

AN INTRODUCTION TO STATISTICAL ENERGY ANALYSIS

1. NOTATION

Vectors are indicated by bold letters, *e.g.* **x**. Averaging is indicated ' $\langle \rangle$ ', *e.g.* $\langle v^2 \rangle$ is the spatial average of the velocity squared.

			Units
A	cross-sectional area	m^2	in^2
$C_{g,i}$	group velocity of incident wave type in plate ' i '	m/s	in/s
D	bending stiffness ($= Eh^3/12(1-\nu^2)$)	$N\ m$	$lbf\ in$
E	Young's modulus of elasticity	N/m^2	lbf/in^2
E_i	energy of subsystem i	J	$lbf\ in$
e_i	energy density	J/m^2	lbf/in
F	force	N	lbf
f	frequency	Hz	Hz
h	thickness of plate	m	in
I	moment of inertia	m^4	in^4
i	index		
j	index		
L	length of beam	m	in
l_i	length of plate junction	m	in
$M(\omega)$	modal overlap		
m_i	mass of subsystem i	kg	lb
N	number of subsystems		
N_i	number of modes in subsystem i		
ΔN	number of modes within the range $\Delta\omega$		
$n(\omega)$	modal density ($= \Delta N/\Delta\omega$)	$modes/(rad/s)$	$modes/(rad/s)$
n_i	modal density of subsystem i	$modes/(rad/s)$	$modes/(rad/s)$

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$P_{i,in}$	power input to subsystem i	W	lbf in/s
$P_{i,diss}$	power dissipated in subsystem i	W	lbf in/s
$P_{i,j}$	power transmitted from subsystem i to subsystem j	W	lbf in/s
$\mathbf{p}$	vector describing the properties of subsystem i		
p	index for mode		
q	index for mode		
S_i	area of plate subsystem ' i '	m ²	in ²
$\langle v^2 \rangle$	spatial average of velocity squared	m ² /s ²	in ² /s ²
$\mathbf{x}$	vector describing spatial location	m	in
δ	viscous damping ratio		
η	loss factor		
η_i	loss factor of subsystem i		
η_{ij}	coupling loss factor from subsystem ' i ' to subsystem ' j '		
η_{pq}	coupling loss factor from mode ' p ' to mode ' q '		
ν	Poisson's ratio		
ρ	density	kg/m ³	*lb/in ³
$\langle \tau_{ij} \rangle$	diffuse wave transmission coefficient		
ω	radian frequency ($= 2\pi f$)	rad/s	rad/s
$\Delta\omega$	bandwidth	rad/s	rad/s

* A density value expressed in British units as pound per cubic inch has to be divided by 386.4 before it can be used in this Item.

2. INTRODUCTION

Statistical Energy Analysis (SEA) is a method for estimation of noise and vibration levels in complex structures. The method has application to most sectors of engineering, including aerospace systems (aircraft and spacecraft structures), ground transportation (road and rail vehicles), civil engineering (building structures), and marine engineering (surface and sub-surface vessels). The purpose of this Data Item is to provide a concise overview of SEA, including the theoretical background to the method, its range of application, and its limitations. More detailed accounts of the method can be found in References 18 and 20, and the present state of the art is summarised in Reference 13.

The role of SEA can be described by considering the way in which the nature of the dynamic response of a structure depends upon the frequency of the excitation. Excitation sources can be classified broadly as (i) low frequency, (ii) medium frequency and (iii) high frequency. There is no absolute frequency range associated with each of these regimes, as much depends on the structure itself; however the main characteristics of the three types of excitation are the same for all structures and can be summarised briefly as follows.

