

EXAMPLE OF THE APPLICATION OF CONSTRAINED MULTIVARIATE OPTIMISATION TECHNIQUES TO THE DESIGN OF AEROFOIL SECTIONS

Design point : Dual
Geometry variation : Leading and trailing edge flap deflections and camber line variations
Initial aerofoil : RAE 2822
CFD : BVGK (viscous flow)

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1. INTRODUCTION

This Data Item presents an example of the application of the constrained multivariate optimisation technique CODAS to minimisation of aerofoil drag coefficient at multiple design points.

Reference 5 illustrates the use of constrained multivariate optimisation techniques in the minimisation of drag by the use of leading and trailing edge flap deflections, or by variation of a section's camber line, at a single design point (M , C_L). The present Data Item demonstrates the combined use of leading and trailing edge flap deflections and camber line variation to optimise aerofoil section drag coefficient at two different design points.

The aim in this example is to minimise a weighted average* of the C_D values at two design points – $M = 0.71$, $C_L = 0.75$ and $M = 0.66$, $C_L = 0.87$; $R = 20 \times 10^6$ and transition is fixed at 5% chord on both surfaces throughout. The flow calculation method used is BVGK for viscous flow, set up to calculate C_D at the required C_L values. The basic section used is RAE 2822 (12.11% t/c , 0.84% ρ_0/c), which was designed to have a pressure distribution with a flat rooftop extending to about 50% chord at $M = 0.66$ and $C_L = 0.56$. Figure 1 shows pressure distributions, also calculated by BVGK, at the section's design condition ($M = 0.66$, $C_L = 0.56$) and at the two conditions for the present optimisation study ($M = 0.71$, $C_L = 0.75$, at which $C_D = 0.008636$ and $M = 0.66$, $C_L = 0.87$, at which $C_D = 0.010544$).

Calculated drag coefficients are generally quoted here to 6 decimal places, not because they are believed to be correct to this level, but in order to indicate adequately the (sometimes small) differences between individual cases. The end results are not considered to be accurate to better than 4 decimal places.

The Data Item is one of a series intended to illustrate the process of using such optimisation techniques, to indicate their advantages and to reveal the kinds of problems that can arise. To facilitate this, the Data Item is in narrative form.

An introduction to the application of constrained multivariate optimisation techniques to the design of aerofoil section shapes is given in Data Item No. 99019 (Reference 4).

* i.e. the objective function = $(W_1 C_{D1} + W_2 C_{D2})$, where $W_1 = 0.7$ and $W_2 = 0.3$.

2. NOTATION AND DEFINITIONS

2.1 Algebraic Notation

C_D	drag coefficient
C_{DW}	wave drag coefficient
C_L	lift coefficient
C_m	pitching moment coefficient (about quarter-chord point, positive nose-up)
C_p	pressure coefficient
c	aerofoil chord
$\bar{H}$	transformed boundary layer shape parameter
M	free stream Mach number
R	Reynolds number, based on chord and free stream conditions
t	maximum thickness of aerofoil, <i>i.e.</i> maximum value of $z_U(x) - z_L(x)$
W	weighting applied to C_D values in the objective function
x	chordwise coordinate
z	normal coordinate
z_c	normal coordinate of incremental camber line (in the Figures this includes flap deflections)
$z_L(x)$	normal coordinate of lower surface of aerofoil at x
$z_U(x)$	normal coordinate of upper surface of aerofoil at x
α	angle of incidence of reference line (<i>i.e.</i> for aerofoil section with leading and trailing edge flaps undeflected) degree
ρ_0	leading edge radius

Subscripts

1	denotes values at first design point
2	denotes values at second design point

2.2 Computational Notation

NFE	number of calls by RQPMIN to CFD routine
NFEMAX	maximum number of calls by RQPMIN to CFD routine (set in input data)

2.3 Definitions

CFD	"Computational Fluid Dynamics", programs used to calculate the flow around aerofoil sections. In the examples in this Data Item the chosen CFD method is BVGK (see Reference 1). The generic term "CFD" is used except where the behaviour under discussion is specific to BVGK.
CODAS	"Constrained Optimisation Design of Aerodynamic Surfaces", a design method for aerofoils and wings which couples CFD and numerical optimisation methods (see References 2 and 4).
Optimiser	Software capable of solving constrained multivariate optimisation problems.
RQPMIN	The chosen optimiser in the examples in this Data Item (see Reference 3).

