individual components that form part of a section)
<i>face</i>	face plate of sandwich
<i>i, k</i>	indices
<i>l</i>	local
<i>Nf</i>	number of flats
<i>riv</i>	rivet

<i>sk</i>	skin
<i>snw</i>	sandwich
<i>st</i>	stringer
<i>x, y</i>	in <i>x</i> -and <i>y</i> -directions respectively

1.2.2 Sign Convention

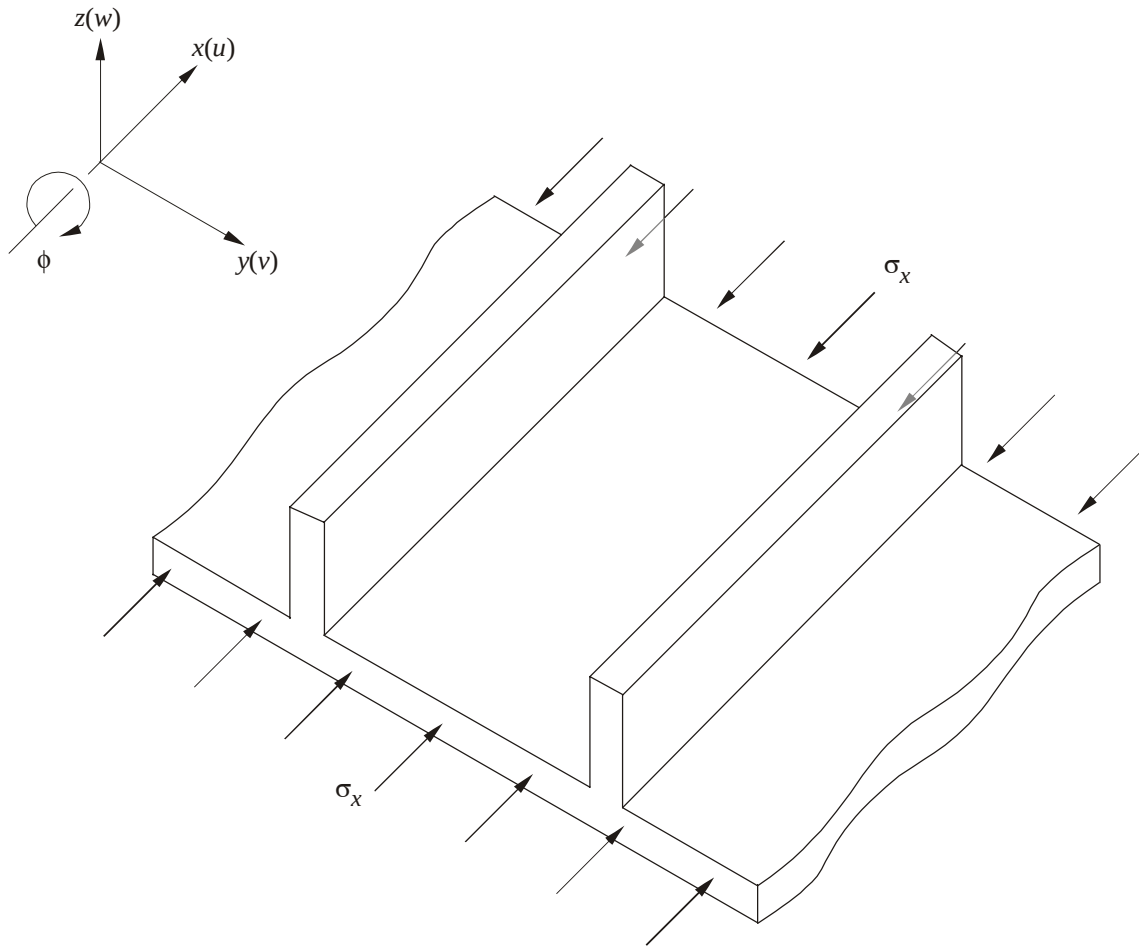
Tensile loads, stresses and strains are positive; compressive loads, stresses and strains are negative.

1.3 OUTLINE OF THE STRIP METHOD AND MODEL DETAILS

The finite strip method may be used to advantage for structural analysis in cases where the analysis pertains to models of prismatic structures that exhibit periodic deformation behaviour. These conditions prevail when considering buckling phenomena that occur in stiffened panels under compressive loading. The uniformity of the pre-buckling state and the usually prismatic character of the panel allow the adoption of the assumption of periodicity for the buckling deformations. The method is a generalised variant of the approach used by Timoshenko for solving various plate buckling problems, see Derivation 2, Article 9.4. The same assumption of periodicity is employed in the derivation of the data presented in Data Item No. 73007, see Reference 7.

The assumption of periodicity of the buckling mode for a panel offers the possibility that the displacements u , v and w may each be expressed as product functions of two factors, the first consisting of appropriately chosen trigonometric functions that represent the respective displacement modes in the axial (x) co-ordinate direction, and the second describing the deformation of the strip in the yz -plane, see Sketch 1.1. The selection of trigonometric expansions in the x -direction, together with the implied possibility of *a priori* integration, reduces the numerical problem to one in only two dimensions instead of three. This is the basic advantage of the strip method compared to full-scale finite element analysis for the type of problems considered.

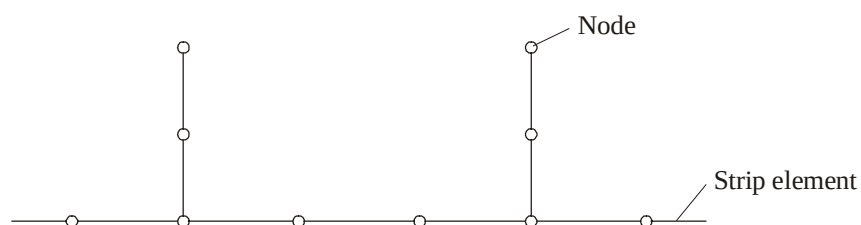
It remains to describe the cross-sectional behaviour of the panel through a choice of functions in the remaining co-ordinate directions. This is accomplished by envisaging the cross-section as an assemblage of plates consisting of narrow strips, for each of which simple deformation behaviour as a function of its transverse in-plane co-ordinate is presumed. Both in-plane and out-of-plane displacements are determined by expanding them as simple polynomial functions of this transverse co-ordinate (compare with $f(y)$ in Equation (b) in Article 9.4 of Derivation 2). The strip element matrices can then be derived and assembled into the system matrix. This is an equivalent approach to that adopted in finite element methods and the generation of the stiffness matrix of the system proceeds in a similar fashion.



Basic co-ordinate system



Section of panel



Finite strip model of section

Sketch 1.1 Basic co-ordinate system and strip model

In the formulations used for developing the present program, the in-plane displacement functions are taken to be linear, while a cubic variation is selected for the out-of-plane deflection. This implies four degrees of freedom per node to define the displacements of the strip. Compatibility requirements between adjacent strips yield the algebraic equations, which in the present application represent the eigenvalue problem of determining the buckling load and its associated mode which in turn defines the normalised buckling displacement pattern of the cross-section of the panel. The eigenvalue problem is formulated on the condition that the assembled finite strip elements undergo equal end-shortening, that is, equal pre-buckling axial compressive strain, so as to ensure compatibility between the strips.

It is assumed that there is no restraint on the strip in the lateral direction. Hence, in the text that follows any reference to boundary conditions applies to the displacements in the buckling mode. The adoption of trigonometric functions for the buckling displacement fields in the axial direction implies that the loaded-edge boundary conditions cannot be specified independently. In the present version of the program, these functions are such that the in-plane axial displacements u are equal to zero at $L/2$, where L denotes the buckling mode half-wavelength, whereas the other displacement components, v and w , are zero at the panel ends, that is, at $x = 0$ and L_p . Note that in this Data Item L is defined as the buckling half-wavelength; the panel length is denoted by L_p . The boundary conditions at the longitudinal, or unloaded, edges are prescribed by the user. These edges may be free, simply supported or clamped and these boundary conditions are realised through imposing the condition that the relevant degrees of freedom are zero.

