



# ESDU 99015

Issued November 1999  
With Amendments A to C  
April 2005  
Re-issued September 2010

## Statistical analysis of wet runway friction for aircraft and ground-test machines

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## STATISTICAL ANALYSIS OF WET RUNWAY FRICTION FOR AIRCRAFT AND GROUND-TEST MACHINES

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## STATISTICAL ANALYSIS OF WET RUNWAY FRICTION FOR AIRCRAFT AND GROUND-TEST MACHINES

### 1. NOTATION AND DEFINITIONS

The units given in this section are a coherent set – as is required for the equations throughout the Data Item. However, where numerical values are quoted, customary British units are used for  $d_{tex}$ ,  $p$  and  $V_G$ .

#### 1.1 Notation

|                |                                                                                                     |                  |                     |
|----------------|-----------------------------------------------------------------------------------------------------|------------------|---------------------|
| $C_V$          | coefficient of variation of $\kappa$ , $C_V = \sigma[\kappa]/\bar{\kappa}$                          | –                | –                   |
| $D_G$          | equivalent mean value of aerodynamic drag for aircraft in ground roll                               | N                | lbf                 |
| $d_{tex}$      | runway surface macro-texture depth                                                                  | m                | ft                  |
| $E[ ]$         | expected or calculated value of quantity in square brackets, <i>i.e.</i> corresponds to $P_E = 0.5$ | –                | –                   |
| $G_B$          | tangential force on tyre due to braking                                                             | N                | lbf                 |
| $g$            | acceleration due to gravity                                                                         | m/s <sup>2</sup> | ft/s <sup>2</sup>   |
| $n$            | size of sample                                                                                      | –                | –                   |
| $P_E$          | probability of exceedence                                                                           | –                | –                   |
| $p$            | tyre inflation pressure                                                                             | N/m <sup>2</sup> | lbf/ft <sup>2</sup> |
| $q$            | kinetic pressure of runway surface water ( $= \rho_W V_G^2/2$ )                                     | N/m <sup>2</sup> | lbf/ft <sup>2</sup> |
| $R_{braked}$   | sum of normal (to runway) loads on braked wheels                                                    | N                | lbf                 |
| $s$            | aircraft stopping distance from speed $V_{bo}$ to stop                                              | m                | ft                  |
| $s_{dry}$      | value of $s$ in dry-runway conditions                                                               | m                | ft                  |
| $s_{wet}$      | value of $s$ in wet-runway conditions                                                               | m                | ft                  |
| $V_{bo}$       | ground speed for “brakes-on”                                                                        | m/s              | ft/s                |
| $V_G$          | ground speed                                                                                        | m/s              | ft/s                |
| $W$            | aircraft weight                                                                                     | N                | lbf                 |
| $Z$            | normal (to runway) load on wheel                                                                    | N                | lbf                 |
| $z$            | percentage point in Standard Normal Distribution                                                    | –                | –                   |
| $\beta$        | parameter defined in Equation (3.1)                                                                 | –                | –                   |
| $\Delta[ ]$    | residual about correlation of quantity in square brackets                                           | –                | –                   |
| $\kappa$       | tyre-to-wet-runway interaction parameter, $(\beta d_{tex})^{1/2}$                                   | m <sup>1/2</sup> | ft <sup>1/2</sup>   |
| $\bar{\kappa}$ | mean value of $\kappa$ in an aircraft or ground-test machine Friction Data Base                     | m <sup>1/2</sup> | ft <sup>1/2</sup>   |
| $\mu$          | coefficient of friction, $G_B/Z$                                                                    | –                | –                   |
| $\mu_{datum}$  | datum value of $\mu$ at $V_G = 0$ on dry runway                                                     | –                | –                   |

Issued November 1999 – 30 pages

Re-issued with Amendments A to C, September 2010 – 35 pages

This page Amendment A

|                  |                                                                                                   |                   |                      |
|------------------|---------------------------------------------------------------------------------------------------|-------------------|----------------------|
| $\mu_{eff}$      | effective tyre-runway coefficient of friction for aircraft (includes effects of anti-skid system) | —                 | —                    |
| $\mu_m$          | coefficient of friction for ground-test machine                                                   | —                 | —                    |
| $\rho_W$         | density of water (taken as 1000 kg/m <sup>3</sup> or 1.94 slug/ft <sup>3</sup> )                  | kg/m <sup>3</sup> | slug/ft <sup>3</sup> |
| $\sigma[\kappa]$ | standard deviation of $\kappa$                                                                    | —                 | —                    |
| $\Phi$           | Standardised Normal Cumulative Distribution Function, see Reference 4 for numerical values        | —                 | —                    |

