

VGK METHOD FOR TWO-DIMENSIONAL AEROFOIL SECTIONS

PART 4: ESTIMATION OF EXCRESCENCE DRAG AT SUBSONIC SPEEDS

1. INTRODUCTION

This Item is the fourth in a sequence dealing with the VGK aerofoil method (for Parts 1 to 3, see Refs. 1, 2 and 3). It describes the use of an extended VGK program (termed VGKE) to provide values of the various parameters needed in order to estimate the excrescence drag appropriate to isolated two-dimensional excrescences, such as those produced by skin joints and joints between flaps or slats and the main wing. The method may be used for small excrescences located on an aerofoil where the boundary layer is locally turbulent. The method is appropriate to aerofoils operating at subsonic freestream speeds, for values of lift coefficient for which the upper- and lower-surface boundary layer flows are attached or have only very limited regions of separation. The excrescences must be small enough for the available methods for estimating local excrescence drag coefficient increments (such as those given in Refs. 4, 5 and 6) to be applicable. The types of two-dimensional excrescence treated in Refs. 4, 5 and 6 cover various shapes of grooves, steps and ridges, and transverse rows of spherically-headed rivets.

The parameters needed for the estimation of excrescence drag that are provided by the use of VGKE are (a) the local values of Mach number, Reynolds number, skin friction coefficient and non-dimensional boundary layer thickness, and (b) the excrescence drag magnification factor, m_d . The first set of parameters is needed in order to estimate the local excrescence drag increment (using methods such as those in Refs. 4, 5 and 6), and the excrescence drag magnification factor is required to relate the overall drag produced by an excrescence to this local value.

The Item explains the usefulness of the excrescence drag magnification factor, m_d , in estimating the required excrescence drag. It describes how VGKE is used to represent the effects of an excrescence by means of an increment in the boundary layer momentum thickness, $\Delta\theta_0$. The steps needed to obtain the overall excrescence drag are given in detail. The use of VGKE to obtain consistent results for m_d for subcritical and supercritical flow cases is explained. Typical results are presented and discussed, and include the effects of variations in excrescence location and $\Delta\theta_0$. Comparisons with results from the use of a comparably extended version of BVGK (see Refs. 1 and 7) are shown and suggest a prediction accuracy for m_d likely to be adequate for aircraft drag estimation studies, whilst Ref. 7 itself provides a rapid guide to the values of m_d for a range of typical pressure distributions that may be useful in their own right.

The use of the various programs entailed in the calculation method is described in detail.

2. NOTATION

Algebraic notation is printed in italics or has Greek symbols. Notation as used in program and data files is given in Roman type. Notation sequencing is based on the algebraic terms (except where none exists); some program and data file notation therefore appears out of alphabetical order. All linear quantities made non-dimensional are based on aerofoil chord, c .

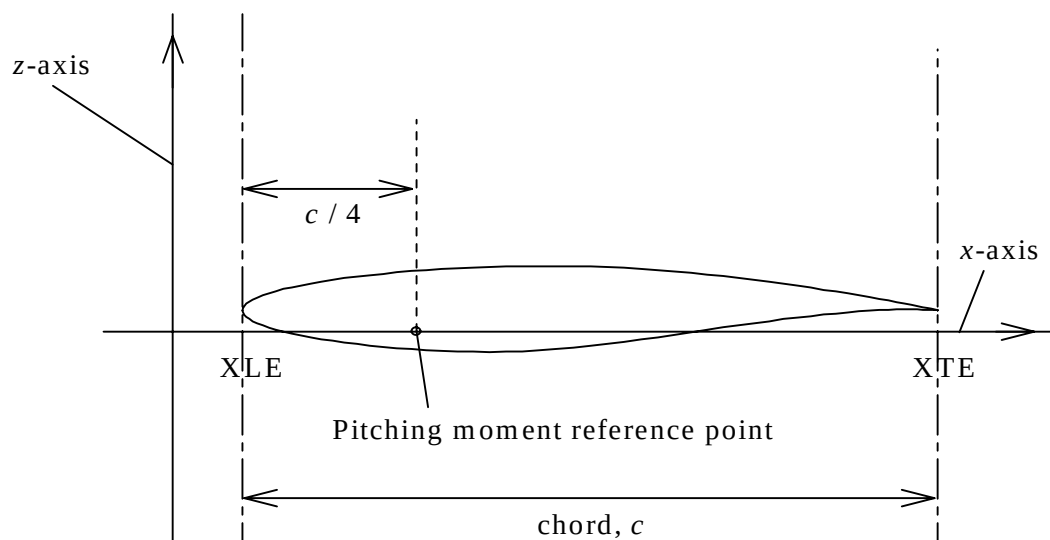
C_D	CD	overall aerofoil section drag coefficient
C_{D1}	CD1	wave drag coefficient determined from 'first-order method' (see Parts 1 and 2)
C_{D2}	CD2	wave drag determined from 'improved method' (see Parts 1 and 2)