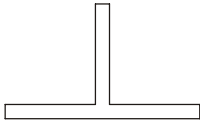
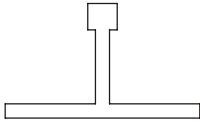
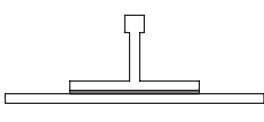
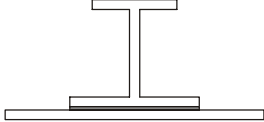
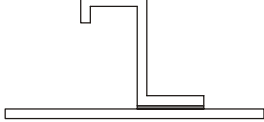
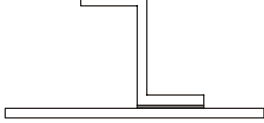
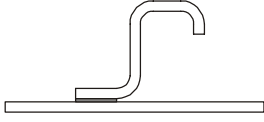
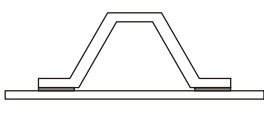
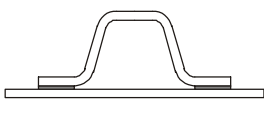
1.3.1 Stringer Section Models

1.3.1.1 Predefined skin-stringer sections

One of the features of the program is a pre-processor routine that simplifies the input of data for a number of commonly used stringer sections by generating the strip model data file for a panel consisting of a skin and a number of stringers. This feature frees the user from the need to generate a usually large strip model data file. The stringer types for which this feature is available are illustrated in Table 1.2. Detailed geometrical and model information for these skin-stringer sections is presented in Appendix A and input details are presented in Tables 2.2 and 2.3 of Section 2.3.2.

In most cases, use of the element meshes that are given for the predefined sections is expected to give reasonably accurate results; if, however, a very local buckling mode arises, it is suggested that the strip density in the region of the buckle is increased. See Section 1.4.2 for more detailed comments on this aspect of the strip models.

TABLE 1.2 Predefined Skin-stringer Sections

Type No.	Section	Relevant section of Appendix A
1		A2.1
2		A2.2
3		A2.3
4		A2.4
5		A2.5
6		A2.6
7		A2.7
8		A2.8
9		A2.9
10		A2.10

1.3.1.2 General panel model

It is emphasised that the user may still generate the strip model data file for a panel of completely arbitrary cross-section without utilisation of the predefined sections, see Table 1.3. The input file definitions required in this instance are provided in Section 2.4.2. When analysing struts it will often be necessary to generate the strip model data file in this way.

In many cases, however, modification of a strip model data file generated for one of the predefined sections may satisfy the user's needs. A detailed example of adapting such a file to specific requirements is presented in Appendix C, where the strip model data file for an I-section stiffened panel is modified to create space in the cross-section of the panel to accommodate hydraulic system parts.

1.3.1.3 Special single-stringer section

An added option for use with the predefined skin-stringer sections is the possibility of representing a wide panel having many stringers by a special single skin-stringer model with predefined boundary conditions.

The special single-stringer model consists of a piece of skin of width equal to the stringer pitch with the web of the stringer located centrally (except for the top-hat sections which are simply located centrally on the skin).

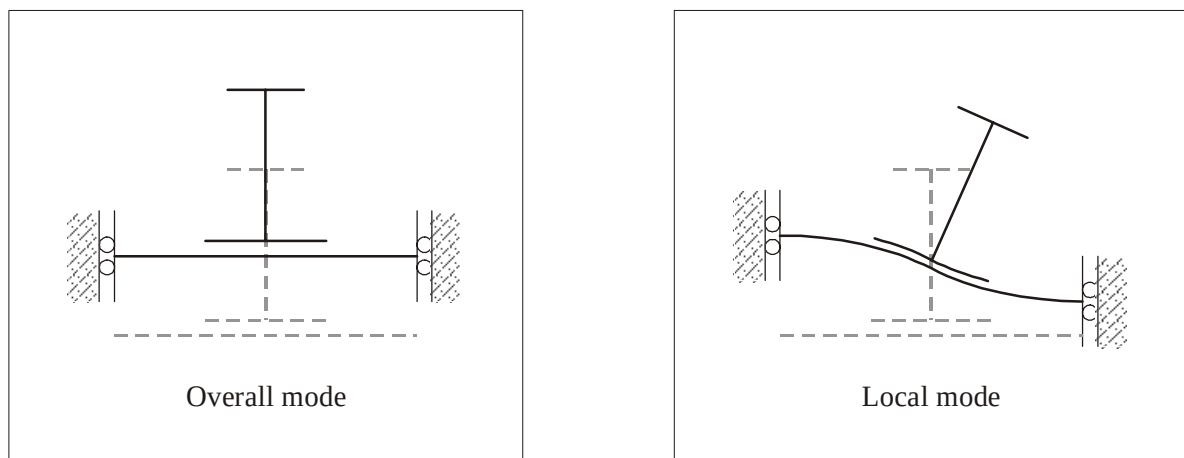
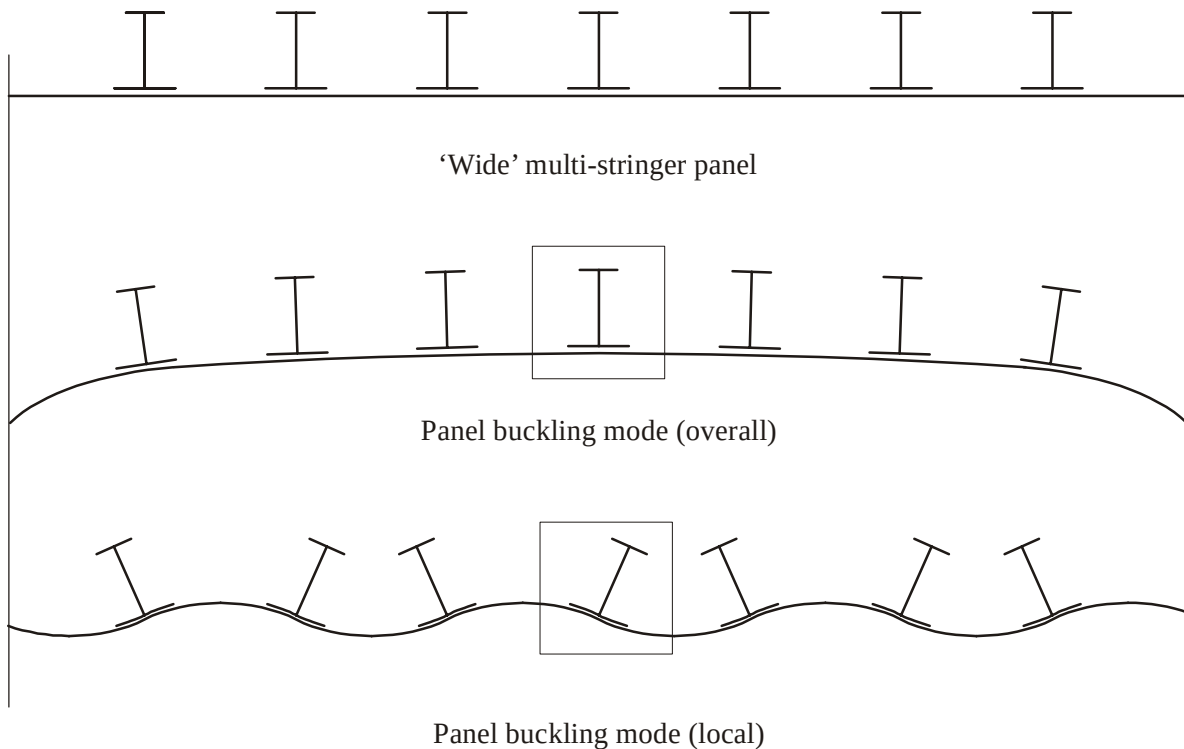
The predefined boundary conditions have been selected in such a way as to give complete freedom in the z-direction whilst not allowing lateral in-plane displacements and rotations in the buckling mode of the outer edges of the skin section, see Sketch 1.2. This option allows quick exploration of the variation of buckling loads for ranges of geometrical and physical parameter values that may occur in a new design.