Superscript

— denotes mean value

Subscripts

*dry* denotes value for dry conditions  
*wet* denotes value for wet conditions

1.2 Definitions

1.2.1 Ground-test machine

Any device giving a measure of runway friction conditions that is suitable for subsequent use in predicting aircraft stopping performance (includes towed devices and those that are integral with a vehicle).

1.2.2 Coefficient of friction

Reference 1 gives an introduction to tyre-runway friction. For many purposes it is acceptable to represent various measures of coefficient of friction (for both ground-test machines and aircraft) by a common symbol,  $\mu$ . However, in presenting and using the method of this Data Item it is necessary to distinguish between measurements made by ground-test machines and values deduced from aircraft tests.

**For ground-test machines,**  $\mu( \equiv \mu_m )$  represents whatever coefficient of friction is recorded by any particular machine. In general,  $\mu_m$  can be expected to be one of  
locked-wheel coefficient of friction,  
coefficient of friction at prescribed tyre “slip” condition,  
maximum coefficient of friction at optimum tyre “slip” condition,  
sideforce coefficient of friction for free rolling at prescribed tyre yaw angle.

**For aircraft,**  $\mu( \equiv \mu_{eff} )$  is the effective coefficient of friction deduced from braked ground runs and includes the effects of the aircraft anti-skid system. For the present method to be applicable it is *essential* that those runs provide values of  $\mu_{eff}$  that relate to “friction-limited” braking and not values constrained by the effects of brake torque or energy limits or the intervention of a pre-set level of “auto-braking”.

1.2.3 Runway wetness conditions

**Natural wetness:** tests conducted during or following rain.

**Simulated-natural wetness:** tests conducted following spraying of water on to extensive length of runway.

**Self-wetting:** tests conducted with ground-test machine equipped with means of depositing a prescribed flow of water (and hence depth) immediately in front of the test tyre(s).

## 2. INTRODUCTION

This Data Item provides a method for representing – and, if necessary, relating – the braking performances of aircraft and ground-test machines in wet conditions. The method is essentially statistical and implies that there is a clearly-defined level of probability that can be deduced from test runs of an aircraft (or ground-test machine) in a given set of wetness conditions on a particular runway. If the runway macro-texture depth is known, then the parameter that defines the variation of effective coefficient of braking friction in the given conditions can be readily calculated from a pre-determined *Friction Data Base*.

**Section 3** defines the model used to represent wet runway braking friction. **Section 4** describes procedures for generating a Friction Data Base for aircraft and ground-test machines and introduces examples based in existing test data. **Section 5** discusses several ways in which Friction Data Bases can be used. Within the context of this Data Item the primary uses are,

to represent measured aircraft braking performance data on a statistical basis,

to provide means of *relating* the braking performances of ground-test machines and aircraft.

**Section 6** addresses the possibilities of interpreting existing Friction Data Bases for use elsewhere. **Section 7** reviews the limitations of the method and various aspects of its implementation.

If the method is to be used to *relate* the braking performances of a given combination of aircraft and ground-test machine, the procedure comprises two distinct phases. The first is the establishment of separate *Friction Data Bases* for the aircraft and for the ground-test machine. The second is the prediction of operational braking performance of the aircraft from day-to-day measurements from the ground-test machine.

ESDU 99016 and ESDU 99017 (References 7 and 8) provide two examples of the derivation of Friction Data Bases for individual aircraft. The former of these considers an aircraft for which an extensive set of test data is available; the latter considers an aircraft for which a limited set of test data is available. Reference 9 provides a detailed example for a ground-test machine.

See Reference 42 for methods appropriate to runways contaminated with significant depths of water, slush, snow or ice. Reference 3 gives a general introduction to many aspects of aircraft ground performance.

