

Transonic data memorandum

Constrained multivariate optimisation techniques for the design of aerofoil sections

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CONSTRAINED MULTIVARIATE OPTIMISATION TECHNIQUES FOR THE DESIGN OF AEROFOIL SECTIONS

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CONSTRAINED MULTIVARIATE OPTIMISATION TECHNIQUES FOR THE DESIGN OF AEROFOIL SECTIONS

1. INTRODUCTION

Many varied methods exist for the derivation of optimum aerofoil shapes by the combined use of numerical optimisation techniques (*optimisers*^{*}) and Computational Fluid Dynamics (CFD). Some are capable of optimisation at more than one design point (*multi-point design*). This Data Item deals with the principles of the technique in general and, for illustration, the application of one such multi-point method (CODAS) in particular.

Section 3 gives an overall view of the optimisation design method. Section 4 presents a general description of constrained multivariate optimisation. Section 5 deals with some aspects of geometry definition and Section 6 discusses CFD methods involved. A brief survey of current optimisation methods is given in Appendix A. Various topographical features which might be met in design surfaces are described in Appendix B.

References 12, 13, 14, 15, 16 and 17 provide illustrations of various aspects of the constrained multivariate optimisation process. In this Data Item the description of the general process is in unboxed text, whereas reference to specific methods used by CODAS and individual examples from CODAS **appear in boxed text**.

2. NOTATION[†]

a_i, b_i	lower and upper bounds of constraint function $c_i(X)$
C_D	drag coefficient
C_L	lift coefficient
$C_{Ltarget}$	required lift coefficient
$c_i(X)$	constraint functions ($i = 1, \dots, n$)
$f(X)$	objective function, <i>i.e.</i> function to be optimised
$f'(X)$	objective function revised to include constraint penalty functions, see Equation (4.7)
i	numbers identifying constraint functions
j	numbers identifying independent variables
k	number of independent variables
M	Mach number
l, m, n	numbers defining the number of equality, single inequality, and double inequality constraints, see Equations (4.2), (4.3) and (4.4)
t/c	maximum value of aerofoil thickness/chord ratio

* In this Data Item, the term 'optimiser' is used to describe any software capable of solving constrained multivariate optimisation problems.

† This Data Item departs from the usual practice of providing a comprehensive Notation and Units Section. This is done since the only equations quoted are used in general terms (*i.e.* not dimensioned) or are non-dimensional.

$p(X)$	penalty function, see Equation (4.6)
X	vector of independent variables, $(x_1, \dots, x_k)$
x	chordwise coordinate
x_j	independent variable, $j = 1, \dots, k$
z	normal coordinate
$z_L(x)$	normal coordinate of lower surface of aerofoil at x
$z_U(x)$	normal coordinate of upper surface of aerofoil at x
α	angle of incidence
η_i	weighting factor for constraint i in Equation (4.6)
ρ_0	leading edge radius
Superscripts	
L	denotes lower bound
U	denotes upper bound

3. OPTIMISATION DESIGN METHOD

3.1 Main Modules

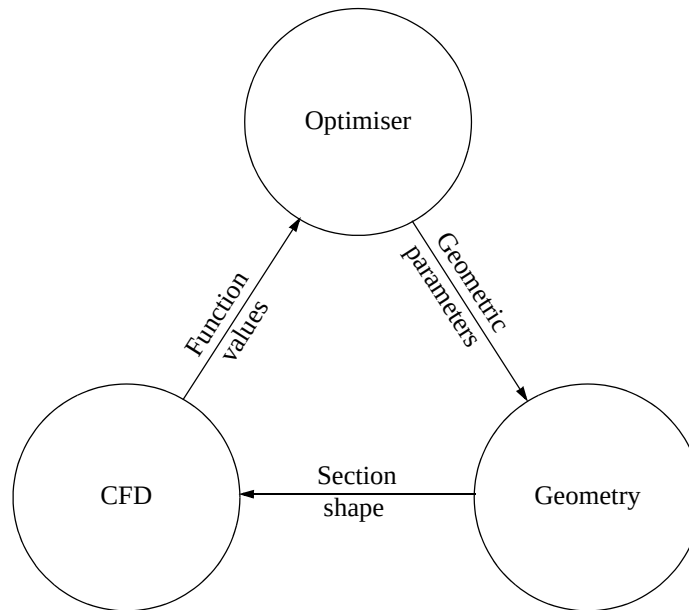
In broad terms optimisation design consists of linking three main elements (Sketch 3.1),

