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Estimation of transmission efficiencies of coupled plates

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ESTIMATION OF TRANSMISSION EFFICIENCIES OF COUPLED PLATES

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ESTIMATION OF TRANSMISSION EFFICIENCIES OF COUPLED PLATES

1. NOTATION

Vectors and matrices are indicated by bold characters, *e.g.* $\mathbf{R}_j$. Transposed vectors and matrices are indicated by the superscript ' T ', *e.g.* $\mathbf{R}_j^T$.

		<i>Units</i>	
		<i>SI</i>	<i>British</i>
A	area of cross section of beam element	m^2	in^2
$\mathbf{a}$	displacement vector of beam element	— [*]	— [*]
b	width or height of square beam element	m	in
$\mathbf{C}$	dynamic stiffness matrix for beam element	— [*]	— [*]
$C_{g,i}$	group velocity of wave type ' p ' in plate ' i '	m/s	in/s
c_B	phase velocity of bending waves	m/s	in/s
c_L	phase velocity of longitudinal waves	m/s	in/s
c_S	phase velocity of shear waves	m/s	in/s
E	Young's modulus of elasticity of beam element	N/m^2	lbf/in^2
E_j	Young's modulus of elasticity of plate ' j '	N/m^2	lbf/in^2
$\mathbf{F}_i$	force vector corresponding to incident wave in plate ' i '	— [*]	— [*]
F_w	warping constant with pole at shear centre	m^6	in^6
f	frequency	Hz	Hz
Δf	constant frequency bandwidth	Hz	Hz
G	shear modulus of elasticity	N/m^2	lbf/in^2
h_j	thickness of plate ' j '	m	in
I_p	polar moment of inertia of beam element	m^4	in^4
I_y	moment of inertia with respect to y-axis of beam element	m^4	in^4
I_z	moment of inertia with respect to z-axis of beam element	m^4	in^4
i	integer	—	—
J	torsion constant of beam element	m^4	in^4

For footnotes refer to end of Notation section.

j	integer	—	—
$\mathbf{K}_j$	dynamic stiffness matrix for plate 'j'	— *	— *
l_i	length of plate junction	m	in
N	number of plates (can be any number between 2 and 6)	—	—
N_{bands}	number of constant bandwidth bands	—	—
N_{steps}	number of steps	—	—
O	output code for frequency bands, see Section 5.1.2	—	—
p	incident wave type: bending, shear or longitudinal	—	—
$\mathbf{R}_j$	projection matrix for forces and displacements of plate 'j'	— *	— *
R_{pr}^{ij}	transmission loss of wave type 'p' in plate 'i' being transmitted to wave type 'r' in plate 'j', see Equation (2.2)	dB	dB
r	transmitted wave type: bending, shear or longitudinal	—	—
S_i	area of plate 'i'	m ²	in ²
t	thickness of beam element	m	in
x	coordinate in x-direction, see Sketch 2.1	m	in
y	coordinate in y-direction, see Sketch 2.1	m	in
y_0, y_j	y-coordinate of shear centre and offset connection of plate 'j'	m	in
z	coordinate in z-direction	m	in
z_0, z_j	z-coordinate of shear centre and offset connection of plate 'j'	m	in
α_y	shear coefficient in y-direction, see Equation (3.1)	—	—
α_z	shear coefficient in z-direction, see Equation (3.1)	—	—
β	angle of incident wave measured from normal incidence, see Sketch 2.1	degree	degree
θ_j, θ_N	angles of plates 'j' and 'N', see Sketch 2.1	degree	degree
η_{pr}^{ij}	coupling loss factor of wave type 'p' in plate 'i' being transmitted to wave type 'r' in plate 'j'	—	—
λ_B	bending wavelength in beam element	m	in
$\lambda_{B,j}$	bending wavelength in plate 'j'	m	in
ν	Poisson's ratio of beam element	—	—

For footnotes refer to end of Notation section.