(i) *Low frequency excitation*

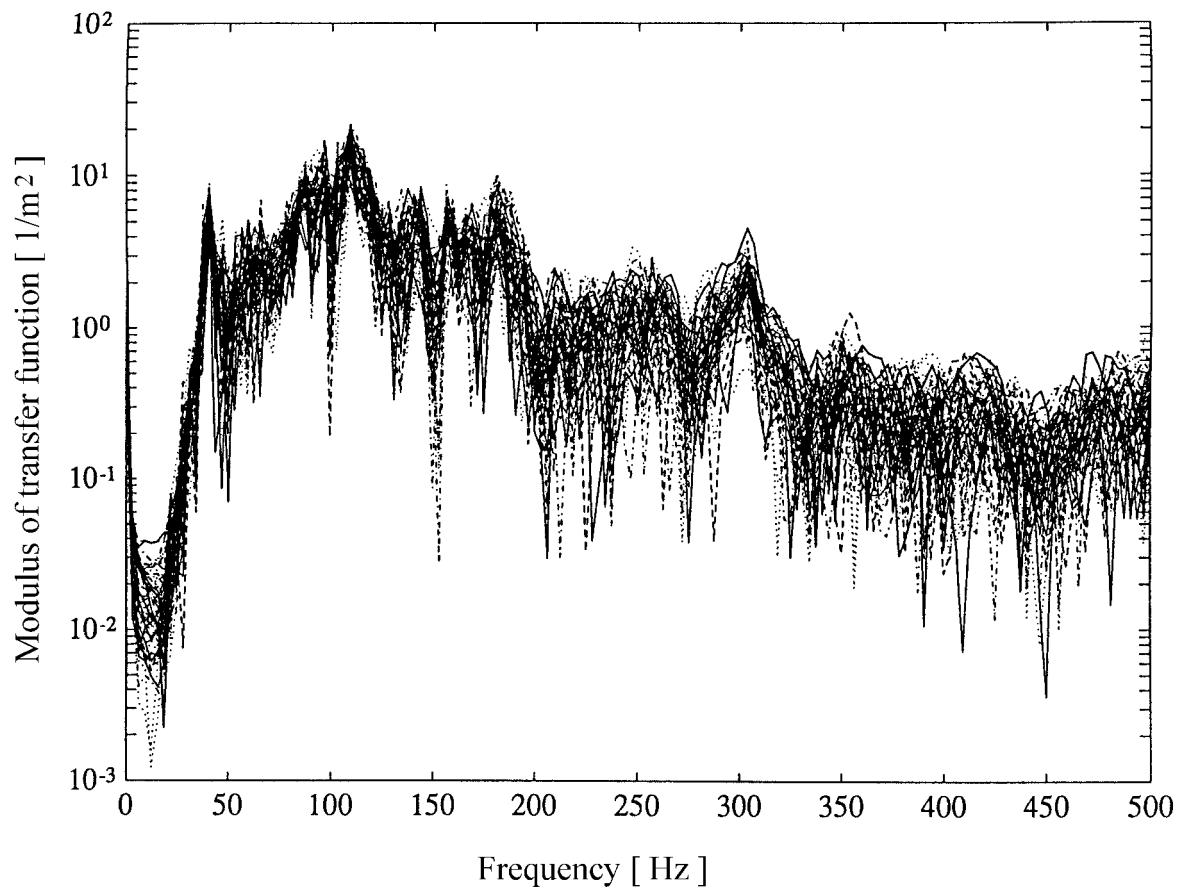
In this case the lower modes of vibration of the system are excited (up to perhaps the first ten modes) and the noise and vibration response can generally be predicted accurately by using the Finite Element Method (FEM) (Reference 4).

(ii) *Medium frequency excitation*

This involves excitation of the higher modes – these modes are less well predicted by the finite element method, and a large computational model may be required to capture in detail the relatively short wavelength of the structural deformation. In this regard, in Reference 12 it has been shown that over half a million degrees of freedom are needed to capture accurately the modes of vibration which fall below 225 Hz for a 2 m length of the fuselage of the Fokker F27 aircraft. Even with models of this size, finite element response predictions can in some cases be inaccurate due to structural uncertainties; the response of the structure can be sensitive to small fabrication details, so that two nominally identical structures may display significant modal differences – even to the point where disassembly and reassembly of the same structure can significantly alter the natural frequencies. For example, it can be deduced from Sketch 2.1 that the noise levels inside 57 nominally identical pick-up trucks can vary by more than 10 dB at medium frequencies.

(iii) *High frequency excitation*

In this case the higher modes of vibration are excited and the characteristic wavelength of the structural deformation is much less than the overall dimensions of the structure. It is generally recognised that a detailed finite element model of the structure is not feasible in this frequency range due to the excessive number of degrees of freedom that would be required to capture the short wavelength deformation. For example it has been estimated (Reference 13) that the Saturn launch vehicle possessed over 500 000 natural frequencies below 2 kHz. At higher frequencies the response tends to become less sensitive to small changes in the structural properties due to the phenomenon of modal overlap. The bandwidth of a resonant mode becomes large compared to the average modal spacing, so that the modal peaks in the frequency response function merge together and no single mode dominates the response. The response thus tends to display less (although in some case still significant) variability due to structural uncertainties than at medium frequencies.