## 3. FRICTION DATA BASE: DEFINITION AND BASIS

### 3.1 Definition

A Friction Data Base is a Probability Model that describes the behaviour of a braking system on wet runways. It is completely defined by the values of,

a datum value of friction coefficient,  $\mu_{datum}$ ,

the mean value,  $\bar{\kappa}$ , of a tyre-to-wet-runway interaction parameter,  $\kappa$ ,

the standard deviation,  $\sigma[\kappa]$ , of  $\kappa$ .

## 3.2 Representation of Wet Runway Friction Data

For any aircraft or ground-test machine, the variation of  $\mu$  with speed on a single wet runway can be represented by the simple expression

$$\mu = \frac{\mu_{datum}}{1 + \beta \frac{q}{p}} \quad (3.1)$$

This relationship is developed in Appendix A from the “three-zone” model of the tyre-runway contact area and is shown in Reference 6 to represent in a satisfactory manner measured data for a wide range of aircraft ( $\mu \equiv \mu_{eff}$ ) and ground-test machines ( $\mu \equiv \mu_m$ ), provided that

braking force is limited solely by friction,

runway water depths are small ( $\leq 3$  mm) so that no significant retarding force is developed due to displacement of water by the tyre.

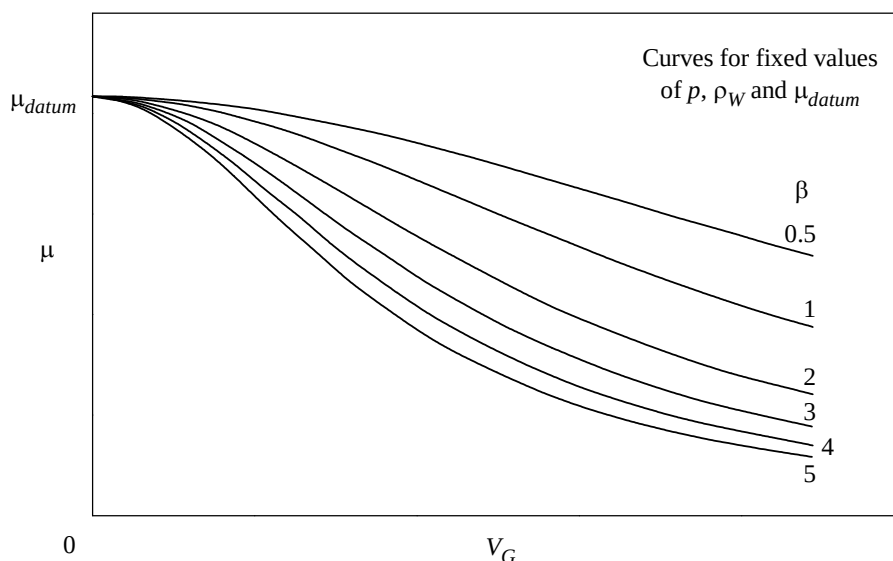
The form of the relationship represented by Equation (3.1) is illustrated in Sketch 3.1 for a typical range of values of the parameter  $\beta$ .

Provided that sufficiently comprehensive test data are available for a particular aircraft or ground-test machine, each value of the parameter  $\beta$  can be combined with the macro-texture depth,  $d_{tex}$ , of the wetted runway to define a variable  $\kappa$  such that

$$\kappa = (\beta d_{tex})^{1/2} \quad (3.2)$$

conforms to the Normal Distribution, with mean value  $\bar{\kappa}$  and standard deviation  $\sigma[\kappa]$ . This variable is a characteristic of the tyres, wheels and brakes for the type of aircraft or ground-test machine.

**The values of  $\mu_{datum}$ ,  $\bar{\kappa}$  and  $\sigma[\kappa]$  constitute the Friction Data Base for the aircraft or ground-test machine.**



**Sketch 3.1 Examples of curves generated from Equation (3.1) representing variation of  $\mu$  with ground speed**

## 4. CONSTRUCTION OF FRICTION DATA BASE FROM MEASURED DATA

In practice, the construction of a Friction Data Base for an aircraft or ground-test machine translates into the need to establish a value for  $\mu_{datum}$  from tests on dry runways and to determine the distribution of  $\kappa$  (that is values of  $\bar{\kappa}$  and  $\sigma[\kappa]$ ) from tests on wet runways. The overall procedure is most readily understood by reference to the schematic presentation in Figure 1. Uses of Friction Data Bases are described in Section 5.