- an optimiser to select, at each step in its search for the optimum, values for the independent variables, such as the geometric parameters which modify the aerofoil section,
- a geometry module to calculate the new section profile using these parameters, and
- a CFD module to calculate the flow field around the section and to determine the value of the objective function (*i.e.* the function which is to be minimised) together with any variables which are subject to constraints. Note that as many flow fields have to be calculated as there are design points.

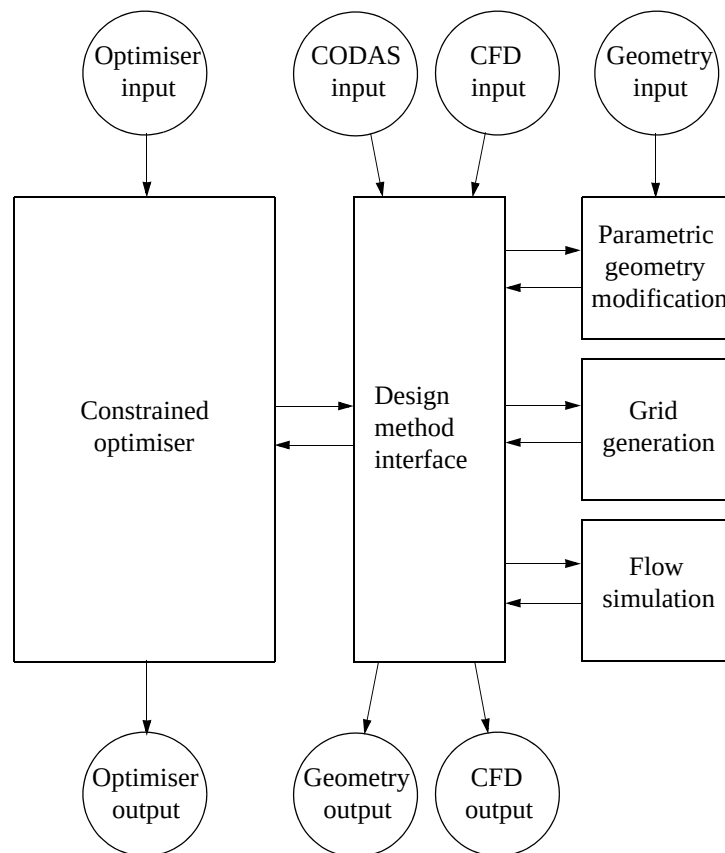
3.2 Design Method Interface

In practice, a further module is required to provide overall control of the process. This module controls input and output of design data, transfers data between the modules, and provides a means of monitoring progress of the design process. It acts as a front end to the optimisation algorithm, so that the design problem is properly posed. The existence of this interface module allows modified or new geometry, grid generation, or flow solution procedures to be incorporated easily.

The particular program used to produce the examples in References 12, 13, 14, 15, 16 and 17 is CODAS (Constrained Optimisation Design of Aerodynamic Shapes) developed at DERA, Farnborough. This uses the constrained multivariate optimisation program RQPMIN Version 3.1 (Reference 8), also developed at DERA, Farnborough. The CFD methods available within the two-dimensional version of the CODAS program are a viscous-coupled full-potential code (BVGK, Reference 3) and an inviscid Euler code (Reference 2). The overall structure of the optimisation design method is, then, of the form shown in Sketch 3.2.



