

EXAMPLE OF SIMILARITY METHODS APPLIED TO BASE DRAG COEFFICIENT OF CONICAL AFTERBODIES IN AXISYMMETRIC FLOW (JET OFF)

1. NOTATION

		<i>SI</i>	<i>British</i>
a_1	coefficient defined in Equation (6.4)		
a_2	coefficient defined in Equation (6.5)		
C_{D_b}	base drag coefficient		
C_p	pressure coefficient		
$\hat{C}_p$	transformed pressure coefficient		
$C_{p_{cyl}}$	pressure coefficient at base of right circular cylinder in axisymmetric flow		
$\hat{C}_{p_{cyl}}$	transformed pressure coefficient at base of right circular cylinder in axisymmetric flow		
$C_{p_{local}}$	local pressure coefficient defined in Equation (5.1)		
$C_{p_{min}}$	minimum pressure coefficient at given set of free-stream conditions		
d_b	base diameter	m	ft
d_m	maximum diameter	m	ft
$f_1[\quad], f_2[\quad]$	arbitrary functions of quantities in square brackets		
l_a	afterbody length	m	ft
M	free-stream Mach number		
p	ambient static pressure	N/m ²	lbf/ft ²
p_{local}	local static pressure	N/m ²	lbf/ft ²
q	free-stream kinetic pressure, $\gamma p M^2 / 2$	N/m ²	lbf/ft ²
S_b	base area	m ²	ft ²
S_{REF}	reference area	m ²	ft ²

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β_a	half angle of conical afterbody or boat-tail angle	deg	deg
$\beta_n^m [\]$	beta function of quantity in square brackets, see Appendix A		
γ	ratio of specific heat capacities of air		
$\Delta [\]$	residual about correlation of quantity in square brackets, see Appendix B		
$\hat{\Delta} [\]$	transformed residual about correlation of quantity in square brackets, see Appendix B		
Θ_0	transformed Mach number, see Equation (5.3)	radian	radian
Θ_1	transformed Mach number, see Section 6.1.1	radian	radian
Θ_2	transformed Mach number, see Equation (6.2)	radian	radian
Θ_3	transformed Mach number, see Section 6.2		
Φ	probability function, see Reference 1		
τ	thickness parameter		
χ^2	“Chi-squared” test statistic, see Reference 1		

2. INTRODUCTION

This Data Item provides an example of the application of the similarity rules introduced in Reference 4 (Data Item No. 97025) to treat a complex aerodynamic case in a relatively simple way. The particular case is the correlation of measured values of base drag coefficient of conical afterbodies in axisymmetric flow with no jet.

Similarity rules are used only to identify the *form* of variables that are to be used in a process of correlation or curve-fitting. In this Data Item, the process of curve fitting is approached intuitively and is not based on any specific statistical process. The exercise can therefore be adapted to other equally complicated aerodynamic phenomena without compromising any part of the approach.

In studying base drag coefficient of afterbodies in axisymmetric flow two initial considerations arise. Firstly, the geometric parameters that govern the flow over an afterbody have to be identified; secondly, the range of Mach number over which a correlation can be sought must be determined. In identifying the geometric parameters it may be noted that any geometric shape may result from the compromises that arise during design. A relatively crude approximation to the shape of the afterbody on conventional aircraft is to use a truncated cone to envelop the aft fuselage or the after parts of engine nacelles. In the absence of jet flow, and based on the idea that such an afterbody tends to a right circular cylinder as its diameter ratio tends to unity (or the boat-tail angle tends to zero), a correlation can be sought in terms of a base pressure coefficient that also tends to the value for a right circular cylinder as diameter ratio approaches unity.

Although there is no general solution to the (simplified) differential equation for *transonic axisymmetric flow* (see Reference 4), it is supposed that correlations may be sought for measured base drag coefficient data throughout the subsonic, transonic and supersonic ranges of Mach number. Figure 1 shows a typical collection of measured data; the appearance of these data is similar to that of many other collections of information for different afterbody geometries. There may exist a formulation in terms of similarity parameters that will serve to describe such material to tolerable precision. However, all the similarity rules are based on the assumption that no strong shocks or significant separations exist in the flows. Thus any correlations for base drag may turn out to be purely empirical.