### 4.1 Measured Data for $\mu$

Sections 4.2 and 4.3 deal with the analysis of test data obtained on dry and wet runways respectively, presupposing the recording of sufficient information, as follows.

#### 4.1.1 Aircraft

Values of  $\mu_{eff}$  can be deduced from any braked ground run during which the variations of accelerations and ground speed with time are recorded together with sufficient other parameters to permit solution of the longitudinal equation of motion. Alternatively, in some instances, aircraft braking performance is represented in terms of the parameter  $s_{wet}/s_{dry}$ . Section 4.5 shows that such data can be related to the scheme used in the current method.

There is no need for the data to be recorded concurrently with ground-test machine readings.

#### 4.1.2 Ground-test machines

For most ground-test machines the values of coefficient of friction,  $\mu_m$ , are available as a direct output so that the only additional information required is ground speed and tyre inflation pressure.

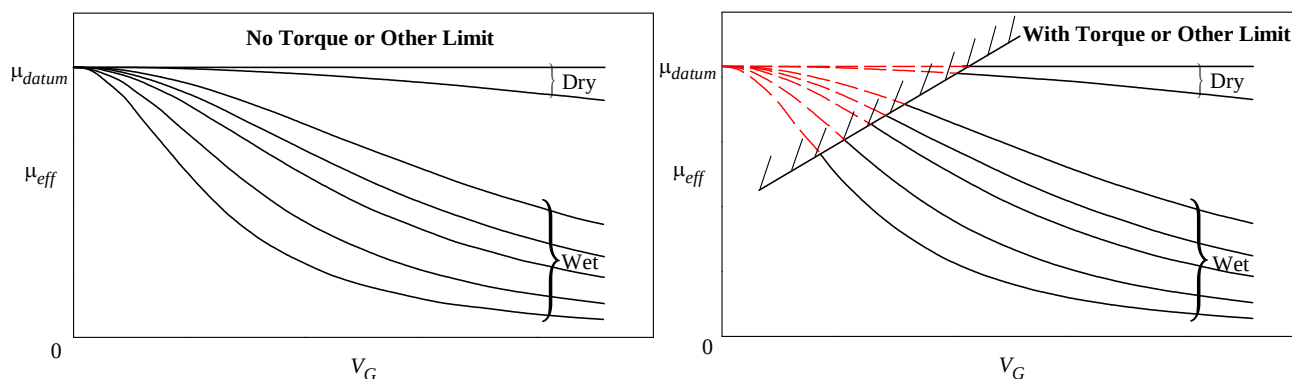
### 4.2 Selection of Value for $\mu_{datum}$ (Dry Runway)

Datum friction coefficient is that value of effective coefficient of friction that would be present on a *dry runway* with ground speed tending to zero. In the majority of cases considered when deriving and validating the method, there is no evidence of a variation of effective braking friction coefficient with speed. However, it is possible that a braking system could exhibit such a tendency. In that case, the datum value could be chosen in such a manner that it would not be achievable in practice. This consequence is not a disadvantage because, although the rationale of Appendix A for the choice of characteristic equation stems from elementary physics, the method is wholly statistical so a primary aim is *to fit the experimental data*.

Sketch 4.1 illustrates the principles involved in selecting a value of  $\mu_{datum}$  for an idealised case where the measured data are not subject to torque or other limits and for a case where a torque limit has significant effects on both dry and wet runway data.

If tests are performed on more than one runway, then it will be possible to confirm that  $\mu_{datum}$  is independent of  $d_{tex}$ . However, there is sufficient experimental evidence to show that runway surface macro-texture depth has no significant effect on friction coefficient in dry conditions, so that such additional tests are not essential.

In general the dry-runway tests needed to provide the required information will be part of those carried out in the normal course of aircraft certification work.



