

VGK METHOD FOR TWO-DIMENSIONAL AEROFOIL SECTIONS

PART 5: DESIGN TO A SPECIFIED UPPER-SURFACE PRESSURE DISTRIBUTION

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PART 5: DESIGN TO A SPECIFIED UPPER-SURFACE PRESSURE DISTRIBUTION

1. INTRODUCTION

This is the fifth Data Item related to the VGK aerofoil method, and deals with the aerodynamic design of aerofoils. For the first four Items related to VGK (Parts 1 to 4) see Refs. 1 to 4.

One of the tools often employed in the process of wing aerodynamic design is a method of obtaining the shape of aerofoil which generates a specified (*i.e.* target) upper-surface pressure distribution at a given freestream Mach number. In Ref. 5 guidance is given concerning the application of such a two-dimensional (*i.e.* aerofoil section) design method in the overall wing design process. Ref. 5 also provides information on the selection of appropriate design (or target) two-dimensional pressure distributions for various situations. This present Item gives a method for obtaining the shape of aerofoil which yields (according to the VGK aerofoil method) a specified upper-surface pressure distribution, at a specified subsonic freestream speed, and which has a specified thickness distribution. Since VGK is employed, viscous effects are taken into account in the design process. If desired, however, design with inviscid flow assumptions may be undertaken (see Section 4.8).

In the aerodynamic design of aerofoils with transonic flow fields, common features of the target upper-surface pressure distribution (see Sketch 1.1) are (a) the isentropic recompression region, which may be immediately followed by (b) a weak shock, located typically near or somewhat aft of the mid-chord position, and followed again by (c) a region of roughly linear pressure rise to the trailing-edge. At the conditions of Mach number and incidence which maximise $M_\infty L/D$ (which correspond, in the case of an aircraft, roughly to maximum cruise efficiency) there is a weak shock present in the upper-surface flow and a small proportion of the aerofoil drag is wave drag. Under these precise conditions, any decrease in the Mach number or incidence leads to a decrease of $M_\infty C_L$ that outweighs the decrease in C_D , whilst any increase in Mach number or incidence leads to an increase in C_D that outweighs the gain in $M_\infty C_L$. The inclusion of a shock in the target pressure may lead to some benefits in the aerofoil performance characteristics (by virtue of positioning the shock 'favourably', at the design condition), see Section 4.3.

Two types of upper-surface target pressure distribution are dealt with in this Item, referred to respectively as (a) **shock-free** and (b) **weak shock**. Further comments on features of two-dimensional target pressure distributions which may lead to beneficial characteristics in various situations are given, for example, in Refs. 6 and 7.

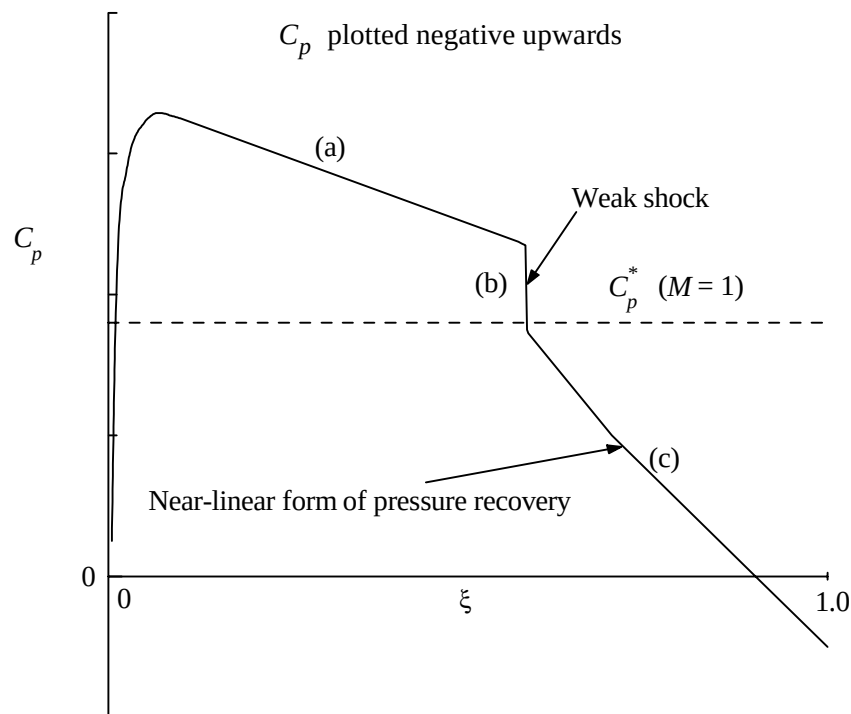
Those important aspects of aerofoil design which concern the 'best' choice of thickness distribution and target pressure distribution for a particular application are not addressed in detail in this Item, but references to the effects of changes in thickness distributions and target pressure distribution are made in the examples. Further comments on the selection of appropriate forms of thickness distribution and target upper-surface pressure distribution are given in Refs. 5, 6, 7 and 8, for example.