A correlating parameter may be sought from the solutions to the subsonic and supersonic (linearised) cases considered in Reference 4, Equation (3.26). If even an oblique theoretical basis for a correlation can be shown, confidence in using the correlation for interpolation (if not for limited extrapolation) is increased.

3. SIMILARITY RULES FOR AXISYMMETRIC FLOW

3.1 Linearised (Subsonic and Supersonic) Cases

The similarity rule for *linearised axisymmetric flow* can be represented (see Reference 4, Equation (3.26)) by

$$C_p(1 - M^2) = f_1[\tau(1 - M^2)^{1/2}]. \quad (3.1)$$

This form of the rule is not very convenient if a practical extension to cover the whole Mach number range is sought since, for a Mach number of unity, the dependent variable in the equation is always zero. In such cases, when values of pressure coefficient are reconstituted from the correlation, the value of C_p will not change over a range of values of the thickness parameter, τ . An alternative representation can be found by dividing both sides by $\tau^2(1 - M^2)^*$ and rearranging[†] to obtain

$$\frac{C_p}{\tau^2} = f_2[\tau^2(1 - M^2)]. \quad (3.2)$$

Equation (3.2) can be used to seek a correlation for linearised (subsonic and supersonic) flow. However, it must be borne in mind that no subsonic flow can be similar to a supersonic flow. This follows because the forms of the solutions to the approximate differential equations are different between the two cases. See Section 4 of Reference 4.

3.2 Non Linear Transonic Case

In principle, the case of transonic, axisymmetric flow has to be treated separately and the reasons for this are summarised in Section 3.7 of Reference 4. However, because there is no general treatment available for this case, it is proposed to use here the *linearised* version of the solutions as the basis of correlation. In consequence, an assessment must be made of any significant limitations of the resulting correlation.

* The operation is justifiable because the ratio of two functions of a variable is still a function of that variable.

† Part of this procedure may not be *formally* justified because it is implied that the domain of the independent variable is being limited in an unspecified manner. If, however, the procedure leads to a feasible correlation over the available range of information then this is sufficient justification for an *empirical* application.

4. BASE DRAG COEFFICIENT AND BASE PRESSURE COEFFICIENT FOR CONICAL AFTERBODIES

4.1 Relation Between Drag Coefficient and Pressure Coefficient

Base drag coefficient is defined as

$$C_{D_b} = - \frac{\int_b (p_{local} - p) dS_b}{qS_{REF}} \quad (4.1)$$

which can be written

$$C_{D_b} = - \frac{\int_b C_{p_{local}} dS_b}{\int_b dS_b} \times \frac{\int_b dS_b}{S_{REF}}, \quad (4.2)$$

$$i.e. \quad C_{D_b} = - \frac{S_b}{S_{REF}} \times C_{p_b} \quad (4.3)$$

$$or \quad C_{D_b} = - \left(\frac{d_b}{d_m} \right)^2 \times C_{p_b}. \quad (4.4)$$

In Equations (4.3) and (4.4) C_{p_b} is the mean value of the base pressure coefficient, d_b is the base diameter and d_m is the maximum diameter of the body (which is most often used as a reference). It follows that a correlation of C_{p_b} can be readily converted into one for C_{D_b} ; this is the procedure followed in Sections 5 and 6.

4.2 Limiting Case

When the afterbody length, l_a , tends to zero or to infinity, or the boat-tail angle, β_a , tends to zero or 90 deg, the afterbody becomes a right circular cylinder. Because it is useful when establishing a scheme for correlation to deal with variables that tend to predictable limits, and the right circular cylinder is the limiting case here, a transformed pressure coefficient, $\hat{C}_p$, is defined where

$$\hat{C}_p = C_{p_b} - C_{p_{cyl}} \quad (4.5)$$

and is the independent variable to be correlated. By doing this and couching the correlation in terms that ensure that the limits to which it tends are zero at the extremes of the geometric variation, the value of C_{p_b} will tend to $C_{p_{cyl}}$. It follows that, as a first step, it is necessary to establish a sound correlation for the base pressure coefficient on a right circular cylinder.