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C_{DF}	CDF	contribution to C_D determined from integration of surface shear stresses
C_{DP}	CDP	contribution to C_D determined from integration of surface pressures
C_{DV}	CDV	viscous drag coefficient, determined from calculated momentum thickness of wake far downstream
C_{DW}	CDW	wave drag coefficient
C_{Dloc}	CDLOC	contribution to C_D due to local effects of excrescence, and omitted from C_D determined from $C_{DF} + C_{DP}$
C_{f0}	CF0	local skin friction coefficient (<i>i.e.</i> based on kinetic pressure at edge of boundary layer) at excrescence location, in absence of excrescence
C_L	CL	aerofoil lift coefficient
C_{Lpot}	CLPOT	value of C_L determined in VGK from jump in potential at trailing edge
C_p	CP	pressure coefficient
c	C	aerofoil chord
H	H	boundary layer shape factor ($= \delta^*/\theta$)
h		height of protruding excrescence or depth of groove- or cavity-type excrescence
j	NSG	number of radial lines in computing plane (see Part 1, Sketch 3.1)
K	K	number of circumferential grid lines in computing plane (see Part 1, Sketch 3.1)
l		excrecence characteristic length (<i>e.g.</i> height of step or length of chamfered step)
M	EML	local Mach number (at edge of boundary layer) based on isentropic flow relations
M_∞	EM	freestream Mach number
M_0	EM0	Mach number at edge of boundary layer at excrescence location, in absence of excrescence
m_d	MD	excrecence drag magnification factor (see Equation (3.2))
q	Q	freestream kinetic pressure
q_0	Q0	kinetic pressure at edge of boundary layer at excrescence location, in absence of excrescence
R	R	Reynolds number based on section chord and freestream values of velocity and kinematic viscosity
R_{c0}	RC0	Reynolds number based on c , and local velocity and kinematic viscosity at edge of boundary layer at excrescence location, in absence of excrescence

R_{l0}	RL0	Reynolds number based on l , and local velocity and kinematic viscosity at edge of boundary layer at excrescence location, in absence of excrescence, $= R_{c0} \times (l/c)$
	XLE	value of x -coordinate at leading-edge
	XTE	value of x -coordinate at trailing-edge
x		'chordwise coordinate' in physical plane (see Sketch 2.1)
z		'normal coordinate' in physical plane (see Sketch 2.1)
α	ALP	aerofoil incidence, <i>i.e.</i> angle between x -axis and freestream direction, positive nose-up
δ	DEL	boundary layer thickness, including the effect of excrescence (if present), made non-dimensional by dividing by c
δ^*	DELSTAR	boundary layer displacement thickness, including the effect of excrescence (if present), made non-dimensional by dividing by c
ΔC_D	DCD	increment in section drag coefficient due to excrescence, based on section chord and freestream kinetic pressure
$(\Delta C_D)_0$	(DCD)0	increment in drag coefficient of element of flat plate due to two-dimensional excrescence, based on length c and local kinetic pressure at edge of boundary layer
$(\Delta C_D)'_0$	(DCD)'0	increment in drag coefficient of element of flat plate due to two-dimensional excrescence, based on length l and local kinetic pressure at edge of boundary layer
$\Delta \theta_0$	DTH0	increment in θ representing effect of excrescence
θ	THETA	boundary layer momentum thickness, including the effect of excrescence (if present), made non-dimensional by dividing by c
θ_0	THETA0	boundary layer momentum thickness in absence of excrescence, made non-dimensional by dividing by c
θ_c	T	angular coordinate in computing plane (see Part 1, Sketch 3.1)
ξ	X	non-dimensional chordwise coordinate, $\xi = 0$ at leading-edge and $\xi = 1$ at trailing-edge
ξ_{TL}	XTL	value of ξ corresponding to boundary-layer transition location on lower surface
ξ_{TU}	XTU	value of ξ corresponding to boundary-layer transition location on upper surface
ζ	Z	non-dimensional normal coordinate in physical plane
Suffices		
0	0	denotes value or conditions at location of excrescence
<i>clean</i>	CLEAN	denotes value in absence of excrescence

e	E	denotes value with excrescence present
L	L	denotes lower surface
U	U	denotes upper surface