Sketch 2.1 Modulus of transfer functions between force input at front left wheel and sound pressure level at the location of driver's head measured on 57 nominally identical vehicles*

SEA is directed primarily at predicting high frequency noise and vibration levels for which the finite element approach is not a feasible option, although the method can also be applied (with less confidence) to medium and low frequency problems. The modelling approach adopted in SEA differs fundamentally from that of the Finite Element Method (FEM); initially it can be noted that all approximate analytical methods describe the response of a system in terms of a set of degrees of freedom, and then a set of equations that govern these degrees of freedom is derived on the basis of certain physical laws. In the FEM, the degrees of freedom are the displacements and rotations of certain points on the structure (the “nodes”), and the governing equations are the equations-of-motion, which arise from a combination of Newton’s Second Law and the principles of structural mechanics. In contrast, the SEA degrees of freedom consist of the vibrational (or acoustical) energies of various regions of the system, and the governing equations are derived on the basis of the conservation of energy flow. Whereas Newton’s Second Law and the principles of structural mechanics are valid under conditions that are likely to be encountered in engineering, certain SEA hypotheses regarding the energy flow are of restricted validity, and this places a number of limitations on the method. However, the method is very efficient in terms of the required computational effort, and it can be used at medium or high frequencies where the FEM (although “theoretically” valid) is impractical or unrealistic. It is now clear that the “Energy” in Statistical Energy Analysis stems from the use of energy as a degree of freedom; the “Statistical” aspect is related to the fact that the method employs certain averaging procedures and also attempts to predict a measure of the variability of the response, as explained in the following sections.

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In what follows, Section 3 provides an overview of the SEA modelling procedure while Section 4 describes how the various parameters that appear in the equations can be found. Section 5 then describes a number of applications of SEA and includes a list of various commercial software packages that are available.