Though extremely useful for practical purposes, especially in cases where symmetric stringer sections are analysed, care should be exercised in cases where asymmetry of the section, or other irregularities, may give rise to buckling deformations for the full panel cross-section that do not fit the predefined boundary conditions for this special single-stringer case. This is shown more explicitly in the example problem in Section 1.4.4.2. It is recommended that the results of special single-stringer analyses of panels with asymmetric stiffeners always be verified by a multi-stringer analysis.

A model for a single stringer with user-specified boundary conditions can obviously be analysed as well. The typical differences in the definitions for the two single-stringer models are discussed in Section A1.1 of Appendix A.

1.3.1.4 Rolled-sheet and machined sections

In the definitions of the various predefined skin-stringer section types shown in Table 1.2 and listed in Table 2.3, a distinction is made between 'rolled-sheet' sections and other types, for example, 'machined' sections. The 'rolled-sheet' type sections have a different finite strip model for the stringer corners which takes the value of the bend radius into account. This aspect is elaborated in Section A1.2 of Appendix A where information on how to model a corner when not using the predefined skin-stringer sections is provided. The effect of different bend radii on the critical stress is illustrated by the example in Section 1.4.4.1. For the non-rolled-sheet cases, the effect of a corner radius is ignored.



Sketch 1.2 Special single-stringer, wide panel model

1.3.2 Skin-stringer Joint Models

The use of the finite strip method allows the scope of the computational models to be extended beyond that of the Data Items listed in Table 1.1 with little additional effort. One way in which this has been done is through the development of adhesive layer and rivet line elements. These elements function as joining elements between the stringer flange and the skin and allow the creation of a more realistic panel model.

For the adhesive layer, simplified strain-displacement relationships are adopted on the basis that the stress distribution is assumed to be approximately constant through the thickness of the adhesive layer. The strain energy can then be formulated in terms of the degrees of freedom of the two plate strips that constitute the joint. Also, in order to accommodate a wide range of adhesive tension/compression and shear stress-strain

behaviour, the Young's modulus, E_{adh} , and shear modulus, G_{adh} , are uncoupled in the stress-strain relationship for the adhesive layer.

A more detailed account of the specific features of the adhesive layer element, and of the mechanical joint model to be described next, may be found in Appendix B of this Data Item.

To make some allowance for a rivet line connection between the skin and stringer, a rivet line element is derived for the model depicted in Sketch B2.2 of Appendix B. The two-dimensional model of the panel cross-section allows the connection of adjacent skin and stringer nodes via a simple beam element. This element is presumed to function strictly as a tension-compression-bar/bending-in-two-directions beam element. Since the edge displacements of the strip are represented by the nodal point degrees of freedom, together with the stipulated trigonometric functions for displacement variation in the axial direction, the strains in a rivet element at location x along the assumed line of connection can be expressed completely as a function of the nodal degrees of freedom of the endpoints of the beam. Then, a summation of all energy contributions of the discrete rivet elements provides the basis for deriving the rivet line stiffness matrix as a function of the cross-sectional area and second moment of area of the rivet (which are expressed in terms of the diameter, D_{riv}), the rivet pitch, p_{riv} (which defines any typical rivet location along the joint), and the Young's modulus of the rivet material, E_{riv} . Again, the complete behaviour may be expressed in terms of the degrees of freedom of the two plate strips that constitute the joint.

In practical cases, the rivet line is usually located about halfway across the skin flange of the stringer. Unless preventive measures are taken (see below), the finite strip approximation for the actual panel model may show a displacement pattern in the buckling mode in which the node point at the stringer heel or the node point at the flange tip displaces beyond the adjacent skin nodal point. In the real structure that is not possible, and the buckled shape is less extreme because the skin and flange constrain each other.

This freedom of the model stringer to rotate and for the edges to penetrate the skin will result in a conservative estimate of the buckling load, but will be physically unrealistic. Some alleviation of the phenomenon can be introduced by placing additional elements between the skin and the stringer, and over the otherwise unsupported nodes, to represent the contact stiffness or the stiffness of an adhesive or sealant layer when this is present.

Note that neither of these connection elements contributes to the load-carrying cross-sectional area of the panel.

It is emphasised again here that no attempt has been made to equip the present finite strip model with the means to analyse inter-rivet buckling phenomena.

1.3.3 Analysis Options

For all the models described in the previous sections, the following three options for obtaining buckling loads are provided.

- 1 Calculation of the buckling load for the buckle half-wavelength equal to the specified panel length, that is, $L = L_p$. (Note that lower buckling loads associated with buckling half-wavelengths less than L_p will be ignored.)
- 2 Calculation of the complete P - L curve with L ranging from a 'small' value up to the actual panel length.
- 3 A more precise search, starting from a user-specified value, L_0 , for a minimum in the load – length characteristic of the panel.

The three options are explained fully in Section 2.4.1.

The second option is available because the buckling phenomenon may be rather complicated on account of the various deformation patterns that may occur at different load levels, such as overall panel buckling, torsional, flexural, or a combination of the latter two, and/or local inter-stringer buckling, *et cetera*.

It is envisaged that, in cases where the first option does not provide sufficiently detailed results, the second option will be used to identify areas of the P – L curve that are of interest for a particular panel; the last option will then be used to perform more detailed analyses of these areas.

A summary of the various models and analysis options is given in Table 1.3.

TABLE 1.3 Finite Strip Models for Panels and Struts

Parameter/Feature	Model		Remarks
	Predefined Skin-stringer Section	Panel of Arbitrary Geometry	
Number of Stringers	- Special single-stringer as ‘wide panel’ model, or N_{st} stringers with $N_{st} = 1$ to 10–18 (depending on stringer type, see Section 2.5.2)	User-defined	The maximum value of N_{st} depends on the program parameter constants (see Section 2.5.2)
Boundary Conditions	Program-defined for the buckling displacements: - special single-stringer with ‘wide panel’ boundary conditions, or - multi-stringer models, with panel edges – free, – simply-supported, or – clamped	User-defined	The user may create other types of support conditions by adding or removing nodal degrees of freedom
Skin-stringer Joint Models	Adhesive layer and/or rivet line/sealant layer		The user can modify the joint model generated for the predefined sections
Analysis Options	- Single buckling load calculation - Generation of complete P–L curve - Search for particular minimum in P–L curve		

1.4 RESULTS

1.4.1 Introduction

This section presents a number of example cases. The purpose of these examples is to demonstrate the wide scope of the finite strip method and to draw attention to various aspects of the finite strip models. In order to put the finite strip method into perspective with respect to the methods used in other Data Items in the Structures Series, the results for a number of cases selected from those Data Items are presented in Section 1.4.2 and summarised in Table 1.4. In most of the example cases four strips were used to model each flat of the various sections.

Section 1.4.3 treats examples that illustrate the special features that arise as a consequence of using the finite strip method.

Section 1.4.4 presents two more examples. The first investigates the effect on the results of varying the stringer corner bend radius. The second example compares the results obtained when the special single-stringer model is used with those obtained when multi-stringer models are used for the case of a Z-section stiffened panel.

1.4.2 Comparison Cases from the Data Items of Table 1.1

This section presents values for the buckling stresses of panels and struts. All the cases were taken from Data Items mentioned in Table 1.1. Assumptions that were basic to the derivation of the methods employed in those Data Items have been incorporated, for the purposes of comparison, in the strip models for these cases. These assumptions entail, among other things, the omission of a detailed geometry for the corner radii of stringer sections and ignoring the offset of the skin flange of the stringer relative to the skin. It was also assumed that there was no special mechanical joint connection between skin and stringer elements.

The first six entries quoted in Table 1.4 treat the problem of computing the local buckling stress of a stiffened panel section. In the present Data Item the local buckling stress is defined as the minimum buckling stress relating to a half-wavelength that is of the order of a 'local' dimension of the cross-section of the panel, for instance, a web height or a stringer pitch.

The last two examples in Table 1.4 represent the computation of the 'buckling characteristic' of struts, that is the P - L curve, which gives the relationship between the buckling load of the strut and its half-wavelength.