5. CORRELATION OF BASE PRESSURE COEFFICIENT FOR RIGHT CIRCULAR CYLINDER IN AXIAL FLOW

5.1 General

The database for the base pressure coefficient for a right circular cylinder in axial flow is extensive but of uncertain quality in some cases. Some of the data are of considerable age, of unknown provenance or obtained from tests in very small transonic wind tunnels with large blockages and possible problems with shock reflections creating interference fields in the wake close to the body. Since the exercise undertaken is empirical*, it is appropriate to include all the available information. The data are plotted in Figure 1 as a function of free-stream Mach number. Inspection shows that at low subsonic Mach number the data tend to some limit that is not readily identifiable. At high supersonic Mach number, the data tend to zero or to some small negative value of pressure coefficient.

5.2 Identification of an Independent Variable

The process followed in this Section is not an exercise in similarity. The only independent variable to be considered is the free-stream Mach number – no variation in geometry is treated. To identify, at least, the theoretical lower limit to the pressure coefficient as described in Section 5.1, consider the definition of local pressure coefficient,

$$C_{p_{local}} = \frac{(p_{local} - p)}{q} = \frac{2(p_{local} - p)}{\gamma M^2 p}. \quad (5.1)$$

The absolute minimum value of local pressure is zero (vacuum). The pressure coefficient is then

$$C_{p_{min}} = \frac{-2}{\gamma M^2} \quad (5.2)$$

and is often called the *vacuum limit*. This is the absolute minimum value of pressure coefficient which can ever be achieved under any conditions. As Mach number increases, the vacuum limit tends to zero. Because this reflects the trend shown by the measured pressure coefficient data, it is reasonable to try to use $2/\gamma M^2$ as the basis for correlation.

In seeking to use a quantity as an independent variable in a correlation it can be inconvenient if that quantity is not finitely bounded – as is the case in Equation (5.2) where the vacuum limit tends to minus infinity as Mach number tends to zero, and tends to zero as Mach number tends to infinity. There are several strategies that can be used to transform the limit to a form that has finite bounds. The strategy used here is to introduce the “transformed Mach number”, Θ_0 , given by

$$\Theta_0 = \tan^{-1} \left[\frac{2}{\gamma M^2} \right] \quad (5.3)$$

so that when $0 \leq M < \infty$ then $\pi/2 \geq \Theta_0 > 0$. In this interval of Θ_0 , $1 \geq \sin[\Theta_0] > 0$, and so it is worthwhile to explore the possibility of using $\sin[\Theta_0]$ as the basis of a relation.

* The theory underlying the study is approximate.

5.3 Establishing a Relationship Between Base Pressure Coefficient and Transformed Mach Number

The first step is to plot base pressure coefficient against $\sin[\Theta_0]$ and, to this end, the data from Figure 1 are replotted in Figure 2. The general trend in the data suggests that there is a reciprocal relationship between the two variables. So, by transforming the dependent variable into $\sin[\Theta_0]/C_{p_{cyl}}$ a pattern emerges – as shown in Figure 3. The function $-(1 + 6.5 \sin^{1.2}[\Theta_0])$ is also shown on Figure 3 – where it can be seen to represent the data at each end of the range of values of $\sin[\Theta_0]$.