Sketch 2.1 Aerofoil axes and notation

3. INCREMENT IN SECTION DRAG COEFFICIENT DUE TO EXCRESCENCE AND EXCRESCENCE DRAG MAGNIFICATION FACTOR

The drag coefficient, ΔC_D , due to a small two-dimensional excrescence located at a particular position on the surface of an aerofoil, is the difference between the aerofoil drag coefficient with the excrescence present and with the excrescence absent, with the flow conditions of Mach number, lift coefficient, Reynolds number and boundary layer transition locations being the same in the two situations. Thus

$$\Delta C_D = C_{De} - C_{Dclean}, \quad (3.1)$$

where C_{De} and C_{Dclean} relate to the same flow conditions. An excrescence could also affect the locations of natural transition but this effect is not considered.

The value of ΔC_D , for a particular combination of aerofoil flow conditions and excrescence type, size and location, is related to the local excrescence drag, expressed as $(\Delta C_D)_0$, which is the increment in drag coefficient experienced by a short element of flat plate due to the same form of excrescence and for which the local conditions of the turbulent boundary layer are the same as those local to the excrescence on the aerofoil. The relationship between ΔC_D and $(\Delta C_D)_0$ defines the excrescence drag magnification factor, m_d :

$$\Delta C_D = m_d (\Delta C_D)_0. \quad (3.2)$$

It should be noted that although the coefficients ΔC_D and $(\Delta C_D)_0$ are based on the same reference length (namely the chord, c), ΔC_D is based on the freestream kinetic pressure, q , whilst $(\Delta C_D)_0$ is based on the local kinetic pressure, q_0 . Thus m_d incorporates two effects; namely that due to drag coefficient differences resulting from q_0 differing from q , and that resulting from the changes the excrescence produces in the

non-local viscous and inviscid flow fields around the aerofoil. The latter effect may be considerable, particularly for aerofoils at moderate and high lift coefficients and for flows with a significant wave drag contribution. Although approximate methods exist for the estimation of m_d (see Ref. 7), clearly the only schemes that are likely to be reliable in principle are those that accurately calculate the effects produced by the excrescence on the overall flow. One such scheme is to use a coupled viscous/inviscid calculation method, in which the excrescence is represented by an increment, $\Delta\theta_0$, in the non-dimensional boundary layer momentum thickness at the location of the excrescence (see Ref. 7). This is the scheme adopted in this Item, which utilises VGKE, a suitably extended version of the VGK program, as described in Section 4.

The usefulness of the parameter m_d lies mainly in the fact that, for a given excrescence location, m_d is effectively independent of $\Delta\theta_0$ (provided $\Delta\theta_0$ is not too large). Calculation of the local drag and of m_d may therefore be made independently, and once m_d has been determined, changes in the size of the excrescence do not require any further VGKE calculations to be made in order to obtain the consequent changes in the excrescence drag.

3.1 Turbulent Boundary Layer Parameters Affecting $(\Delta C_D)_0$

The coefficient $(\Delta C_D)_0$ is the increment in drag coefficient (based on the aerofoil chord, c , and the local kinetic pressure, q_0) resulting from the excrescence being placed on a short element of a long flat plate, when the flow over the element is equivalent to that local to the excrescence on the aerofoil. The word short here applies to the streamwise length of the element of flat plate on which the drag is considered to act, and signifies only that (in the absence of the excrescence) the boundary layer parameters may be considered to remain effectively constant over this length (see Ref. 8). The equivalence of the local turbulent boundary layer flow over the aerofoil and that over the element of flat plate requires the values of the three dimensionless parameters M_0 , R_{l0} and C_{f0} to be the same for the two flows.