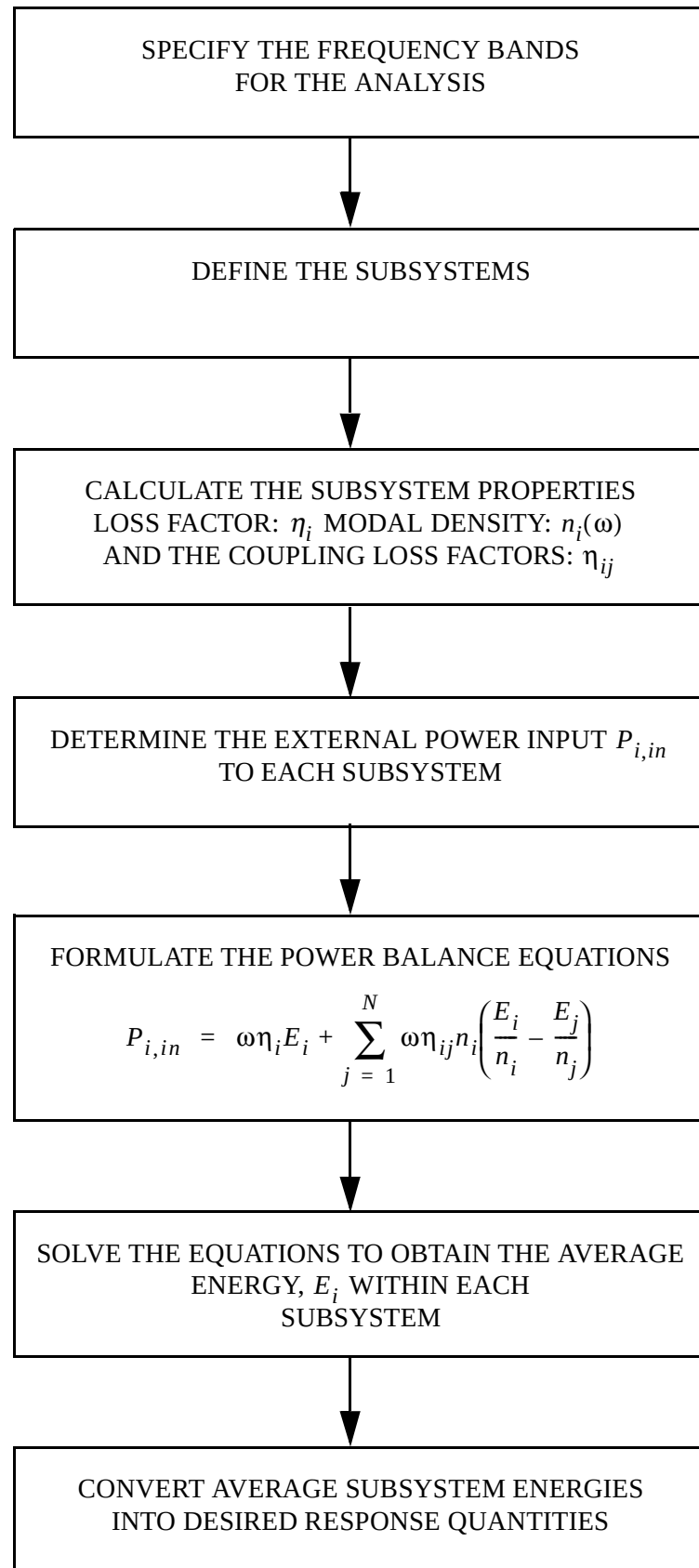
3. FUNDAMENTALS OF SEA MODELLING

3.1 Types of Averaging: Spatial, Ensemble, and Frequency

SEA does not attempt to predict the detailed spatial pattern of the response of a structure at every single frequency of excitation. Rather the method predicts the *average* response in three senses: the spatial average, the ensemble average, and the frequency average. The *spatial* average is a consequence of the fact that the method predicts the total vibrational energy stored in a region of the structure, rather than the vibrational energy density at a particular point. The ensemble average refers to an average taken over a number of apparently identical structures, *i.e.* structures that visually appear to be identical but which in reality have random dynamic properties due to manufacturing tolerances. This average is not taken explicitly, but rather it arises as a consequence of a number of the assumptions made in deriving the SEA equations, as detailed in the following sections. The *frequency* average arises from the fact that an SEA analysis is performed in frequency bands, which may be of constant bandwidth (for example 100 Hz), or may consist of 1/3 octave bands or even octave bands. The method predicts the total vibrational energy stored in each band, or equivalently the frequency band-average energy. Thus the output of an SEA analysis would be the average vibrational energy stored in various regions of the system in each of a set of specified frequency bands - an overview of the complete SEA modelling and analysis procedure is given in Sketch 3.1. The way in which the various “regions” of the structure are selected is detailed in the following subsection.

3.2 The Concept of a Subsystem

It was stated in Section 2 that the degrees of freedom adopted in SEA consist of the vibrational (or acoustical) energies of various regions of the system. To be more precise, the system is considered to be an assembly of “subsystems”, and each subsystem is assigned a single (energy) degree of freedom. The idea of a subsystem is actually a fairly abstract concept, and there are numerous ways in which a given structure can be broken down into subsystems; no one way is necessarily “correct”, although some representations will yield much better SEA models than others. So what then is a subsystem? The most simplistic answer would be to say that a subsystem is a physical component of the system: the interior volume of a car for example, or the roof, or the floor, or the passenger door assembly. The system may then be represented as an assembly of these various components. The situation can become more complicated than this however; the roof of the car might actually be represented by three subsystems – one representing out-of-plane (bending) motion, and the other two representing in-plane motion (stretching motion and shearing motion). Thus a subsystem might not represent a complete physical component, but rather just one aspect of the motion exhibited by the component – for example, bending motion or shearing motion for the car roof.



Sketch 3.1 An outline of the steps involved in SEA

To explain fully the concept of a subsystem it is necessary to consider a number of wider issues. SEA lends itself to both a *modal* view of structural dynamics (*i.e.* the response is composed of motion in the various modes of vibration of the system) and a *wave* view (*i.e.* the response is composed of elastic and acoustic waves that propagate around the system). These two views are not fundamentally different, in the sense that a mode can be considered to be a set of standing waves, and the equivalence of the two views is sometimes referred to as “wave-mode duality”. From the modal viewpoint, a subsystem is a *group of modes of vibration*; for example the complete set of modes of the car roof, or the bending modes of the car roof, or the in-plane shearing modes of the car roof. Note that the modes under consideration are those of the roof alone when decoupled from the rest of the structure, and the boundary conditions to be applied to the uncoupled system are arbitrary from the point of view of the subsystem definition. From the wave viewpoint a subsystem is a *particular type of wavefield in a particular structural component*; for example bending waves in the car roof, or shear waves in the roof, or longitudinal waves in the roof. If required, wave subsystems can be combined, so that the bending, longitudinal, and shear waves can be considered to belong to the same subsystem, namely the complete car roof, which is equivalent to the modal view of the complete set of modes of the roof.