**Sketch 4.1** Selection of value of  $\mu_{datum}$  for cases without and with torque (or other) limit

## 4.3 Derivation of Values of $\bar{\kappa}$ and $\sigma[\kappa]$ (Wet Runway)

For wet conditions it is very desirable (though not essential) for tests to be conducted on runways having a range of values of macro-texture depth,  $d_{tex}$ , and measured values of  $d_{tex}$  for each runway used are required. It is *essential* that a representative range of natural and/or simulated-natural wetness conditions is used and that the same level of braking effort (usually, the maximum) is applied as in the dry-runway tests.

To obtain a value of  $\beta$  for each braked run on each wet runway, Equation (3.1) is applied to all the test results for that run until a “best-fit” value of  $\beta$  is found. Several different mathematical strategies can be adopted to deduce that “best-fit” – the method used in the example of Data Item No. 99016 (Reference 7) is as follows.

- (i) Calculate values of  $q/p$  for each data point.
- (ii) Guess a value of  $\beta$  and use Equation (3.1) to calculate corresponding expected values,  $E[\mu_{eff}]$ , for each data point. Guidance as to the broad relationship between the desired curve shape and an approximate value of  $\beta$  is given in Sketch 3.1.
- (iii) Calculate the difference,  $\Delta[\mu_{eff}]$ , between  $E[\mu_{eff}]$  and  $\mu_{eff}$  for each data point.
- (iv) Calculate the sum,  $\Sigma[\Delta[\mu_{eff}]]$ , of the differences between the expected values and the measured values for all the points in the run.
- (v) Repeat the sequence in steps (ii) to (iv) until a value of  $\beta$  is found so that the sum of the differences (*i.e.*  $\Sigma[\Delta[\mu_{eff}]]$ ) is sufficiently close<sup>†</sup> to zero.
- (vi) The final, “best-fit” value of  $\beta$  is used to calculate the corresponding value of  $\kappa$  using Equation (3.2). Consistent units must be used for  $d_{tex}$  and  $\kappa$ .
- (vii) The expected (or mean) value,  $E[\kappa]$  (or  $\bar{\kappa}$ ), and its standard deviation,  $\sigma[\kappa]$ , are calculated from the data obtained in step (vi) using the definitions of these two statistics given in Sections 2.4 and 2.5 of ESDU 91017 (Reference 4).

<sup>†</sup> In the example in ESDU 99016 (Reference 7), “sufficiently close”, has been taken to imply

$$\left| \Sigma[\Delta[\mu_{eff}]] \right| < 0.00005.$$

## 4.4 Expansion of Friction Data Base for Specified Probability of Exceedence

Once the Friction Data Base is derived in the form of values of  $\mu_{datum}$ ,  $\bar{\kappa}$  and  $\sigma[\kappa]$ , it may be expanded into any desired form at any specified probability of exceedence,  $P_E$ . For a specified value of  $P_E$ , the corresponding value of the standardised Normal Cumulative Distribution Function,  $\Phi$ , is deduced from

$$\Phi = 1 - P_E. \quad (4.1)$$

The values of  $\kappa$  that are exceeded at the chosen probability of exceedence are calculated from

$$\kappa = E[\kappa] + z\sigma[\kappa] \equiv \bar{\kappa} + z\sigma[\kappa], \quad (4.2)$$

where  $z$  is the value of the percentage point of the Normal Distribution corresponding to the value of  $\Phi$  from Equation (4.1). Values of  $z$  and  $\Phi$  are given in the relevant Table of ESDU 91017 (Reference 4).

The equation for  $\beta$  is obtained by rearranging Equation (3.2) so that

$$\beta = \frac{\kappa^2}{d_{tex}}. \quad (4.3)$$

Hence the values of  $\beta$  for various values of texture depth, at the chosen levels of probability, can be calculated from

$$\beta = \frac{(\bar{\kappa} + z\sigma[\kappa])^2}{d_{tex}} \quad (4.4)$$

and the corresponding values of  $\mu$  calculated using Equation (3.1).

