



# ESDU 99016

Issued June 1999  
With Amendment A  
November 2000  
Re-issued September 2010

## Example of statistical analysis of wet runway friction: aircraft with extensive set of test data

Endorsed by  
The Royal Aeronautical Society

 **Documents**  
Document Sharing Hub

## ESDU DATA ITEMS

Data Items provide validated information in engineering design and analysis for use by, or under the supervision of, professionally qualified engineers. The data are founded on an evaluation of all the relevant information, both published and unpublished, and are invariably supported by original work of ESDU staff engineers or consultants. The whole process is subject to independent review for which crucial support is provided by industrial companies, government research laboratories, universities and others from around the world through the participation of some of their leading experts on ESDU Technical Committees. This process ensures that the results of much valuable work (theoretical, experimental and operational), which may not be widely available or in a readily usable form, can be communicated concisely and accurately to the engineering community.

We are constantly striving to develop new work and review data already issued. Any comments arising out of your use of our data, or any suggestions for new topics or information that might lead to improvements, will help us to provide a better service.

## THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item was monitored and guided by the Performance Committee. This Committee first met in 1946 and now has the following membership:

|                    |                                                                   |
|--------------------|-------------------------------------------------------------------|
| Chairman           |                                                                   |
| Mr K J Balkwill    | — Independent                                                     |
| Members            |                                                                   |
| Mr P V Aidala*     | — Northrop Grumman ESID, Bethpage, New York, USA                  |
| Mr T Bartup        | — Avro International Aerospace, Woodford                          |
| Mr M Broad         | — Independent                                                     |
| Mr G M J Davis     | — Defence Evaluation and Research Agency, Boscombe Down           |
| Mr N J Herniman    | — Airbus Industrie, Toulouse, France                              |
| Mr R G Humpston    | — Rolls-Royce plc, Aero Division, Derby                           |
| Mr T S R Jordan    | — Independent                                                     |
| Mr R J Orlowski*   | — Boeing Commercial Airplane Company, Seattle, Washington, USA    |
| Dr P Render        | — University of Loughborough                                      |
| Mr D N Sinton      | — Independent                                                     |
| Mr G J R Skillen   | — Civil Aviation Authority, Safety Regulation Group, Gatwick      |
| Mr G E Smith       | — Independent                                                     |
| Mr A Stanbrook     | — Independent                                                     |
| Mr R Storey        | — British Aerospace, Military Aircraft and Aerostructures, Brough |
| Prof. E Torenbeek* | — Delft University of Technology, The Netherlands                 |
| Mr Y D Traeger*    | — Israel Aircraft Industries, Jerusalem, Israel                   |
| Mr C J Turner*     | — Boeing, Douglas Products Group, Long Beach, Calif., USA         |
| Mr M Wilson        | — Britten-Norman Ltd, Isle of Wight                               |
| Mr R D Younger*    | — Raytheon Aircraft Company, Wichita, Kansas, USA                 |

\* Corresponding Member

The construction and subsequent development of the Data Item were undertaken by

Mr K J Balkwill — under contract to ESDU.

The person with overall responsibility for the work in this subject area is Mr D.J. Mitchell, Head of Performance Group.

EXAMPLE OF STATISTICAL ANALYSIS OF WET RUNWAY FRICTION:  
AIRCRAFT WITH EXTENSIVE SET OF TEST DATA

CONTENTS

|                                                                             | Page  |
|-----------------------------------------------------------------------------|-------|
| 1. NOTATION, UNITS AND DEFINITION                                           | 1     |
| 1.1 Notation                                                                | 1     |
| 1.2 Definition                                                              | 2     |
| 2. INTRODUCTION                                                             | 2     |
| 3. DESCRIPTION OF AIRCRAFT, TESTS AND AVAILABLE DATA                        | 2     |
| 4. ANALYSIS                                                                 | 3     |
| 4.1 Dry Runway Data                                                         | 3     |
| 4.2 Wet Runway Data                                                         | 4     |
| 4.3 Graphical Representation of Friction Data Base for the Example Aircraft | 7     |
| 4.4 Concluding Remarks                                                      | 7     |
| 5. REFERENCES                                                               | 9     |
| FIGURES 1 to 5                                                              | 10-12 |

## EXAMPLE OF STATISTICAL ANALYSIS OF WET RUNWAY FRICTION: AIRCRAFT WITH EXTENSIVE SET OF TEST DATA

### 1. NOTATION, UNITS AND DEFINITION

The units given in this section are coherent sets – as are required for equations throughout the Data Item. However, where numerical values are quoted, customary British units are used for  $d_{tex}$ ,  $p$ ,  $q$  and  $V_G$ .