From the foregoing discussion it is clear that there is considerable flexibility in the choice of the subsystems that make up an SEA model. There are however a number of clear guidelines as to how the subsystems should be selected:

- (i) As mentioned in Section 3.1, an SEA analysis is performed in frequency bands. For any particular band each subsystem should contain a minimum number of modes whose natural frequency falls within the band. The exact value of a “minimum number” is the subject of some debate, but the general view is that this lies somewhere between three and seven. A fundamental assumption of SEA is that the response of the structure is dominated by resonant modes, and the formulation requires each subsystem to exhibit resonant behaviour.
- (ii) There should ideally be equi-partition of vibrational energy between the modes of a subsystem. This means that each mode should contribute more or less equally to the energy of the subsystem, so that no single mode or small group of modes dominates the response. Referring back to the example of the car roof, this explains why it might be advantageous to consider the bending modes and the in-plane modes (shear and longitudinal) to be separate subsystems – these modes are very different in character and they are unlikely to support an equal amount of energy.
- (iii) The subsystems should ideally be weakly coupled. This means that if only one particular subsystem is subjected to excitation, then the response of that subsystem should be significantly greater than that of any other subsystem.

The above guidelines are phrased in terms of the modal approach to SEA. Near equivalent statements in terms of the wave approach would be: (i) the structural wavelength must be significantly less than the dimensions of the subsystem, (ii) the wavefield in a subsystem should be diffuse, so that waves propagate equally in all directions, and (iii) the wave transmission coefficient should be small at a subsystem boundary (so that most of the energy in an incident wave is reflected), or the damping must be sufficiently high to ensure that most of the energy input to a particular subsystem is dissipated within that subsystem. It should be emphasised that the wave and modal approaches usually lead to the same SEA equations and hence the same response predictions – they just represent two different physical views of SEA modelling.

The reasons for guidelines (i)-(iii) are related to various assumptions made in formulating the SEA equations, as discussed in the following sections. The three guidelines cannot always be met for every subsystem, and in some cases SEA can still yield a good response prediction if one or more of the guidelines is violated.

3.3 Properties of Subsystems: Modal Density and Modal Overlap

One of the key physical properties of a subsystem is the *modal density*, $n(\omega)$, which is defined as the average number of resonant modes per unit frequency band. Thus if the modal density at frequency ω is written as $n(\omega)$ then the expected number of resonant subsystem modes in the frequency band ω to $\omega + d\omega$ is $n(\omega)d\omega$. The evaluation of $n(\omega)$ for various types of subsystem is discussed in Section 4.3; suffice it to say here that $n(\omega)$ can in most cases be estimated with reasonable accuracy. Another important property of a subsystem is the *modal overlap factor*, $M(\omega)$, which is related to the modal density, $n(\omega)$, and the loss factor, η , in the form

$$M(\omega) = \omega\eta n(\omega). \quad (3.1)$$

The loss factor is a measure of the amount of dissipation in the subsystem (discussed more fully in Section 4.2), and it is defined so that the half power bandwidth of a resonant mode is $\omega\eta$ – this is effectively a measure of the “width” of a resonant peak when the response is plotted as a function of frequency. The modal overlap factor is thus the ratio of the width of a resonant peak to the average modal spacing; if the factor is in the region of unity or greater then the frequency response function can be expected to be a fairly smooth function of frequency, since the resonant peaks will tend to merge together.

3.4 Energy as a Variable

As has been stated previously, the degrees of freedom adopted in SEA correspond to the vibrational energies of the various subsystems. The vibrational energy is just the sum of the kinetic and potential (strain) energies associated with the motion of the subsystem, and if the subsystem exhibits resonant behaviour (which is assumed to be the case) then these two energies are equal. Thus the vibrational energy E_i of subsystem i can be considered to be twice the kinetic energy, and it can be written as

$$E_i = m_i \langle v_i^2 \rangle, \quad (3.2)$$

where m_i is the mass of the subsystem i and $\langle v_i^2 \rangle$ is the average value of the velocity squared over the subsystem i . It should be noted that Equation (3.2) is only strictly valid if the subsystem has a homogeneous mass density; in other cases the equation must be modified by averaging the density times the velocity squared, rather than just the velocity squared, over the subsystem.

It is clear from Equation (3.2) that the SEA energy variable is a definite physical entity that can in principle be measured on a real structure by measuring the velocity distribution. Conversely, if the subsystem energy has been found from an SEA analysis then the space averaged velocity (which is also ensemble and frequency averaged, as discussed in Section 3.1) can be estimated from Equation (3.2). Furthermore, if the subsystem contains a reasonable number of resonant modes then the value of the velocity squared will tend to be spatially homogeneous, and the value yielded by Equation (3.2) will provide a reasonable estimate of the velocity at any particular point. In some cases the SEA subsystem energy can also be used to recover an estimate of stress levels, as discussed in References 9 and 18.

In order to calculate the subsystem energies it is necessary to formulate a set of governing equations, and, as explained in the following sub-section, this is done on the basis of a power (or energy flow) balance for each subsystem.