ν_j	Poisson's ratio of plate 'j'	—	—
ρ	density of beam element	kg/m ³	[†] lb/in ³
ρ_j	density of plate 'j'	kg/m ³	[†] lb/in ³
ρ_{pr}	reflection efficiency of wave type 'p' being reflected as wave type 'r'	—	—
τ_{pr}^{ij}	transmission efficiency of wave type 'p' in plate 'i' being transmitted to wave type 'r' in plate 'j', see Equation (2.1)	—	—
$\tau_{pr,m}^{ij}$	mean transmission efficiency of wave type 'p' in plate 'i' being transmitted as wave type 'r' in plate 'j'	—	—
ω	radian frequency (= $2\pi f$)	rad/s	rad/s
ω_w	warping function at shear centre	m ²	in ²

* These are not specified here as the elements of the vectors/matrices have mixed units.

[†] A density value expressed in British units as pounds per cubic inch has to be divided by 386.4 before it can be used in this Item.

2. NOTES

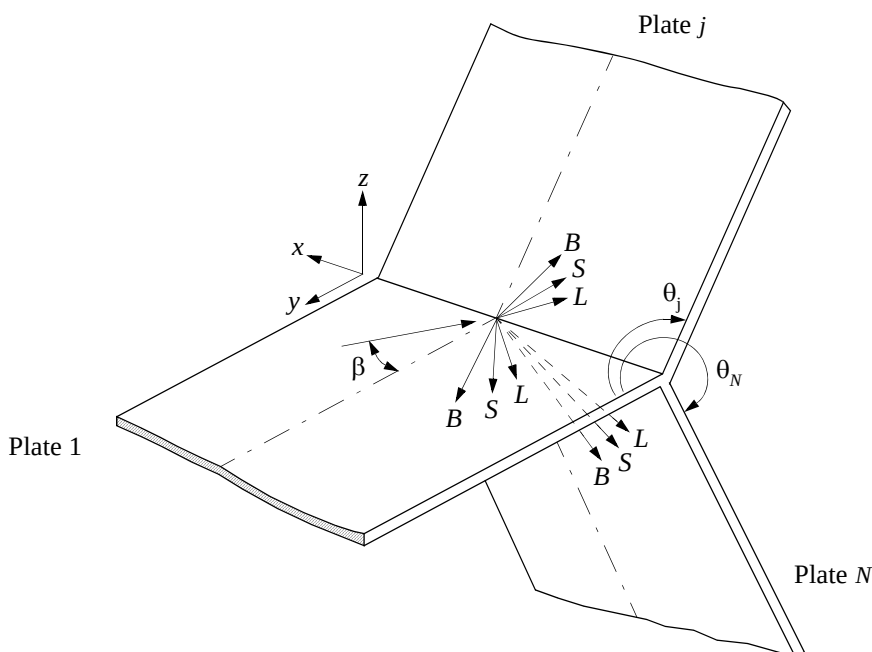
2.1 General

The three fundamental wave types that can propagate in plates are bending, shear and longitudinal waves. A wave type that is incident on a junction can be transmitted and reflected as all three types as illustrated in Sketch 2.1. This wave propagation in structures is sometimes referred to as structure-borne sound (Derivation 3).

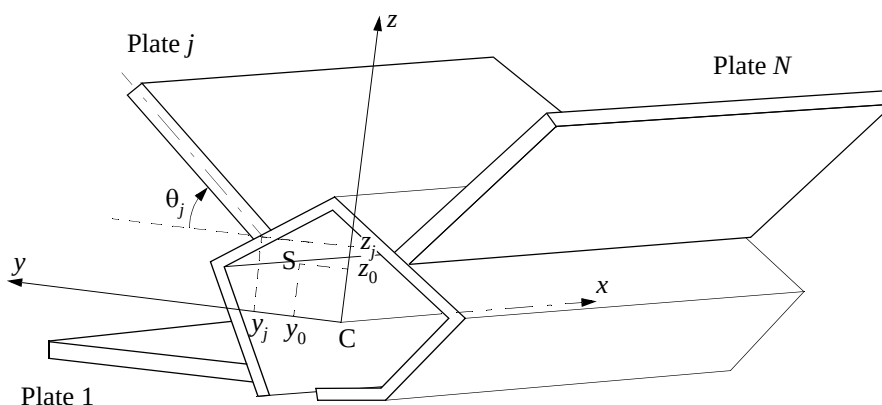
This Item permits estimation of the ratio of transmitted to incident wave power, that is, the transmission efficiencies of an arbitrary junction of between two and six coupled thin isotropic plates. The plates can either be coupled directly as in Sketch 2.1, or via a beam element as shown in Sketch 2.2.