In this Item, a method is provided for generating the shape of the aerofoil profile having a specified thickness distribution, so that, at some value of incidence, α , a specified target upper-surface pressure distribution is achieved at given flow conditions; namely for given values of Mach number, Reynolds number, and for given boundary-layer transition locations. A run of the design method yields the change in camber required to be applied to the 'starting aerofoil' (which is an aerofoil that has the desired distribution of thickness, and which may or may not be cambered) and provides values of C_L , C_D , C_M (and α) for the designed aerofoil at the given flow conditions. These results enable the adequacy of the design at the given flow conditions to be assessed. Further assessment of the suitability of any such designed aerofoil (for example at 'off-design' flow conditions) would also generally be necessary.

The design method used for determining the camber shape needed to achieve the target upper-surface pressure distribution is **inverse**, i.e. an aerofoil flow calculation method is used repeatedly in an iteration loop which includes a procedure, or algorithm, to adjust the aerofoil profile in the light of the current and target pressure distributions. The aerofoil flow calculation method used is a slightly extended version of VGK denoted by VGKD (see Parts 1 and 2 for a description of VGK, and Section 3.2.4 for a description of the extension).

In a typical run of the design method the number of design iterations employed is 300. These design iterations are referred to as **design cycles** so as to avoid confusion with VGK flow iterations. There are four separate Fortran programs utilised in the design method, and these are controlled either by a 'Batch' file (in MS-DOS or MS Windows) or a Shell procedure (for use with UNIX operating systems). With a current PC, computing times vary between a few minutes and an hour (or so) for each design undertaken. The use of a Windows operating system makes it possible to monitor the computation entailed in a particular run of the design method, and permits adjustment of the controlling parameters (such as those which influence the convergence) should this be desired.

The elements of the design method are described in Section 3. In Section 4, a number of examples of application of the design method are given. Known limitations and a number of precautions are presented in Section 5. A brief description of the programs employed in the design method and their use is given in Appendix A. Details of the design algorithm are given in Appendix B. The effects of geometry smoothing are described in Appendix C.



Sketch 1.1 Common features of target upper-surface pressure distribution

2. NOTATION

Algebraic notation is printed in italics or has Greek symbols. Notation used in associated computer files is given in Roman type. Notation sequencing is based on the algebraic terms (except where none exists); some file notation therefore appears out of alphabetical order. All linear quantities made non-dimensional are based on aerofoil chord, c .

C_D	CD	overall (<i>i.e.</i> total) aerofoil drag coefficient (evaluated as $C_{DV} + C_{D1}$, see Part 1)
C_{DV}	CDV	viscous drag coefficient, determined from momentum thickness of wake far downstream, see Part 1.
C_{DW}	CDW	wave drag coefficient
C_{D1}	CD1	value of C_{DW} determined from 'first-order method', see Part 1
C_L	CL	aerofoil lift coefficient
C_M	CM	pitching moment coefficient, see Part 1
C_p	CP	pressure coefficient
C_p^*		pressure coefficient for local Mach number of unity
c	C	aerofoil chord, see Part 1
c_f	CF	local skin friction coefficient, see Parts 1 and 2
D		overall aerofoil drag (corresponding to C_D)
f	F	scaling factor applied to $C_p \sim \xi$ graph
	ICYCLE	number of design cycles completed
	IDES	total number of design cycles in design run
i_G	IG	parameter governing chordwise extent of region of imposed equality of C_{pt} and C_{pc}
	ISAV	switch used in program DES ISAV=1 results in .PDO files being saved
	ISHOCK	switch used in program DES ISHOCK=0 or 1 depending on whether or not target pressure distribution contains a shock (see Appendix A)
	ISMOOTH	number of smoothing 'passes' employed in smoothing process
L		aerofoil lift (corresponding to C_L)
M	EML	local Mach number (at edge of boundary-layer) based on isentropic flow relations
M_∞	EM	freestream Mach number
R	RE	Reynolds number based on chord length