Sketch 3.1 The main elements of the method



Sketch 3.2 The structure of CODAS

4. OPTIMISER PROGRAMS

Constrained multivariate optimisation is used to solve the problem of optimising a function (the *objective function*) subject to the variables lying between simple upper and lower bounds and satisfying user-specified constraints (the *constraint functions*). Objective and constraint functions can be any functions which can be extracted from the geometric model or CFD solution. Examples include t/c , ρ_0/c , C_L , C_D and MC_L/C_D .

4.1 General Description

Mathematically, the problem to be solved can be stated as follows.

$$\text{Minimise}^* \text{ the objective function, } f(\mathbf{X}), \quad (4.1)$$

where $\mathbf{X}$ is the vector of the independent variables, $(x_1, \dots, x_k)$,

subject to equality constraints

$$c_i(\mathbf{X}) = 0 \quad (i = 1, \dots, l), \quad (4.2)$$

inequality constraints

$$c_i(\mathbf{X}) \leq 0 \quad (i = l+1, \dots, m), \quad (4.3)$$

double inequality constraints

$$a_i \leq c_i(\mathbf{X}) \leq b_i \quad (i = m+1, \dots, n), \quad (4.4)$$

and to upper and lower bounds on the independent variables

$$x_j^L \leq x_j \leq x_j^U \quad (j = 1, \dots, k). \quad (4.5)$$

* Note that a problem requiring maximisation of $f(\mathbf{X})$, e.g. if the objective function was MC_L/C_D , can be expressed in the same form by replacing $f(\mathbf{X})$ by $-f(\mathbf{X})$.

For example, the problem might be to minimise C_D at a given value for $C_L (= C_{Ltarget})$ by varying the thickness distribution.

In this case

- the objective function, $f(\mathbf{X})$, is C_D ,
- the independent variables are
 - ◇ the various parameters defining changes to the thickness distribution, and
 - ◇ the angle of incidence,
- the constraints are
 - ◇ an equality constraint to maintain C_L constant, defined in the form

$$c_1(\mathbf{X}) = C_L - C_{Ltarget} = 0, \quad \text{and}$$

- ◇ an inequality constraint to avoid crossover of the upper and lower surfaces, defined in the form

$$c_2(\mathbf{X}) = z_L - z_U < 0.$$

The k -dimensional space defined by the k independent variables x_j is known as the *design space*. As the k independent variables are varied, the objective function can be considered as defining a surface in $(k+1)$ -dimensional space. This surface is referred to as the *design surface*.

The design surface cannot, in general, be visualised but an example for the limited case of $k = 2$ (Reference 12) is shown in Sketch 4.1. This limited case gives several advantages in understanding the optimisation process and many references to it will be used in the following sections.

A discussion of the topography of the design surface and its effect on the ease of solution of the optimisation problem is given in Appendix B. The region where the constraints are satisfied is known as the *feasible region*. The periphery of the feasible region is known as the *constraint boundary*.

A *minimum* is a point where increasing and decreasing each parameter both result in higher objective function values. If a function has more than one minimum, the smallest is described as the *global minimum* and the others as *local minima*. Although the required solution is the global minimum, it must be remembered that no optimisation method can guarantee to find the global minimum. The values of the independent variables corresponding to the solution are referred to as their *optimum values*.

4.1.1 Scaling factors

The variables are normalised by the introduction of scaling factors. In principle, these scaling factors are chosen so that all variables are reduced to the order of unity. Each variable then has equal influence on the design surface search. In practice, it is found to be necessary to choose other values for the scaling factors in order to alter the relative weight of the variables in the optimisation process. However, this has to be done with great care to avoid distorting the contours of the design surface and, thereby, interfering with an efficient progress towards the optimum.

Reference 12 (see Section 3.3 and Figure 13 therein) shows an example where different scaling factors were applied to one of the two primary independent variables, either flattening or elongating the design surface relative to equal scaling. The result was three different paths to a solution; in one of the distorted cases the search still reached the global optimum whereas in the other case the search went in a totally different direction, ending up near a local optimum.