There are two main applications of this Item. Firstly, it allows the evaluation, of the attenuation expressed as the transmission loss, R_{pr}^{ij} , of vibrational power through different plate junctions, and the determination of the percentage of the incident wave power that is being transmitted as bending, shear and longitudinal wave power. Secondly, the Data Item allows the calculation of diffuse field transmission efficiencies in 1/3-octave bands, octave bands or at a user defined constant bandwidth. These can be used to calculate coupling loss factors for SEA (Statistical Energy Analysis) models.

Transmission efficiencies of coupled plates are functions of the angle of incidence, β . If the movement of the junction is not restricted, as is assumed in this Item, then bending waves and at least one of the other wave types (longitudinal or shear in the plates or torsion or longitudinal in the beam element) will normally be present, and then the transmission efficiencies will also be functions of frequency.



Sketch 2.1 N plates* coupled directly. The plane of the x - and y -axes of the coordinate system coincide with the mid-plane of the plate designated 1. The position of plate ' j ' is given by the angle, θ_j , measured positive from the x - y plane in the direction shown. A wave incident in plate 1 at angle β is being transmitted as bending (B), shear (S) and longitudinal (L) waves.



Sketch 2.2 N plates* coupled via a beam element.† Here the x -axis of the coordinate system goes through the centroid, indicated by 'C', of the cross section of the beam element. The direction of the y -axis is chosen so the x - y plane is parallel to the mid-plane of plate 1. The position of the plate ' j ' is given by the coordinates (y_j, z_j) and the angle θ_j . The shear centre, indicated by 'S', is specified by the coordinates (y_0, z_0) .

* Though the plates are shown having finite dimensions, in the analysis used in this Item they are assumed to be semi-infinite in extent.

† The beam element is also assumed to be infinite in extent.

The transmission efficiencies for coupled plates have also some application for coupled curved plates and for coupled beams. Above the ring frequency, a curved plate behaves with good accuracy as a flat plate. Transmission efficiencies for flat plates can therefore be used to describe a similar coupling between curved plates for frequencies above their ring frequencies. For coupled beams, the transmission efficiencies are identical to those of a similar plate system at normal incidence ($\beta = 0$).

2.2 Definitions

A wave that is incident on a junction will be partly transmitted to the other plates and partly reflected as shown in Sketch 2.1. The transmission efficiency, τ_{pr}^{ij} , is defined as

$$\tau_{pr}^{ij} = \frac{\text{transmitted power per unit length of wave type 'r' in plate 'j'}}{\text{incident power per unit length of wave type 'p' in plate 'i'}} , \quad (2.1)$$

and is thus a real quantity. The transmission efficiency of the plate with the incident wave is frequently referred to as reflection efficiency and can be designated by ρ_{pr} . These reflection and transmission efficiencies must not be confused with reflection and transmission coefficients which describe the corresponding ratios of wave amplitudes. The reflection and transmission coefficients are complex quantities.

The transmission loss, R_{pr}^{ij} , is defined as

$$R_{pr}^{ij} = 10 \log_{10}(1/\tau_{pr}^{ij}) \text{ dB} , \quad (2.2)$$

and describes the attenuation of vibrational power through the coupling. For example, a transmission efficiency $\tau_{pr}^{ij} = 0.1$ corresponds to a transmission loss $R_{pr}^{ij} = 10 \text{ dB}$.