For the short element of flat plate, the drag coefficient increments are more conveniently based on the appropriate characteristic length of the excrescence, l , than on c . Thus, it is usual to express the drag increment on the short element of flat plate in terms of $(\Delta C_D)'_0$ rather than $(\Delta C_D)_0$, where

$$(\Delta C_D)'_0 = (c/l)(\Delta C_D)_0. \quad (3.3)$$

This coefficient convention then corresponds to the one adopted in the Items dealing with the drag of three different types of two-dimensional excrescence (Refs. 4, 5 and 6 respectively), for each of which the excrescence drag is presented in the functional form,

$$(\Delta C_D)'_0 = F(M_0, R_{l0}, C_{f0})C_{f0}. \quad (3.4)$$

The data given in these Items represent correlations of a large number of experimentally determined drag values. Use of the Items permits $(\Delta C_D)'_0$ to be found for specific values of the parameters M_0 , R_{l0} and C_{f0} , within relatively wide ranges of these parameters, but the excrescence must be small enough not to extend beyond the wall region of the turbulent boundary layer (see, for example, Ref. 9). In broad terms, this means that the height of a protruding excrescence (or the depth of a groove) should be no more than about one tenth of the boundary layer thickness, *i.e.*,

$$h/c < 0.1\delta_0. \quad (3.5)$$

3.2 Steps Entailed in Determining Excrescence Drag

The detailed process of determining excrescence drag for the two-dimensional case may be split into the following steps.

- (1) Specify the following features and parameters which affect the excrescence drag:

Aerofoil profile

Flow conditions, namely $M_\infty, C_L, R, \xi_{TU}, \xi_{TL}$

Excrescence location: either ξ_{0U} (for an excrescence on the upper surface)
or ξ_{0L} (for an excrescence on the lower surface)

Type of two-dimensional excrescence (*e.g.* chamfered ridge)

Size parameters of excrescence relative to aerofoil chord, l/c and h/c

- (2) Use the VGKE method to find:

$$M_0, R_{c0}, C_{f0}, \delta_0 \text{ and } m_d$$

(Details are given in Sections 4 and 5.)

- (3) Find R_{l0} from

$$R_{l0} = (l/c)R_{c0}$$

- (4) Check the condition expressed by the inequality (Equation (3.5)), and if it is satisfied, use the values of M_0, R_{l0} and C_{f0} found in Steps (2) and (3) and the relevant Data Item (either Ref. 4, 5 or 6 depending on the type of excrescence), or other source, to find:

$$(\Delta C_D)_0'$$

A suggestion as to how one might proceed if the inequality expressed by Equation (3.5) is not satisfied is given in Section 5.1.

- (5) Use the value of m_d from Step (2), and the value of $(\Delta C_D)_0'$ from Step (4), to find the overall excrescence drag coefficient, ΔC_D , from

$$\Delta C_D = m_d(\Delta C_D)_0' = m_d(l/c)(\Delta C_D)_0' \quad (3.6)$$

Notes:

- (a) If a further excrescence at the same location, but either of a different type or size, is to be considered, Step (2) does not need to be repeated.
- (b) If further excrescences at other locations are to be considered (again for the same aerofoil and flow condition), then all the steps need to be repeated. However, as described in Section 4, even if only one excrescence location is of interest, it is desirable at Step (2), particularly for supercritical flows, to carry out a series of VGKE runs covering a range of excrescence locations, and more than one value of $\Delta\theta_0$. Such a series of runs provides an indication of the levels of inconsistency in the

computed values of m_d and of the nature of the variation of m_d as the excrescence location is varied (see Section 4 and Figure 1). Should such a series of VGKE runs covering a sufficient range of excrescence locations have been undertaken at Step (2), then, again, this step does not need to be repeated.

- (c) The check referred to in Step (4) may be carried out by examining the .BLP file appropriate to the flow without any excrescence (see Section 4), **before** any values of m_d are determined.

3.3 Drag Due to the Simultaneous Presence of a Number of Excrescences

Within the degree of accuracy of the methods of predicting the excrescence drag, provided the excrescences are all small, in the sense of Equation (3.5), the increment in drag produced by a number of excrescences simultaneously present on an aerofoil is simply the sum of the excrescence drag produced by the individual excrescences.