In order to put the computed values quoted in Table 1.4 into perspective, it should be noted that ESDUpac A9816 computes the local buckling stress of a simply-supported plate to within 0.05 per cent of the theoretical value using a model consisting of only four strips across the plate. In the light of this, the good correspondence between the two sets of results quoted in Table 1.4 is to be expected. In addition, the simplicity of most of the panel stringer models ensures that the skin deformations are virtually identical to those that occur in the plate problem. This also implies periodicity of the buckling mode in the panel y -direction, which is a presumption in the derivations of the methods used in the Data Items. Furthermore, since the mesh divisions in most models were comparable with those of the plate problem mentioned above, equivalence of the results of both methods might be expected, though the solutions of the algebraic equilibrium equations are arrived at along very different avenues.

It may be concluded that structural models analysed with program A9816 that resemble those treated in the Data Item No. Struct 02.01.3X series, and for which the actual buckling mode is similar to that exhibited in the figures of Data Item No. Struct 02.01.28, should yield buckling stresses very close to the Data Item values. If, however, there is the possibility of significant shear deformation, or of circumstances arising in which model features such as skin-flange offsets and connection play a significant role, more distinctive differences in critical stress values may be expected.

TABLE 1.4 Test/Comparison Cases

Data Item No.	Model	L/h^*	Critical Stress		Units
			Data Item Value	A9816 Result	
Struct 02.01.34	Blade-stiffened panel		–30 000	–29 830	lbf/in ²
Struct 02.01.35	Z-stiffened panel		–40 900	–41 547	lbf/in ²
Struct 02.01.36	Top-hat-stiffened panel		–47 700	–48 029	lbf/in ²
Struct 02.01.37	Corrugated panel		–20 400	–20 144	lbf/in ²
70003	Blade-stiffened panel		–48 953	–48 320	lbf/in ²
71014	Z-stiffened panel		–382	–381	MN/m ²
76023	Lipped channel-section	2	–285	–292	MN/m ²
		4	–223	–223	MN/m ²
		10	–206	–205	MN/m ²
77030	Lipped Z-section	0.5	–62	–62	MN/m ²
		1	–48	–49	MN/m ²
		10	–115	–117	MN/m ²
		40	–23	–23	MN/m ²

* L = buckle half-wavelength and h = stringer or section height.

1.4.3 Illustrative Examples

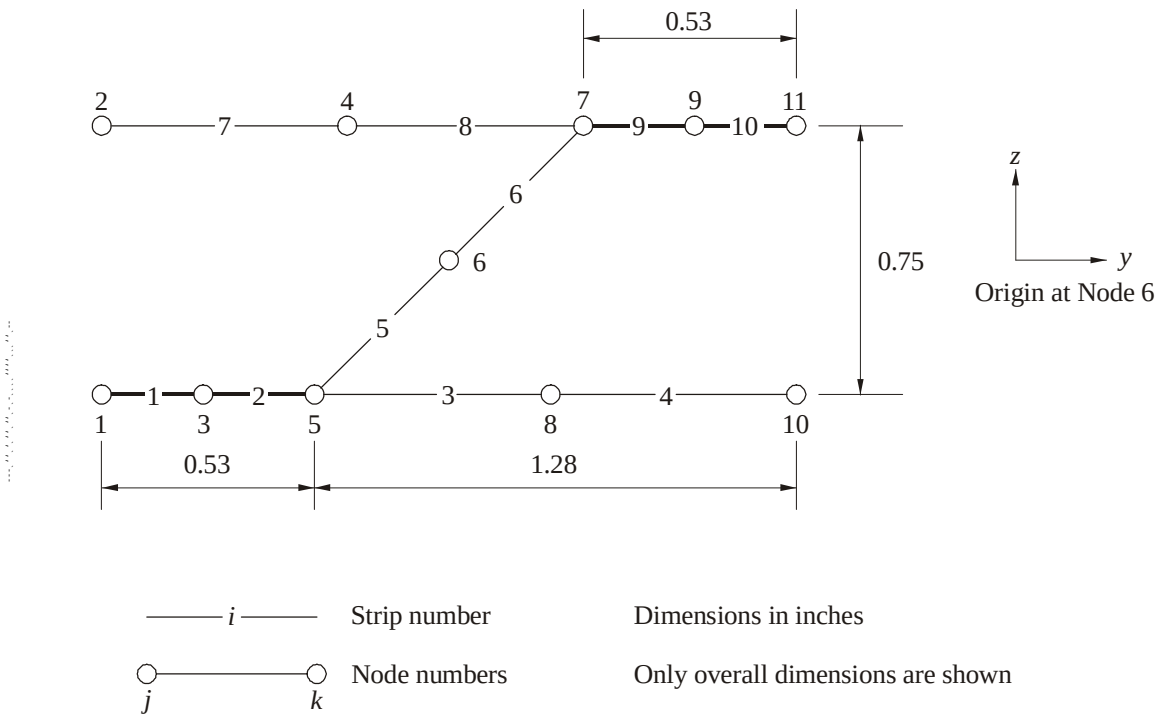
This section brings together some results that illustrate

- (a) features that may be exploited to obtain buckling stresses for models that depart in geometry from the usual stiffened panel layouts,
- (b) uncommon modes of deformation that may implicitly arise with the finite strip formulation, such as shear deformation,
- (c) the application of the adhesive layer element to sandwich-type problems, and
- (d) the computation of the P – L curve for a strut.

1.4.3.1 A corrugated sandwich panel

In this first example a corrugated panel model is analysed to demonstrate the versatility of the modelling process within the capabilities of the program. The case selected is the example problem of Data Item No. Struct 02.01.37. Firstly, a simplified model that is equivalent to the example in the Item is analysed. Secondly, the results are presented for a model that incorporates an adhesive layer between the skin and the corrugation.

The finite strip model depicted in Sketch 1.3 is used to represent the actual corrugated panel and the value of the computed critical stress is then compared with the value given in the Data Item. The strip model data file for this example can be found in Section 2.4.2.



Sketch 1.3 Corrugated sandwich panel model

Node 6 is taken as the origin of the y- and z-axes. The simplification mentioned above amounts to the fact that no attempt has been made to model the exact eccentricity of the parts of the corrugation that adjoin the faces. Elements 1, 2, 9 and 10 have the total thickness of the two plates that constitute the face and corrugation. All corner points have boundary conditions imposed to represent symmetry of the buckling mode, presumed to occur at the mid-points of the flats (that is, the v -displacements and the rotation, ϕ , at nodes 1, 2, 10 and 11 in Sketch 1.3 are set equal to zero in the buckling mode). The mid-points of the flats are indicated by B, F, G and H in the sketch in Section 3 of Data Item No. Struct 02.01.37. The results given by the program are reproduced below.

Results for Local Buckling				
Step	L in	Eps-cr	Sigma-cr lbf/in ²	Buckling Load lbf
1	.1000E+01	-.003011	-.301141E+05	-.753165E+04
2	.1200E+01	-.002466	-.246648E+05	-.616876E+04
3	.1400E+01	-.002187	-.218674E+05	-.546912E+04
4	.1600E+01	-.002055	-.205474E+05	-.513899E+04
5	.1800E+01	-.002014	-.201438E+05	-.503804E+04
6	.2000E+01	-.002036	-.203603E+05	-.509219E+04

The value given by Data Item No. Struct 02.01.37 is

$$f_x (= \sigma_x) = -20\,400 \text{ lbf/in}^2,$$

which should be compared with the value of σ (sigma) of $-20\,144 \text{ lbf/in}^2$ in the penultimate line of the program results above, for which $L = 1.80 \text{ in}$, which is equal to the Data Item value.