2.3 General Assumptions

The transmission of vibrational power through a coupling in a real structure depends not only on the properties of the coupling but also on the resonant behaviour of the whole structure. To make possible comparison of different types of couplings, independently of the dynamic properties of the structure, two assumptions are made. Firstly, it is assumed that the coupled elements are semi-infinite in extent. Thus, plate width and length do not enter into the calculations. Secondly, it is customary to assume in the calculation of transmission efficiencies that no power is dissipated in the coupling, though much of the dissipation of power in real structures is due to friction in the joints. As a result, the sum of the transmission efficiencies is always unity. For an incident wave of type 'p' in plate 'i' of N coupled plates, this can be expressed as

$$\sum_r \sum_{j=1}^N \tau_{pr}^{ij}(\omega, \beta) = 1 , \quad (2.3)$$

where 'r' represents summation over bending, shear and longitudinal waves. An example of this is illustrated in Sketch 7.2.

2.4 Method of Analysis

The method for the calculation of transmission efficiencies is described in Derivations 1, 3 and 4 and in Reference 12. Supplementary information can be found in References 7 to 10. In principle, the method consists of deriving the displacements of each element due to incident and reflected or transmitted wave types and then matching the forces, moments, displacements and angles at the junction. This approach leads to a number of complex equations from which the reflection and transmission coefficients can be calculated. The complex reflection and transmission coefficients can then be converted into reflection and transmission efficiencies by considering the power of the different wave types.

The main difficulty in calculating transmission efficiencies is in minimising the amount of algebraic manipulation involved. The approach that is used in this Item is based on Derivation 4. There the equations of motion for a coupling consisting of a beam and N plates are formulated using a dynamic stiffness approach. This approach leads to a set of equations which in matrix form can be written as

$$\left\{ C + \sum_{j=1}^N R_j K_j R_j^T \right\} \mathbf{a} = R_i F_i, \quad (2.4)$$

where C and K_j are dynamic stiffness matrices for the beam element and plate 'j', respectively. R_j is a matrix that projects the forces and the displacements of plate 'j' on the main axes and F_i is a force vector that corresponds to the excitation due to the incident wave type in plate 'i'. The superscript 'T' indicates a transposed matrix. This set of equations yields the vector $\mathbf{a}$ of the beam displacements from which the reflection and transmission coefficients and efficiencies can be calculated.

2.5 Average Transmission Efficiencies

The transmission efficiency, τ_{pr}^{ij} , is a function of both frequency, f , and angle of incidence, β . In order to reduce the quantity of data it is frequently assumed that the vibrational energy is diffuse, i.e. that the energy is distributed uniformly over all angles of incidence and the various wave components can be considered to be uncorrelated. The assumption of a diffuse field permits the mean of the transmission efficiency, $\tau_{pr,m}^{ij}$, for two-dimensional elements to be calculated from

$$\tau_{pr,m}^{ij} = \int_0^1 \tau_{pr}^{ij}(\beta) d(\sin \beta), \quad (2.5)$$

where $\sin \beta$ corresponds to the projection of the incident wave field on the junction. Because a $\sin \beta$ weighting appears in Equation (2.5), $\tau_{pr}^{ij}(\beta)$ is normally plotted against $\sin \beta$ as in Sketch 7.2.

For plate elements (two-dimensional elements), the assumption of a diffuse field will generally only be fulfilled in the high frequency range. In the low and medium frequency range, the field seldom is diffuse and the directivity ought to be taken into account in Equation (2.5), see Reference 11. However, when the directivity is difficult to predict, the vibrational field is assumed to be diffuse.

2.6 Coupling Loss Factors

The mean transmission efficiency, $\tau_{pr,m}^{ij}$, can be employed to calculate SEA coupling loss factors describing the coupling between plates of finite dimensions. For a so-called line coupling, the coupling loss