4. USE OF THE VGK AEROFOIL METHOD TO DETERMINE DRAG MAGNIFICATION FACTOR, m_d

The VGK aerofoil method (see Refs 1 and 2) has been extended to include the representation of an excrescence by an increment in the non-dimensional momentum thickness. Since this local increment, $\Delta\theta_0$, is related to the local excrescence drag coefficient by

$$\Delta\theta_0 = (\Delta C_D)_0 / 2$$

the value of m_d (for a particular case) may be found from:

$$m_d = (C_{De} - C_{Dclean}) / 2\Delta\theta_0, \quad (4.1)$$

see Equations (3.1) and (3.2).

The details of the changes required to extend the VGK program are presented in Ref. 10. The extended VGK program (named VGKE) permits an excrescence to be represented on either the upper or lower surface or on both surfaces. The excrescence locations and the magnitudes of the increments in non-dimensional momentum thickness are values specified by the user in the *.VIN file (see Part 2, Section 2.1.1 for a description of the files used in VGK and their purposes; the VGKE files are very similar to those of VGK, except for the three additional output files that are produced by VGKE, with extensions CDO, BLP and EXC, see Section 5.1).

Generally, the specified excrescence locations do not coincide with any of the grid points on the aerofoil boundary. In VGK, these grid points are the only points around the aerofoil profile where, for the purposes of calculation, the boundary layer parameters are defined. Thus in VGKE an extra calculation point is generally introduced (or two extra points when excrescences are to be represented on both upper and lower surfaces).

Because of the way in which the boundary layer development is calculated in the VGK program, it is not possible to obtain reliable results for excrescence locations that are within a few percent of the chord of the appropriate transition location, nor within a few percent chord of the trailing edge. It is suggested that extrapolation of the results obtained for other excrescence locations may be used to supply results of comparable accuracy, for excrescence locations in these 'inaccessible' regions.

Although, as described, extra calculation points are introduced in the VGKE method in those cases where the excrescence location does not coincide with a grid point, it has been found that results for parameters such as m_d are very slightly inconsistent between the cases where such an extra calculation point is introduced and the cases where it is not (*i.e.* in the cases where the excrescence location coincides with a grid point). An example is shown in Figure 1. The reasons for these inconsistencies are associated with the details of various numerical integrations that are carried out, and in the interpolations that they effectively embody, and also with the representation of the excrescence by a jump in boundary layer thickness. To remove the inconsistencies would entail a considerable number of program changes, and would provide only a very marginal benefit. **As a result, it is considered preferable to calculate the desired excrescence parameters for excrescence locations which coincide with grid points and subsequently interpolate (or extrapolate) to obtain values for any other excrescence locations of interest, if this is considered necessary.** Details of how such calculations are made are given in Section 5.2.

In addition to the inconsistencies described, two further aspects need to be considered in order to ensure that consistent and (so far as is possible) reliable results are obtained from the use of VGKE. These are dealt with in Sections 4.1 and 4.2 which also cover respectively the effects of varying the size of the modelled excrescence (*i.e.* the magnitude of $\Delta\theta_0$) and the choice of method of calculating the drag increment due to an excrescence.

4.1 Results from VGKE for Excrescences Represented by Various Values of $\Delta\theta_0$

The example results shown are typical of those of interest, and are defined as follows:

Aerofoil profile:	RAE 2822	(see Refs. 1 and 2)
C_L	0.5	
R	20×10^6	
ξ_{TU}	0.02	
ζ_{TL}	0.05	
M_∞	0.675 (subcritical case),	0.75 (supercritical case)

4.1.1 Results for subcritical case

In Figure 2, pressure distributions over the aerofoil upper surface are shown for an excrescence located on this surface at $\xi_0 = 0.20001$ (which is a grid point), for $\Delta\theta_0 = 0$ (*i.e.* the 'clean aerofoil'), for $\Delta\theta_0 = 0.0002$ and for $\Delta\theta_0 = 0.0004$. The VGKE calculations correspond to all the default parameters being set to their default values, see Part 1. The default values are the same as those for VGK, except in the case of NSF, the number of fine mesh iterations of the inviscid flow element, which for VGKE is set to 1000. As is clear from Figure 2, excrescences of these magnitudes cannot be considered as being small from an engineering point of view, and such pressure disturbances would almost certainly be larger than those normally produced by excrescences on aircraft. They correspond to increases in the aerofoil drag coefficient of 11 and 23 per cent respectively. From the point of view of using VGKE in order to find m_d , however, the values selected for $\Delta\theta_0$ are acceptable, since the pressure differences generated are almost precisely linear with $\Delta\theta_0$, as is shown by the near-collapse of points in Figure 3. It is apparent that such an excrescence, as modelled by a jump in boundary layer momentum thickness, acts like a source (as would be expected), since in the near neighbourhood of the excrescence the flow speeds are decreased upstream, and increased downstream, with the disturbance effect decaying rapidly with distance from the excrescence location. The peak values of the pressure disturbance are related to the grid point spacing. If the grid point