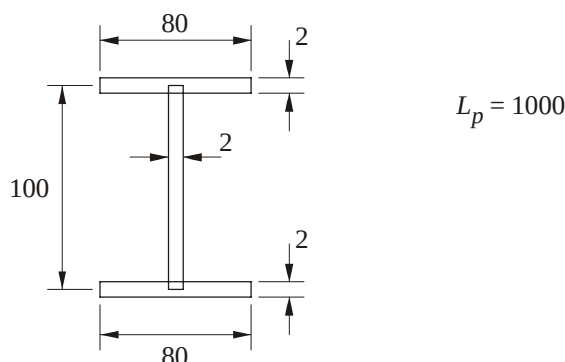
As a matter of interest, program A9816 was also run for the corrugated sandwich plate with a model that has adhesive layers between the corrugations and face plates. Table 1.5 lists critical stresses for several values of the adhesive layer shear modulus, G_{adh} . The modulus of elasticity, E_{adh} , is taken as $2.6G_{adh}$. A value of G_{adh} of $58\,000 \text{ lbf/in}^2$ is an average thick-adherend test value for a typical structural adhesive. The critical stress is seen to have a high rate of variation at low values of G_{adh} , tending asymptotically to the 'infinitely stiff adhesive' value of approximately $20\,100 \text{ lbf/in}^2$ as G_{adh} increases, as indicated in the results above.

TABLE 1.5 Effect of Adhesive Shear Modulus

G_{adh} lbf/in ²	L in	Critical Stress lbf/in ²
14 500	2.00	-18 389
29 000	1.80	-18 788
58 000	1.80	-19 149
116 000	1.80	-19 452
174 000	1.80	-19 598
232 000	1.80	-19 686
290 000	1.80	-19 746

1.4.3.2 Buckling of an I-beam. The influence of shear deformation

As noted in Section 1.1.1, the formulae on which program A9816 is based are derived from classical thin plate theory. However, the models may show effects that are usually only brought out by higher order theories, such as the effect of shear deformation on the buckling loads of struts or panels. A stiffened panel consists, by definition, of skin and stringer parts that intersect at some angle of finite magnitude. This implies that some flats, such as webs or uprights, have their mid-planes in directions other than that of the skin, which is usually the flat having the least second moment of area. When the panel is subjected to, say, a distributed load acting perpendicular to that mid-plane, then the resulting displacement will consist of a part due to pure bending and a part due to 'in-plane' shear deformation of those plate elements that are inclined to the skin mid-plane. This additional deformation reduces the buckling load for the panel when it is loaded in compression. Note that on account of adopting classical thin plate theory, which does not describe 'through-the-thickness' shear strains, no contribution to this 'shear deformation mode' arises from flats running parallel to the plane of minimum bending stiffness. As an example, a simple I-section beam, as shown in Sketch 1.4, is analysed for buckling.



Dimensions in millimetres

Sketch 1.4 I-section as modelled

Derivation 2 treats such cases and computes the critical load with a correction factor $n = A/A_w$ in the following formula (with appropriate changes to account for the different sign convention (P_E being negative in this Data Item)):

$$P_{cr} = \frac{P_E}{1 - n \frac{P_E}{AG}}, \quad (1.1)$$

where, for the model,

$$A = \text{section area} = 520 \text{ mm}^2,$$

$$A_w = \text{'shear' area, in this case the area of the web of the I-section, that is, } 200 \text{ mm}^2,$$

$$I_{yy} = 966\,773 \text{ mm}^4, \text{ and}$$

$$P_E = \text{Euler load, } -\pi^2 EI/L^2 = -954\,167 \text{ N.}$$

Note that the second moment of area used in the calculations is that of the model depicted in Sketch 1.4 which differs slightly from that of the real section, see the footnote in Section 2.4.1.4.

The dimensions of the model of the section are as shown in Sketch 1.4 and the material properties are as follows.

$$\text{Young's modulus, } E = 100\,000 \text{ N/mm}^2$$

$$\text{Poisson's ratio, } \nu = 0.3, \text{ and}$$

$$\text{Shear modulus, } G = 38\,462 \text{ N/mm}^2.$$

With $A_w = 200 \text{ mm}^2$, $n = 2.6$, and the reduction factor is given by

$$\begin{aligned} \frac{P_{cr}}{P_E} &= \frac{1}{1 - n \frac{P_E}{AG}} \\ &= \frac{1}{1 - 2.6 \frac{-954\,167}{520 \times 38\,462}} = 0.89. \end{aligned}$$

Using this value, Equation (1.1) gives the reduced critical load as:

$$P_{cr} = -954\,167 \times 0.89 = -848\,872 \text{ N},$$

which is approximately 4 per cent greater in magnitude than the value of $P_{cr} = -818\,039 \text{ N}$ given by the program. The difference is attributed partly to the approximate nature of Equation (1.1) and partly to distortion of the cross-section, that is, small flange and web deformations which give rise to an additional reduction in the buckling load that is not accounted for by Equation (1.1).

1.4.3.3 Buckling of a sandwich column - using the adhesive layer to represent the sandwich core

The adhesive layer model provided with the program of ESDUpac A9816 allows the buckling analysis of sandwich-type columns and plates. Although that model is strictly only applicable to isotropic adhesive materials, which do not include, for example, honeycomb core types, a sandwich column with isotropic core properties affords an appropriate test example for the adhesive layer element and the results show its wider applicability, beyond its original role as a skin-stringer joint element. It should be noted that the application of the layer element to sandwich-type structures should be restricted to cases where the stresses may be considered constant over the core thickness, see also Section B2 of Appendix B. This is not usually a very restrictive requirement because non-constant stresses occur mostly only at bond or core layer edges. Moreover, the distances over which such regions extend into the joint amount to no more than a few layer thicknesses.

The formula for the buckling load of a column that accounts for shear deformation is now taken to read (with appropriate changes to account for the different sign convention)

$$P_{cr} = P_E \frac{1}{1 - \frac{P_E}{W_{snw} S}}, \quad (1.2)$$

where P_E = Euler buckling load = $-\pi^2 E_{face} I_{snw} / L^2$,

with $I_{snw} = W_{snw} \left[\frac{t_{face}^3}{6} + t_{face} \frac{(h_c + t_{face})^2}{2} \right]$

and S = shear stiffness = $G_c (h_c + t_{face})^2 / h_c$.

The expressions for S and I_{snw} are taken from Reference 5.

Table 1.6 summarises the results obtained with ESDUpac A9816 for a column with the following dimensions and properties.

Column length, L	=	240 mm
Core thickness, h_c	=	13 mm
Face thickness, t_{face}	=	0.8 mm
Column width, W_{snw}	=	30 mm
Young's modulus, E	=	70 000 N/mm ²
Euler buckling load per unit width, P_E / W_{snw}	=	-914.7 N/mm.

TABLE 1.6 Results for Buckling of a Sandwich Column

$G_{adh}(= G_c)$ N/mm ²	S N/mm	$P_E/W_{snw}S$	P_{cr} for Column N	
			from Equation (1.2)	from A9816
50	732	-1.2488	-12 203	-12 208
125	1831	-0.4995	-18 300	-18 251
525	7691	-0.1189	-24 524	-24 374
1500	21 974	-0.041 63	-26 344	-26 157

To verify convergence of the finite strip model solutions, the results were obtained for two models with three and six strip elements across the column width respectively. There were no significant differences in the two sets of results.

1.4.3.4 Buckling of a lipped channel-section strut. On the minima in the P - L curve

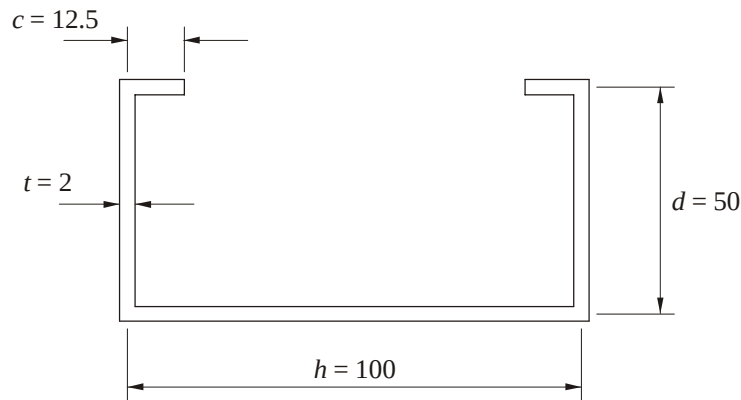
As mentioned earlier, ESDUpac A9816 is equipped with an option to compute the P - L curve for a panel or strut. The P - L curve represents the variation of the buckling load, P , of a panel or strut with the half-wavelength, L , of the associated buckling mode. For arbitrary strut cross-sections this relationship may show significant departures from the 'Euler' relationship, that is, $P_E = -\pi^2 EI/L^2$. The option to compute the curve is particularly useful in panel design cases where many different buckling modes may be expected. This aspect is expounded more fully in Data Item No. 78021 and References 6 and 8.