3. EXAMPLE

3.1 Preliminary Investigation

Before proceeding to the dual design point study, optimisation studies were made, at each of the two design points, for the following cases.

- (a) Camber line variation alone (8 runs),
- (b) flap deflections alone (6 runs), and
- (c) flap deflections and camber line variation combined (9 runs).

Each of the runs corresponds to different starting conditions.

Bar charts comparing the C_D values obtained from these studies are shown in Figure 2 for the first design condition ($M = 0.71$, $C_L = 0.75$) and in Figure 6 for the second design condition ($M = 0.66$, $C_L = 0.87$). It must be emphasised that these preliminary optimisation studies were not as comprehensive as in Reference 5 but are considered to be sufficient to indicate the level of drag optimisation that can be achieved at the design points separately. Not all runs ended in a fully converged solution, some reaching the maximum number (NFEMAX) of calls to the CFD code before the solution converged, one failed in the CFD code at the first call, and one suffered a "program logic error"*. The status of each solution is indicated on the bar chart. There is no difficulty in selecting the best result in each case from the fully converged results; the NFEMAX cases serving merely to confirm the C_D level achieved. These results are compared with the C_D for the RAE 2822 section at the appropriate flight condition in Table 3.1 and convergence histories for them are shown in Figures 3, 4 and 5 for the first design point and Figures 7, 8 and 9 for the second design point.

* The occurrence of a program logic error indicates, in general, a fault in RQPMIN itself. No specific remedies are suggested in Reference 3.

TABLE 3.1 Values of C_D at Separate Design Points

	First design point $M = 0.71, C_L = 0.75$	Second design point $M = 0.66, C_L = 0.87$
RAE 2822 alone	0.008636	0.010544
Camber line	0.007876	0.007973
Flaps	0.007899	0.008003
Flaps and camber line	0.007817	0.007899

The results at each design point are very close, those with flaps and camber line combined being marginally better than either flaps or camber line alone (as is to expected). They all correspond to zero wave drag. Any further drag reduction has to come from reduction of the viscous drag and is, inevitably, small. The long, nearly horizontal tails to the C_D histories in Figures 3 to 5, 7 to 9 shows how slow progress is at this stage.

3.2 Optimisation at Dual Design Points

The dual design point optimisation study was made using a total of 11 runs (starting from different combinations of flap deflections and camber line parameters). The C_D results are shown in Figure 10 and are repeated in Figure 11 to the same scale as in Figures 2 and 6 (which is better suited to the lower C_D values). For each figure, C_D results are shown for the two individual design points and for their weighted mean which is the objective function for the study. The lowest value obtained for C_D from a fully converged solution is higher than the lowest from a case which stopped at NFEMAX, as is seen in Table 3.2.

TABLE 3.2 Values of C_D at dual design points

	First design point $M = 0.71, C_L = 0.75$	Second design point $M = 0.66, C_L = 0.87$	Weighted mean
Flaps and camber line			
Converged solution	0.007868	0.008470	0.008048
NFEMAX solution	0.007828	0.007975	0.007872

For the converged solution, the wave drag is zero at the first design point but non-zero at the second. For the NFEMAX solution the wave drag is zero at both design points. Convergence histories for the two runs are shown in Figure 12 (for the converged result) and Figure 13 (for the NFEMAX result). A continuation run for the NFEMAX case ran for a further 20 iterations (NFE = 440) without any significant decrease in C_D , after which the run suffered a program logic error. It is taken that the NFEMAX solution is, in fact, the required optimum.

The pressure distributions resulting from the dual design point study are compared with those resulting from the two separate design point studies in Figure 14. Because the objective function is weighted towards the first design point (in the ratio 7:3) the pressure distributions are closer at the first design point than at the second.

The slightly irregular form of the pressure distributions (seen, for example, in Figure 14) has no great significance, since smooth forms could be obtained which have the same value of C_D for the given value of C_L . The reason is that the viscous drag coefficient is insensitive to such small irregularities in pressure distribution.