spacing is increased (by selecting a lower value of NSG), the peak disturbances decline. If it were possible to refine the grid indefinitely, the peak disturbances (for any non-zero value of $\Delta\theta_0$) would increase without limit, and would then clearly lead to the situation where the boundary layer would separate, and such a case would be impossible to compute with the VGKE method. To avoid such a situation it would be necessary to refine the representation of the excrescence, and to use (for example) a ramp variation or some even smoother variation of momentum thickness increase in place of a sudden jump. Since there appear to be no such nonlinear effects due to interactions between the viscous and inviscid flow regions for the example shown, and none have appeared in any other case studied (which have involved values of $\Delta\theta_0$ up to 0.0004), it is considered acceptable to represent the excrescence as a jump in boundary layer momentum thickness.

In Figure 4, the variation of the computed drag coefficient C_{DV} with $\Delta\theta_0$ is shown, together with the corresponding variation of $C_{DF} + C_{DP}$. The variations are seen to be both nearly linear. At $\Delta\theta_0 = 0$ the two methods of calculating the overall drag are in good agreement. (See Part 1 for a discussion of the merits of the two methods of drag determination.) As $\Delta\theta_0$ increases, the values of C_{DV} and $C_{DF} + C_{DP}$ diverge, and the reason for this is that the 'near-field' calculation approach (that giving $C_{DF} + C_{DP}$) does not include the local excrescence drag (only the non-local effects are accounted for, through their consequences on the pressure and surface shear stress distributions), whereas the far-field approach (giving C_{DV}) does account both for the local excrescence drag and for the overall effects. To include the local drag effect in the 'near-field' method requires an extra term to be included as shown in the following equation:

$$C_{De} = C_{Dtot} = C_{DF} + C_{DP} + C_{Dloc}, \quad (4.2)$$

where

$$C_{Dloc} = 2(q_0/q)\Delta\theta_0. \quad (4.3)$$

As shown in Figure 5, agreement between C_{DV} and C_{Dtot} is good over the range of $\Delta\theta_0$ studied for this example (see also Section 5).

It may be noted that the effect of the excrescence is to decrease the contribution of the surface shear stress, see Figure 6. This is due to the fact that the increases in boundary layer thickness at positions downstream of the excrescence location (resulting from the presence of the excrescence) lead to decreases in the values of C_f at those positions.

Although the variations of drag coefficient with $\Delta\theta_0$ appear very nearly linear in Figures 4 and 5, they do have some degree of nonlinearity, and give rise to m_d values which increase slightly (and almost linearly) with increasing $\Delta\theta_0$ values, as shown in the example in Figure 7. It is likely that excrescences of interest would generally correspond to values of $\Delta\theta_0$ even smaller than 0.0002, and so the relevant values of m_d would be those corresponding to such small values of $\Delta\theta_0$. However, because of the relatively small variations of m_d with $\Delta\theta_0$, the values of m_d corresponding to any value of $\Delta\theta_0$ up to about 0.0004 would almost certainly be of adequate accuracy. However, values of m_d corresponding to values of $\Delta\theta_0$ less than about 0.00001 could lose accuracy significantly due to rounding errors, and should not be used.

It is apparent from Figure 2 that for this example a value of $\Delta\theta_0$ of about 0.0008 would result in the flow becoming locally supersonic in a small region. For subcritical flows which had higher maximum speeds in the clean condition than for this example, such supercritical conditions would be reached with lower values of $\Delta\theta_0$. In these situations the results for m_d may be expected to behave to some extent like those for a supercritical case (see Section 4.1.2). In particular it would be desirable then to investigate the behaviour of m_d for two or more values of $\Delta\theta_0$ (see Section 4.1.2).