#### 1.1 Notation

|                |                                                                                                     |                   |                      |
|----------------|-----------------------------------------------------------------------------------------------------|-------------------|----------------------|
| $d_{tex}$      | runway surface macro-texture depth                                                                  | m                 | ft                   |
| $E[ \ ]$       | expected or calculated value of quantity in square brackets, <i>i.e.</i> corresponds to $P_E = 0.5$ | –                 | –                    |
| $G_B$          | tangential force on tyre due to braking                                                             | N                 | lbf                  |
| $n$            | size of sample                                                                                      | –                 | –                    |
| $P_E$          | probability of exceedence                                                                           | –                 | –                    |
| $p$            | tyre inflation pressure                                                                             | N/m <sup>2</sup>  | lbf/ft <sup>2</sup>  |
| $q$            | kinetic pressure of runway surface water ( $= \rho_W V_G^2 / 2$ )                                   | N/m <sup>2</sup>  | lbf/ft <sup>2</sup>  |
| $V_G$          | ground speed                                                                                        | m/s               | ft/s                 |
| $Z$            | normal (to runway) load on wheel                                                                    | N                 | lbf                  |
| $z$            | percentage point in Standard Normal Distribution                                                    | –                 | –                    |
| $\beta$        | parameter defined in Equation (4.1)                                                                 | –                 | –                    |
| $\Delta[ \ ]$  | residual about correlation of quantity in square brackets                                           | –                 | –                    |
| $\kappa$       | tyre-to-wet-runway interaction parameter, $(\beta d_{tex})^{1/2}$                                   | m <sup>1/2</sup>  | ft <sup>1/2</sup>    |
| $\bar{\kappa}$ | mean value of $\kappa$                                                                              | m <sup>1/2</sup>  | ft <sup>1/2</sup>    |
| $\mu$          | coefficient of friction, $G_B / Z$                                                                  | –                 | –                    |
| $\mu_{datum}$  | datum value of $\mu$ at $V_G = 0$ on dry runway                                                     | –                 | –                    |
| $\mu_{eff}$    | effective tyre-runway coefficient of friction for aircraft (includes effects of anti-skid system)   | –                 | –                    |
| $\rho_W$       | density of water (taken as 1000 kg/m <sup>3</sup> or 1.94 slug/ft <sup>3</sup> )                    | kg/m <sup>3</sup> | slug/ft <sup>3</sup> |
| $\sigma[ \ ]$  | standard deviation of quantity in square brackets                                                   | –                 | –                    |
| $\Phi$         | Standardised Normal Cumulative Distribution Function (see Reference 2 for numerical values)         | –                 | –                    |

Issued June 1999 – 12 pages

Re-issued with Amendment A, September 2010 – 12 pages

This page Amendment A

## 1.2 Definition

**For aircraft**,  $\mu$  ( $\equiv \mu_{eff}$ ) is the effective coefficient of friction deduced from braked ground runs and includes the effects of the aircraft anti-skid system. For the present method to be applicable it is *essential* that those runs provide values of  $\mu_{eff}$  that relate to “friction-limited” braking and not values constrained by the effects of brake torque or energy limits or the intervention of a pre-set level of “auto-braking”.

## 2. INTRODUCTION

This Data Item is the first of a series of examples of the application of a method for representing – and, if necessary, relating – the braking performances of aircraft and ground-test machines in wet conditions. The method, which is given in Reference 4, is essentially statistical and implies that there is a clearly-defined level of probability that can be deduced from test runs of an aircraft (or ground-test machine) in a given set of wetness conditions on a particular runway. If the runway macro-texture depth is known, then the parameter that defines the variation of effective coefficient of braking friction in the given conditions can be readily calculated from a pre-determined *Friction Data Base*. The concept of a Friction Data Base is explained in Reference 4.

Test data, taken from Reference 1 for a Boeing 727-100QC aircraft, are used to illustrate, by example, the process of establishing a Friction Data Base for that aircraft. The background to those tests is described in Section 3. The method of analysis is summarised and results are presented in Section 4.

A second example, for an aircraft where only a limited data set is available, is given in Reference 5. An example of the application of the method to ground-test machine data is given in Reference 6. Many other examples are considered in Reference 3.

If the method of Reference 4 is to be used to *relate* the braking performances of a given combination of aircraft and ground-test machine, the procedure is comprised of two distinct phases. The first is the establishment of separate *Friction Data Bases* for the aircraft and for the ground-test machine. The second is the prediction of operational braking performance of the aircraft using day-to-day measurements from the ground-test machine.