The option is illustrated by recalculating the curve of Figure 8 in Data Item No. 76023* for an inward-lipped channel-section strut with the parameter values, in the notation of that Data Item, $t/h = 0.02$, $d/h = 0.5$ and $c/d = 0.25$. The dimensions of the strut are given in Sketch 1.5 and the material properties selected for this case are:

$$E = 100\,000 \text{ N/mm}^2$$

$$\nu = 0.3.$$

* Data Item No. 76023 "Buckling of struts. Lipped and unlipped channel sections".



Dimensions in millimetres

Sketch 1.5 Channel-section strut

Table 1.7 presents typical values for comparison. A selection of these values is reproduced in Table 1.4. L is the half-wavelength of the local buckling mode and σ_{cr} is the critical stress.

TABLE 1.7 Results for Buckling of a Channel-section Strut

L/h	Critical stress, σ_{cr} N/mm ²	
	A9816	Data Item No. 76023
2	-291.6	-285
4	-223.0	-223
6	-295.8	-294
8	-299.9	-295
10	-204.7	-206
20	-61.6	-62

Figure 1 depicts the curve for the critical stress, σ_{cr} , versus the half-wavelength, L . The first minimum near $L = 100$ mm is a pure local mode, where the corners of the section do not move. The second minimum at $L = 350$ mm, which is three and a half times the largest 'local' dimension, is associated with a buckling mode in which the lipped flanges both move outwards. The buckling mode of the strut for lengths larger than 700 mm, beyond the second peak of the curve, represents almost all pure torsional deformations. A pure flexural mode, that is the so-called 'Euler' mode, arises at half-wavelengths larger than approximately 3850 mm.

The value of L_{inc} that is chosen (see Section 2.4.1) when exploring the various areas of the P - L curve will obviously determine the accuracy with which a minimum in the curve may be obtained.

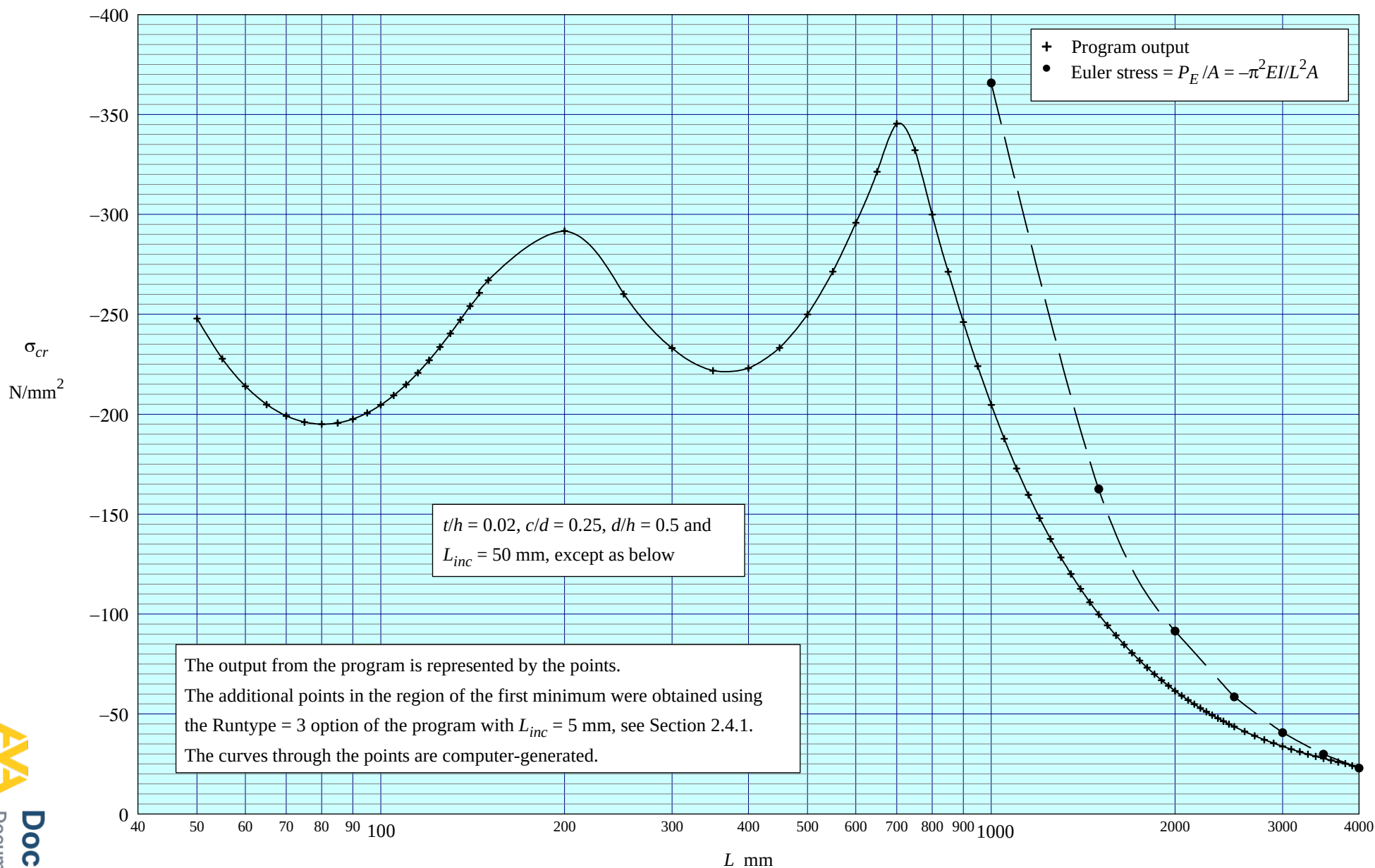


FIGURE 1 BUCKLING STRESS VERSUS HALF-WAVELENGTH FOR INWARD-LIPPED CHANNEL-SECTION STRUT.
 RESULTS FROM ESDUpac A9816. INPUT DATA AS FOR FIGURE 8 OF DATA ITEM NO. 76023

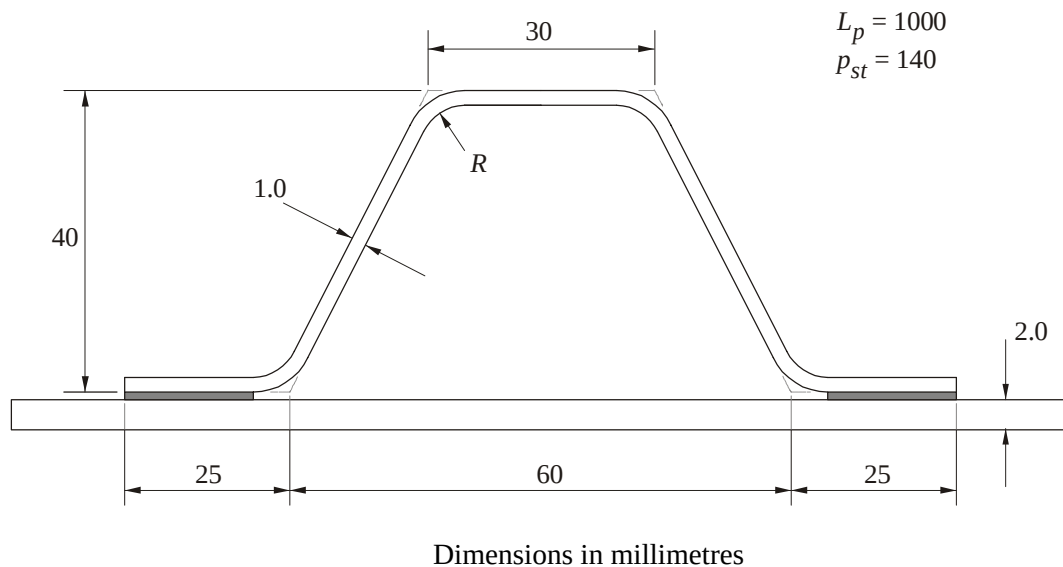
1.4.4 Aspects of the Finite Strip Models

As noted in the introduction, the development of program A9816 serves to unify the methods presented in the Data Items mentioned in Table 1.1. With the adoption of the finite strip approach to the formulation of the buckling problem for panels and struts, that purpose is not only reached, but the options available to the user, and hence the range of cases that may be analysed, are broadened considerably. To illustrate this, two further examples, the first examining the effect of varying the stringer corner bend radius and the second looking at the special single-stringer model, are given in the following sub-sections.