4.1.2 Results for supercritical case

Comparable to Figure 2 for the subcritical case, in Figure 8 supercritical pressure distributions over the aerofoil upper surface are shown for an excrescence located at the grid point on this surface at

$\xi_0 = 0.20001$, for $\Delta\theta_0 = 0$ (i.e. the 'clean aerofoil'), 0.0002 and 0.0004. As for the subcritical case the disturbances attributable to the excrescence are large near the excrescence location, but it is clear from Figure 9, which shows the pressure differences corresponding to the two values of $\Delta\theta_0$, that the nature of the pressure disturbances is quite different from that for the subcritical example. An excrescence not only affects the levels of pressure in the supersonic flow region (where the levels tend to diminish more slowly with distance from the excrescence than in the subcritical case), but influences the shock location, tending to move it forward. The shock movement manifests itself in Figure 9 as a spike in the pressure disturbance level near the shock location. The pressure differences in this spike area do not behave linearly with $\Delta\theta_0$.

Variations with $\Delta\theta_0$ of total drag coefficient determined as (a) $C_{DV} + C_{D1}$, (b) $C_{DV} + C_{D2}$, and (c) $C_{DF} + C_{DP} + C_{Dloc}$ are shown in Figure 10. It is observed that the general trends of these variations are similar, but there are a number of small jumps in the values of the parameters which occur as a result of the details of the numerical treatment of the supersonic flow region and the shock wave. These jumps have a considerable effect on the values of m_d , as shown in Figure 11. It appears that in order to be fairly sure that the results for m_d for supercritical flows are reliable, it is desirable to carry out VGKE calculations with two or more values of $\Delta\theta_0$. This inevitably lengthens the process of estimating the excrescence drag as detailed in Section 3.2.

4.2 Choice of Method of Calculating the Drag Due to an Excrescence using VGKE

When the flow of interest is subcritical it is of little consequence whether the total drag coefficient of the aerofoil (with an excrescence modelled), and the corresponding value of m_d , are evaluated from C_{DV} or $C_{DF} + C_{DP} + C_{Dloc}$. This is demonstrated in Figure 12, where the variation of m_d with excrescence location is shown for two values of $\Delta\theta_0$ and corresponding to the two different methods, termed respectively the 'far-field' approach and the 'near-field' approach.

For supercritical flows, there is a tendency for results from the two approaches to differ significantly, especially, but not solely, for excrescence locations close to any shock. In these cases, the main source of the differences is due to the way in which the wave drag is effectively evaluated. In the far field approach, this drag contribution may be estimated in two ways (giving respectively C_{D1} and C_{D2} , see Part 1, Section 3.4), but in each of these ways the wave drag associated with a shock is a function of the local flow Mach number at a grid point just upstream of the shock. Any variations in the shock strength occurring away from the surface have no influence on the estimated wave drag (although a change in shock location can have an effect). This feature could lead to unsatisfactory estimates of the changes in drag resulting from an excrescence located in the supersonic region, since the flow field changes due to the excrescence could then be quite complex and could produce irregular variations in shock strength over the extent of the shock. The near-field approach does not suffer from this drawback, since the accuracy of any wave drag contribution is related only to the accuracy with which the surface pressure distributions can be evaluated. Whilst this accuracy may not be as high as desired (see Part 1, Section 3.4) it is unlikely to be unduly affected by the excrescence, and for this reason the near-field approach is recommended as the means of determining the drag coefficient when it is desired to find m_d for supercritical flows. Since the 'near-field' approach appears satisfactory also for subcritical flows, this is the approach recommended for both types of flow, and is the approach adopted in Section 5.

In Figure 13, results for m_d are shown corresponding to the two approaches. Also shown is the variation of m_d associated with the viscous drag contribution, C_{DV} . It is clear that, depending on the location of the excrescence, the wave drag may be either increased or decreased by the excrescence. There is no reason to think that such decreases could not be realised in practice, although any practical case would involve a three-dimensional wing, and then there would be only a limited relevance of a single two-dimensional flow condition (see Refs. 12 and 13). In any case, the effects are not likely to be so significant that they would influence the positioning of any excrescence in those situations where there might be some latitude in selecting such a position.