## 3. DESCRIPTION OF AIRCRAFT, TESTS AND AVAILABLE DATA

An extensive series of runway friction trials on various aircraft and ground test machines is reported in Reference 1; data for this example are taken directly from that document. The aircraft used in the part of the trial considered here was a Boeing 727-100QC fitted with 32×1.5-15, 12 PR, type VII aircraft tyres on the dual-wheel nose-undercarriage and 49×17, 26PR, type VII aircraft tyres on the dual-wheel main-undercarriages. Tyres on the main wheels were inflated to  $145 \pm 5$  lbf/in<sup>2</sup> and the nose-wheel tyres were inflated to  $100 \pm 5$  lbf/in<sup>2</sup>. All test runs utilised *maximum braking* and, although the test programme included some runs for which the nose-wheel was braked, none of those runs is considered here. Before the test programme started, the components of the anti-skid system were returned to the manufacturers for complete refurbishment; new wheel-brake units and new tyres were fitted. During the course of the test programme, tyre-wear was monitored and both tyres on a particular leg were replaced when one tyre was noted to have worn by 50%.

Sufficient instrumentation was fitted to enable the equations of motion to be solved for the variation of the effective coefficient of braking friction,  $\mu_{eff}$ , with ground speed,  $V_G$ , for each test run. This solution took into account the variation of drag force, pitching moment and lift force with speed and angle of attack. Thus, the values of  $\mu_{eff}$  obtained were *free from all aerodynamic effects*. (It is assumed that any effects of runway slope and/or wind component along the runway had also been accounted for.)

Tests were conducted at three centres: NASA Wallops, FAA Technical Center and Brunswick Naval Air Station. The runway surfaces ranged from smooth concrete to grooved asphalt and the values of surface macro-texture depth varied from 0.006 to 0.072 inches (0.15 to 1.8 mm approx.) – see Table 4.3. The wetness conditions varied from *dry* (the datum condition) through *damp* to *truck wet* which probably represents a condition intermediate between heavy rain and flooded. The expression *truck wet* is used to indicate that the runway wetting was carried out using water tankers. A total of twenty seven separate wetted runs and eight dry runs has been analysed.

## 4. ANALYSIS

In this Section, the method derived in Reference 4 is summarised and the variables that are used to create the *Friction Data Base* for the “example” aircraft are calculated. The plausibility of the simple modelling is demonstrated and the statistical distributions of the variables in the model are given.

For a single braked run on a wet runway with braking force *limited solely by friction*, it is shown in Reference 4 that the variation of  $\mu_{eff}$  with speed can be represented by the simple expression

$$\mu_{eff} = \frac{\mu_{datum}}{1 + \beta \frac{q}{p}} \quad (4.1)$$

Provided that a sufficiently comprehensive test<sup>†</sup> series is available, it is shown further in Reference 4 that each value of the parameter  $\beta$  can be combined with the macro-texture depth,  $d_{tex}$ , of the wetted runway to define a variable  $\kappa$  such that

$$\kappa = (\beta d_{tex})^{1/2} \quad (4.2)$$

conforms to the Normal Distribution, with mean value  $\bar{\kappa}$  and standard deviation  $\sigma[\kappa]$ . This variable is a characteristic of the tyres, wheels and brakes for the type of aircraft. The parameter  $\kappa$ , together with  $\mu_{datum}$ , is used to create the *Friction Data Base* for the aircraft. The objective of the analysis is, therefore, to establish a value for  $\mu_{datum}$  from tests on dry runways and to determine the distribution of  $\kappa$  from tests on wet runways.

### 4.1 Dry Runway Data

Eight accelerate-stops were performed over the range of dry runways and values of the effective coefficient of braking friction were calculated at several speeds in the braked part of each run. A total of 81 test points was derived and these are shown plotted against texture depth in Figure 1 and plotted against ground speed in Figure 2 (note, in each Figure, some points are coincident). Inspection shows that there is no significant effect of speed or texture depth on the dry runway braking friction coefficient. The mean value ( $\bar{\mu}_{eff,dry} = 0.4543$ ) for the whole set is therefore taken to be applicable as the datum value,  $\mu_{datum}$ , for this aircraft on dry runways. Note that the standard deviation of the data about the mean value is  $\sigma[\mu_{eff,dry}] = 0.0366$ . The standard error of the mean value (see Section 3.2.2 of ESDU 91017 (Reference 2)) is given by

$$\sigma[\bar{\mu}_{eff,dry}] = \frac{\sigma[\mu_{eff,dry}]}{n^{1/2}} = \frac{0.0366}{81^{1/2}} = 0.004067 \equiv \sigma[\mu_{datum}] \quad (4.3)$$

<sup>†</sup> The criterion for sufficiency is related to the range of wetting conditions and to the number of tests in the series. It is advisable to consult Reference 4 to find advice on test planning.