1.4.4.1 The stringer corner bend radius. Local and overall buckling of a top-hat-section stiffened panel

In this section the results of a parameter variation study are presented, the parameter varied being the value of the corner radius. The choice of the number of strip elements used to represent the corner is considered in Section A1.2 of Appendix A.

In Table 1.8 critical stress values are presented for local and overall (Euler) buckling of the top-hat-section stiffened panel shown in Sketch 1.6.



Sketch 1.6 Top-hat-section stiffened panel

The critical stresses for various values of corner bend radius are listed in Table 1.8, together with the half-wavelengths for local buckling and the 'type' of buckling, that is, whether the local mode arises predominantly in the skin or in the stringer, for example, the web. It should be noted that that part of the effect of the stringer corner bend radius on the local buckling stress is due to the fact that that stress is a function of the unsupported skin width under the stringer which, in turn, is here a function of the corner bend radius.

Note also that the variation in critical stress for the overall mode in this case is approximately 8 per cent. This variation is due to small changes in the location of the centroid of the section, the bending stiffness, and the smaller or larger contribution of section deformation and/or shear deformation as the bend radius varies. It can be seen that it may be beneficial to explore the sensitivities of the buckling stresses of the panel to changes in the various geometric parameters.

Panels with other kinds of rolled-sheet-type stiffeners will also show similar effects of corner bend radius on buckling stresses.

TABLE 1.8 Local and Overall Buckling Stresses for Top-hat-section Stiffened Panel

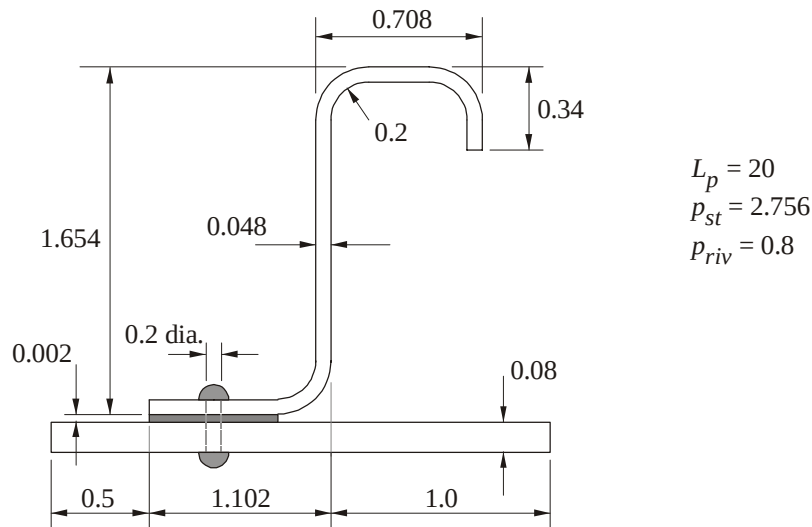
Corner Inside Bend Radius, R mm	Local Buckling			Overall Buckling
	Critical Stress, σ_l N/mm ²	Half-wavelength, L mm	Mode	Critical Stress, σ_E N/mm ²
2.0	–289	30	Web	–166
4.0	–297	40	Web	–164
6.0	–311	40	Web	–162
10.0	–300	60	Skin	–158
14.0	–254	70	Skin	–154

1.4.4.2 The special single-stringer model

As indicated in Section 1.3.1.3, the special single-stringer section is provided so that trends in behaviour of buckling loads as a function of varying parameters can be explored quickly. The special single-stringer models are created by equipping the skin-stringer sections depicted in the sketches in Sections A2.1 to A2.10 of Appendix A with boundary conditions that are intended to simulate the behaviour of a wide panel. The much smaller ensuing data model allows the exploration of the influence of variations in many relevant parameters on the critical stresses of the panel in a very short time.

However, it should be noted that in cases of models with asymmetric stringers, such as Z-type stringers, as shown in Sections A2.6 to A2.8 of Appendix A, the results may sometimes depart significantly from the values computed for the actual multi-stringer model. In general, the asymmetry with respect to the z-axis implies that moments may arise that cause some stringer rotation, the amount of which is determined by the restraining ‘effort’ of the surrounding structural elements. In the actual wide panel, this restraining effect is offered by the adjacent skin-stringer sections. In the special single-stringer model the presumed boundary conditions, together with the specific choice made for the stringer position along the skin, determine the relative freedom of the section to buckle in a different mode compared to its behaviour as a wide panel, which implies different critical stresses.

In Table 1.9, the local and overall buckling stresses for a typical riveted Z-stringer, type = 8, ‘rolled-sheet’ section stiffened panel, as shown in Sketch 1.7, are presented. The Young’s modulus of the skin and stringer plate material is 10.0×10^6 lbf/in² and that of the rivet material is 9.0×10^6 lbf/in². The sealant properties are taken as $G_{adh} = 145$ lbf/in² and $E_{adh} = 377$ lbf/in².



Dimensions in inches

Sketch 1.7 Rolled-sheet, Z-section stringer

In Table 1.9 the results obtained using the special single-stringer model are compared with those for multi-stringer models with longitudinal edges simply supported.

It is seen that, in this case, the special single-stringer analysis underestimates the local buckling stress of the multi-stringer panel by approximately 20 per cent. The reason for this is that the boundary conditions of the special single-stringer model are not closely representative of the wide panel with many stringers for this design. As stated earlier, it is recommended that, for panels with asymmetric stiffeners, the results of a special single-stringer analysis always be verified by a multi-stringer analysis.

TABLE 1.9 Local and Overall Buckling Stresses

Source	Number of Stringers, N_{st}	Local Buckling Stress, σ_l lbf/in ²	Half-wavelength, L in	Overall Buckling Stress σ_E lbf/in ² ($L = 20$ in)
A9816	1 Special single-stringer	-22 987	2.70	-36 248
A9816	5	-29 060	2.65	-37 669
A9816	10	-28 769	2.65	-37 758

A similar exercise to check the special single-stringer option results against those for the multi-stringer case for panels built up from symmetric skin-stringer sections, such as top-hat sections, would not show such significant variation in the critical stresses.

1.5 DERIVATION AND REFERENCES

1.5.1 Derivation

This section lists selected sources that have assisted in the preparation of this Data Item.

1. TIMOSHENKO, S.P.
WOINOWSKI-KRIEGER, S. *Theory of plates and shells.* Second Edition, McGraw-Hill, 1959.
2. TIMOSHENKO, S.P.
GERE, J.M. *Theory of elastic stability.* Second Edition, McGraw-Hill, 1961.
3. VAN DER SLOOT, J.H.
QUILTER, A.C. Elastic buckling of stiffened panels. ESDU Memorandum No. 108. To be issued.

1.5.2 References

The references given are recommended sources of information supplementary to that in this Data Item and, in certain cases, have served as sources of comparison for results.