## 4.5 Use of Aircraft Stopping Distance Measurements to Derive Friction Data Bases

For some aircraft tests the available data are given as values of aircraft weight, braked stopping distance and “brakes-on” (ground) speed. However, many aircraft generate significant aerodynamic lift and drag in the ground-roll attitude – particularly if spoilers are not fitted or not deployed. Consequently, to obtain useful braking friction data from such measurements, it is necessary to account for the effects of those aerodynamic forces on the wheel normal reactions and on the overall decelerating force. If it is assumed that engine residual thrust is negligible, the tyre-runway braking coefficient of friction for the aircraft at any instant can be expressed as

$$\mu = \frac{[\text{resultant longitudinal force on aircraft}] - [\text{aerodynamic drag}]}{\text{normal (to runway) reaction on braked wheels}}. \quad (4.5)$$

If, as is often the case, the deceleration in a braked ground run can be assumed to be constant, then mean values can replace the quantities in Equation (4.5) to give

$$\mu_{eff} = \frac{(WV_{bo}^2 / 2gs) - D_G}{\bar{R}_{braked}}. \quad (4.6)$$

## 4.5.1 Use of dry-runway stopping distance measurements to derive $\mu_{datum}$

For dry-runway test data, Equation (4.6) can be used to determine a value of  $\mu_{eff}$  for each test run and the mean of these values taken as the value of  $\mu_{datum}$ . Stopping distances in dry conditions for any combination of values of  $W$  and  $V_{bo}$  can then be calculated from

$$s_{dry} = \frac{WV_{bo}^2 / 2g}{(\mu_{datum} \bar{R}_{braked}) + D_G} \quad (4.7)$$

## 4.5.2 Use of wet-runway stopping distance measurements to derive $\beta$

An expression for the parameter  $\beta$  is derived here for use with wet-runway tests giving values of the stopping distance,  $s_{wet}$ .

In the case of a wet-runway test, Equation (4.6) becomes

$$\mu_{eff} = \frac{(WV_{bo}^2 / 2gs_{wet}) - D_G}{\bar{R}_{braked}} \quad (4.8)$$

Substituting for  $\mu_{eff}$  from Equation (3.1) and rearranging gives,

$$\frac{WV_{bo}^2}{2g} = \left\{ \left( \frac{\mu_{datum}}{1 + \beta \frac{q}{p}} \right) \bar{R}_{braked} + D_G \right\} s_{wet} \quad (4.9)$$

From Equation (4.7), the corresponding expression (to Equation (4.9)) for the stopping distance,  $s_{dry}$ , in dry conditions for the same values of  $W$  and  $V_{bo}$  is

$$\frac{WV_{bo}^2}{2g} = [(\mu_{datum} \bar{R}_{braked}) + D_G] s_{dry} \quad (4.10)$$

Equating the expressions for  $WV_{bo}^2 / 2g$  in Equations (4.9) and (4.10) gives the required expression for  $\beta$  as

$$\beta = \frac{p}{q} \left\{ \frac{s_{wet} / s_{dry}}{1 - \frac{D_G[(s_{wet} / s_{dry}) - 1]}{\mu_{datum} \bar{R}_{braked}}} - 1 \right\} \quad (4.11)$$

Once values of  $\beta$  have been obtained from Equation (4.11), the procedures of Section 4.3 (steps (vi) and (vii)) and Section 4.4 are followed to derive the Friction Data Base.

## 4.6 Examples of Friction Data Bases for Aircraft and Ground-Test Machines Operating in Natural and Simulated-Natural Wetness Conditions

The procedures described in Sections 4.1 to 4.5 have been applied to measured data for a number of aircraft and ground-test machines operating in natural and simulated-natural wetness conditions (see References 6 and 7 to 9). The resulting values of  $\mu_{datum}$ ,  $\bar{\kappa}$  and  $\sigma[\kappa]$  are summarised here in Table 9.1. For two of the aircraft listed in Table 9.1, the detailed derivations of those values are presented in ESDU 99016 and ESDU 99017 (References 7 and 8). Reference 9 presents the detailed derivation of those parameters for a ground-test machine.

The results presented in Table 9.1 (and the underlying analysis in References 6, 7 and 8) demonstrate that, for the sets of test data considered for each ground-test machine or aircraft, the parameter  $\beta$  can be used, together with texture depth,  $d_{tex}$ , to define a random variable,  $\kappa$ , that describes the interaction of that machine or aircraft with a range of runway surfaces, where

$$\kappa = (\beta d_{tex})^{1/2} \quad (4.12)$$

and has been shown to be Normally Distributed for all the cases investigated.