To get a clearer idea of the way in which the independent variable changes with transformed Mach number, a further calculation is done. So, forming

$$\hat{C}_{p_{cyl}} = \frac{\sin[\Theta_0]}{C_{p_{cyl}}} + (1 + 6.5 \sin^{1.2}[\Theta_0]) \quad (5.4)$$

and preparing Figure 4, it is established by inspection that the data reach a maximum at $\sin[\Theta_0] \approx 0.7$ and are tangential to the horizontal axis at the limits of the independent variable. Referring to Appendix A it appears that the data may well be described by a β -function*. The relationship

$$\hat{C}_{p_{cyl}} = 28 \sin^4[\Theta_0] (1 - \sin^2[\Theta_0])^2 \quad (5.5)$$

is also shown in Figure 4 and represents the data well. It is a β -function of $\sin^2[\Theta_0]$ and can be re-written as

$$\hat{C}_{p_{cyl}} = 1.75 \sin^4[2\Theta_0]. \quad (5.6)$$

The full relation connecting base pressure coefficient on a right circular cylinder is then obtained by substituting from Equation (5.6) into Equation (5.4) and re-arranging so that

$$C_{p_{cyl}} = \frac{\sin[\Theta_0]}{(1.75 \sin^4[2\Theta_0] - 6.5 \sin^{1.2}[\Theta_0] - 1)}. \quad (5.7)$$

This function is shown in Figure 5 together with the measured data and the vacuum limit. Note that, because the denominator of Equation (5.7) is never greater than -1 , $C_{p_{cyl}} \geq -\sin[\Theta_0]$. Since (for Θ_0 in radians) $-\sin\Theta_0 > -\Theta_0$, it follows that $C_{p_{cyl}} > -\Theta_0$ and so the function described always respects the vacuum limit.

The distribution of the data about the function described in Equation (5.7) is discussed in Appendix B. It is shown there that the correlation has a precision of ± 0.0025 at the 0.95 level of confidence.

* A summary of these useful functions is given in Appendix A.

6. CORRELATION OF BASE PRESSURE COEFFICIENT FOR CONICAL AFTERBODIES IN AXIAL FLOW

To implement the similarity principles expressed in Equation (3.2) it is necessary firstly to use the data to identify τ , the relevant thickness parameter for this class of flows.

6.1 Identification of a Thickness Parameter

6.1.1 General

It is not immediately obvious what combination of geometric features of an afterbody may be used to characterise thickness. The flow at the base is strongly influenced by the flow on the boat-tail. This, in turn, is conditioned by the angle through which the flow is turned at the start of the boat-tail and the length of the surface over which it travels before separation occurs at the sharp corner at the base. It may therefore be expected that a study of the effect of afterbody length at a constant value of boat-tail angle may aid the process of identification.

As well as seeking to find a thickness to transform the dependent variable of Equation (3.2), it is also necessary that the thickness be applied to the independent variable there. The form of the transformed Mach number is related to $[1 - M^2]$, see Equation (3.2). To limit the range of the independent variable, use is made of the transformed variable $\Theta_1 = \tan^{-1}[1 - M^2]$.

6.1.2 Effect of ratio of length to diameter

As a simple illustration consider Figure 6. This is a plot of all the available data for a boat-tail angle of 5.6 deg in the form of the parameter $(C_{p_b} - C_{p_{cyl}})$ plotted against the transformed Mach number, Θ_1 . A range of afterbody lengths is covered and there is little obvious organisation in the set of data. However, referring to Equation (3.2), it is noted that the effect of a thickness parameter that is always greater than unity is to move the data (for a given length) towards the origin for (transformed) pressure coefficient and away from the origin for (transformed) Mach number. To ensure that this happens, even for length to diameter ratios that tend to zero, the thickness parameter to be sought must contain the factor $(1 + l_a/d_m)$ or something similar.

In Equation (3.2), the form required is the reciprocal of the square of the thickness so, if the transformation

$$\hat{C}_{p_b} = \frac{(C_{p_b} - C_{p_{cyl}})}{(1 + l_a/d_m)^2} \quad (6.1)$$

is applied, and the result plotted against a further transformed Mach number defined as,

$$\Theta_2 = \tan^{-1}[(M^2 - 1)(1 + l_a/d_m)^2], \quad (6.2)$$

then Figure 7 results. The formulation of Equations (6.1) and (6.2), which is based on Equation (3.2), achieves a collapse of the data so that a single curve can be produced to fit the values (at the chosen fixed value of boat-tail angle). Note that the thickness formulation for the effect of length is the simplest that can be obtained that satisfies the criterion suggested above.