4. ESDU Information on the use of Item No. 73007. Data Item No. Struct 02.01.28, ESDU International plc, London, June 1954.
5. PLANTEMA, F.J. *Sandwich construction.* John Wiley and Sons Inc., New York, 1966.
6. WITTRICK, W.H. A unified approach to the initial buckling of stiffened panels in compression. *Aeron. Quart.*, pp. 265–283, August 1968.
7. ESDU Stiffness of loaded flat strips under sinusoidally distributed bending couples at their edges (for use in local buckling calculations). Data Item No. 73007, ESDU International plc, London, July 1973.
8. PLANK, R.J.
WILLIAMS, F.W. Critical buckling of some stiffened panels in compression, shear and bending. *Aeron. Quart.*, pp. 165–179, August 1974.
9. CHEUNG, Y.K. *Finite strip method in structural analysis.* Pergamon Press, Oxford, UK, 1976.

2. ESDUpac A9816

2.1 INTRODUCTION TO THE PROGRAM

ESDUpac A9816 contains a Fortran 77 computer program for calculating the initial elastic buckling loads of stiffened panels and struts under compressive loading. The basis of the program is explained in Part 1 of the Item. Throughout this section of the Item the text refers mostly to panels but this should be taken to include struts where appropriate.

This Data Item relates to Version 1.2 of ESDUpac A9816v12 and Fortran program A9816v12. Both the ESDUpac number and the program number contain the version number encoded in the three characters v12. Throughout this Data Item the ESDUpac and the program are referred to by the number A9816 and Version 1.2 is taken to apply. The Contents page of the Series Software Volume states the current version number. The program is available in two formats: (a) a Fortran source code and corresponding executable file, and (b) an executable program for use within ESDUview. If the latter version of the program is used, that is, the program is new within ESDUview, Section 2.3 need not be consulted. Both versions of the program, along with virtually all the files that have been used to obtain the results from the program that are quoted in this Data Item, can be found in ESDUscope (the electronic version of the Series) and on disk in the Series Software Volume.

An alternative version of the program, ESDUpac B9816v12, accompanies this Data Item and can also be found in ESDUscope and on disk in the Series Software Volume, though it is not available within ESDUview. Program B9816v12 uses identical input files to program A9816v12 and performs identical calculations, the only difference being the manner in which the file names associated with a particular run are handled; this is described in Section 2.3.1.2.2. There is only that small difference between the two versions and all references to program A9816v12, with the exceptions of those in Sections 2.3.1.2.1 and 2.5.4, may be taken to apply equally to program B9816v12.

Every reasonable effort has been made to ensure that the program of ESDUpac A9816 performs the intended calculations satisfactorily. If, however, any problems are encountered in its use, please inform ESDU and every effort will be made to overcome the difficulties. In common with other software producers, ESDU makes no representation as to the suitability or fitness of the program for any particular purpose and no liability for any loss occasioned by any persons as a direct or indirect result of use of the program, whether arising from negligence or otherwise, can be accepted. In no event shall ESDU or any individuals associated with the development of the program be liable for any damages, including loss of profit or consequential loss, arising out of or in connection with the program.

2.2 PROGRAM OPTIONS AND LIMITATIONS

Version 1.2 of ESDUpac A9816 is capable of the following.

- (i) Elastic analysis for initial buckling of stiffened panels and struts, with the following options:

buckling load calculation for buckling half-wavelength equal to panel length,
analysis to obtain complete P - L curve of the panel, or
computation of local buckling minima.

In all cases either minimal or extended output may be selected. For details of the output and of the analysis options themselves, see Section 2.4.1.

- (ii) The user is not restricted to data models created with the predefined skin-stringer sections listed in Table 2.3. Any other section may be analysed for buckling, though the strip model data file must then be prepared by the user. See **B** and **C** of Section 2.3.1.3 and Entry 4 of Table 2.2.

This page Amendment B

- (iii) Analysis of panels with skin-stringer joint elements modelling adhesive or riveted connections.
- (iv) Analysis of models fabricated from up to ten different isotropic (metallic) materials, that is, up to ten different E , G and ν values per model. Section 2.5 gives directions for increasing the number of different materials that can be handled by the program.
- (v) Skin and stringer elements, such as flanges, webs, *et cetera*, may consist of different materials. However, for rolled-sheet sections the thickness is presumed to be equal for all flats.
- (vi) The program is limited to the analysis of models that comprise up to approximately 10 to 18 stringers. This restriction depends on the value of the relevant constants in the program. Larger models require adaptation of these values. The constants and their values are described in Section 2.5.

2.3 RUNNING THE FORTRAN PROGRAM

2.3.1 Overview

In order to run program A9816 it is necessary to prepare the input files and decide on a running strategy; these aspects are discussed in the following sections. Definitions of the four files that will be encountered in a program run and their naming conventions are given in Sections 2.3.1.1 and 2.3.1.2.

There are three basic approaches to running the program and these are described in Section 2.3.1.3.

Detailed descriptions of the two basic input files are given in Sections 2.3.2 and 2.3.3. The output from the program is described in Section 2.4.

2.3.1.1 File definitions

The four files that are utilised when running program A9816 are defined in Table 2.1. Note that the extensions are optional; those quoted are used throughout this Data Item.

TABLE 2.1 File Definitions

<i>File type</i>	<i>Extension</i>	<i>For details, refer to</i>	<i>Description</i>
Panel Geometry Definition file	PGD	Section 2.3.2, Tables 2.2 and 2.3	A short, user-prepared file containing all the basic geometrical information required to describe a panel or strut.
Material Data file	MAT	Section 2.3.3	A short, user-prepared file containing all the basic materials data.
Strip Model Data file	STO	Section 2.4.2 and Appendix C	A file, usually large, that stores all the detailed model information. This file may be generated by the program from the information contained in the PGD file or it may be prepared by the user, depending on the type of run, see Section 2.3.1.3.
Results file	PRT	Section 2.4.1	An output file containing the results of a particular run.

2.3.1.2 Naming of input and output files

2.3.1.2.1 ESDUpac A9816

ESDUpac A9816 is run using directed input/output, as described in Section 4.4 of the 'Introduction to ESDUpacs' in the Series Software Volume. In running the program in this way, the paths and names of the Panel Geometry Definition (PGD) file and the Results (PRT) file are specified at the DOS prompt by the user.

As mentioned above, two further files are utilised when running the program; those files are the Strip Model Data (STO) file and the Material Data (MAT) file. Lines 4 and 5 of the PGD file are used to name the STO and MAT files as follows.

On the third line of the PGD file the name of that (the PGD) file must be entered. This line may not be left blank.

On the fourth line of the PGD file the name of the STO file may be entered. Alternatively, this line may be left blank in which case the program will name the STO file by taking the stem of the PGD file name and appending to it the extension STO.

On the fifth line of the PGD file the name of the MAT file may be entered. Alternatively, this line may be left blank in which case the program will name the MAT file by taking the stem of the PGD file name and appending to it the extension MAT.

Any file names that are specified should consist of a string of up to 70 alpha-numeric characters followed by an extension of up to three characters and the total length should not exceed 72 characters (that is to say, if the initial string is 70 characters long, the extension can only be one character long (the point between the stem and the extension accounts for the other character); or if the stem is 69 characters long, the extension can only be two characters long). If the files being used for a particular run are in a different location from the program itself, the file names should include their paths.

See, for examples, the PGD files in Appendix A.

2.3.1.2.2 ESDUpac B9816

When running program B9816 the user is prompted to choose one of two options for the naming of the input and output files associated with a particular run. These options are as follows.

- (a) The user enters one string of up to eight alpha-numeric characters, for example, ABCD1234. The four files are then named by adding to this stem each of the four extensions, PGD, MAT, STO and PRT.
- (b) The user enters four file names, one for each of the files associated with a particular run. The only restrictions on these file names are that the stem be no longer than eight characters and the extension be no longer than three characters.

2.3.1.3 Approaches to running the program

There are three basic ways of running the Fortran program. (Note that these should not be confused with the three analysis options alluded to in (i) of Section 2.2 and described in Section 2.4.1.) The first option, A, applies to the analysis of panels consisting of skin and a number of any one of the predefined sections listed in Tables 1.2 and 2.3. Approaches B and C are two ways of constructing a model for a panel of