3.3 Effect of Changing the Weighting in the Objective Function

The above study has been concerned with one set of weighting factors only ($W_1 = 0.7$ and $W_2 = 0.3$). The results of a limited set of runs with a range of weighting factors covering the range $W_1 = 0$ to 1 (with $W_2 = 1 - W_1$) are shown in Figure 15. Graph (a) shows the values of C_{D1} obtained at each value of W_1 , graph (b) shows C_{D2} , and graph (c) shows the weighted mean (i.e. the objective function). Several different starting points were used for runs at $W_1 = 0.3$ and 0.7; only the one with the lowest value for the objective function is plotted.

A number of points should be noted about the results.

- The end points in the graphs (i.e. for $W_1 = 0$ and 1) correspond to (and were, in fact, obtained from) solutions for the single design point cases shown in Table 3.1. These solutions were all fully converged.
 - ◇ At $W_1 = 0$, W_2 takes the value 1 and the objective function consists of C_{D2} only. The resulting optimum aerofoil has zero wave drag at the (second) design point and C_{D2} takes its lowest possible value (0.007899). At the first design point this aerofoil has substantial wave drag but the high value of C_{D1} (0.008684) has no effect on the result. However, as W_1 is increased, this aerofoil still has the same values of C_{D1} and C_{D2} , but the objective function increases due to the influence of the high value of C_{D1} (as shown by the broken line in Figure 15c).
 - ◇ The situation is similar at $W_1 = 1$. The objective function consists of C_{D1} only, the resulting optimum aerofoil has zero wave drag at the (first) design point and C_{D1} takes its lowest possible value (0.007817). At the second design point this aerofoil has substantial wave drag but the high value of C_{D2} (0.008628) has no effect on the result. However, as W_1 is decreased, this aerofoil still has the same values of C_{D1} and C_{D2} , but the objective function increases due to the influence of the high value of C_{D2} (as shown by the broken line in Figure 15c).
- At intermediate values of W_1 , none of the runs resulted in a properly converged solution.
 - ◇ Some stopped when the number of calls to the CFD code reached the maximum set (NFEMAX).
 - ◇ The rest were stopped by a "Program logic error".
 - ◇ C_{D2} results for $0.01 \leq W_1 \leq 0.9$ lie within a very small band, corresponding to zero wave drag. Their standard deviation, 0.000018, indicates the order of accuracy that can be expected from the calculation method.
 - ◇ C_{D1} results obtained at $W_1 = 0.3, 0.7$, and 0.95 are virtually identical despite difference in aerofoil geometries. The corresponding aerofoil configurations (camber line and flap deflections) could, in fact, be used at any value of W_1 . This would result in lower values for the objective function than those found by the actual runs, indicating that those runs had not progressed sufficiently before stopping. C_{D1} can, therefore, be taken as constant for the intermediate values of W_1 , with the value obtained at $W_1 = 0.7$.
 - ◇ The value of the objective function corresponding to this aerofoil configuration then varies linearly with W_1 , as shown by the full line in Figure 15c.

Variation of the objective function with W_1 can now be seen to consist of three separate segments, two corresponding to the end point solutions and that corresponding to the solution in the intermediate region. The results are shown in the lower part of Figure 15, where it is seen that the $W_1 = 0$ end point solution provides the lower value for the objective function for $W_1 < 0.084$ and the $W_1 = 1$ end point solution provides the lower value for $W_1 > 0.984$. Between these two, the actual values for the weighting factors are of no importance.

The reason for this behaviour is that the two design points chosen are fairly similar and allow both to be achieved with zero wave drag conditions. If, on the other hand, one was transonic and the other supersonic (as in the example given in Reference 2) there would be clearer interactions between the two.

4. CONCLUDING REMARKS

The example has demonstrated multi-design point optimisation of an aerofoil section, the use of which provides an impartial approach to the problem of satisfying conflicting demands at the various design points. Fully converged solutions are not necessarily the best. Searches which are stopped when they reach the maximum number of calls to the CFD code can have lower values of C_D , and are acceptable as solutions provided that the convergence histories exhibit an obvious degree of convergence.

The form of objective function used in this example, *i.e.* a weighted mean of the C_D values at the two design points, is the most straightforward. For the two fairly similar design points selected, the actual weighting was found not to be important (apart from values near 0 and 1). More elaborate forms of objective function are possible (see Reference 4).

5. REFERENCES

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