	RELAX	relaxation factor applied to $\Delta\zeta$ values given by design algorithm
r	R	radius of curvature of circular-arc rounding
t		maximum thickness of aerofoil, <i>i.e.</i> maximum value of $z_U(x) - z_L(x)$
x		chordwise coordinate, see Part 1
z		normal coordinate, see Part 1
α	ALP	aerofoil incidence, <i>i.e.</i> angle between x -axis and freestream direction, positive nose-up
ΔC_p		$C_{pt} - C_{pc}$
ΔC_{pM}	DCPM	maximum departure from linear form in rearward segment of target pressure distribution
$\Delta\alpha$	DALP	change in incidence implied by non-zero $\Delta\zeta_{TE}$
$\Delta\zeta$	DZ	increment in ζ (<i>i.e.</i> additional camber) introduced by design algorithm
$\Delta\zeta_e$	DZE	value of $\Delta\zeta$ with allowance for incidence change $\Delta\alpha$, see Equation (3.2)
$\Delta\zeta'$		increment in slope $d\zeta/d\xi$ (<i>i.e.</i> additional camber slope) introduced by design algorithm
ζ	Z	non-dimensional normal coordinate
ζ_f		parameter in design algorithm, see Appendix B
ξ	X	non-dimensional chordwise coordinate: $\xi = 0$ at leading-edge and $\xi = 1$ at trailing-edge
ξ_{TL}	XTL	value of ξ corresponding to boundary-layer transition location on lower surface
ξ_{TU}	XTU	value of ξ corresponding to boundary-layer transition location on upper surface
ρ_{LE}		leading-edge radius of aerofoil profile

SUFFICES

Notation used in files corresponding to the suffices is also shown, again in Roman type. Such notation is aligned with that of the primary quantity and *not* displaced downwards.

a	A	denotes more forward rounding in design pressure distribution
b	B	denotes more rearward rounding in design pressure distribution
c	C	denotes value in current design cycle (relates to upper-surface pressure distribution)
des	DES	denotes value corresponding to design conditions
i	I	denotes value at start of design process
L	L	denotes lower-surface value
M	M	denotes midpoint of rearward segment of target pressure distribution

<i>max</i>	MAX	denotes maximum value (in relation to $ \Delta C_p $)
<i>R</i>	R	denotes point of commencement of rapid pressure recovery (in 'unrounded' target pressure distribution)
<i>rms</i>	RMS	denotes root-mean-square (in relation to ΔC_p distribution)
<i>S1</i>	S1	corresponds to value just upstream of shock
<i>S2</i>	S2	corresponds to value just downstream of shock
<i>s</i>	S	denotes upstream limit of implementation of design algorithm
<i>t</i>	T	denotes value corresponding to target upper-surface pressure distribution
<i>TE</i>	TE	denotes trailing-edge value
<i>U</i>	U	denotes upper-surface value
<i>0</i>	0	denotes leading-edge value in notional (unrounded) target upper-surface pressure distribution

3. DESIGN METHOD

3.1 Objective of Design Method

The objective of the design method is to generate the shape of aerofoil profile, having a given thickness distribution, which, at the **design point**, produces, according to the VGK aerofoil calculation method, an upper-surface pressure distribution in close conformity with a specified target pressure distribution. The design point refers to a particular combination of M_∞ , R , ξ_{TU} , ξ_{TL} and C_L . The freestream speed at the design point must be subsonic, corresponding to $M_{\infty des} < 1$. It is important to note that the value of C_L at the design point, C_{Ldes} , cannot be specified independently of the target upper-surface pressure distribution and thickness distribution. The value of C_{Ldes} depends on both of these distributions, and is determined, in any particular case, as an outcome of the application of the design method. Some examples indicating how C_{Ldes} may be raised or lowered by modification of either the target upper-surface pressure distribution or the thickness distribution are given in Section 4.

3.2 Steps in the Design Method

The various steps in applying the design method are first listed in outline. Further details relating to these steps are given in Sections 3.2.1 to 3.2.9.

- (a) Select the **starting aerofoil geometry**, *i.e.* an aerofoil with its thickness distribution the same as that desired, and set the **current aerofoil geometry** to be same as the starting aerofoil geometry.
- (b) Select the flow conditions of the **design point**. These are principally the values of M_∞ and R , but the transition locations of the upper- and lower-surface boundary layer flows, ξ_{TU} and ξ_{TL} must also be selected - see Part 1.
- (c) Choose the values of the parameters that specify the target upper-surface pressure distribution.
- (d) Calculate the flow about the current aerofoil geometry, at the design point flow conditions, using VGKD. Thus determine $C_{pUc} \sim \xi$, the **current upper-surface pressure distribution**.
- (e) Terminate the design run if the current upper-surface pressure distribution, $C_{pUc} \sim \xi$, is sufficiently close to the target pressure distribution, $C_{pUt} \sim \xi$.
- (f) Use the design algorithm to determine the profile geometry changes (camber changes, $\Delta\zeta \sim \xi$) intended to result in an upper-surface pressure distribution closer to the target pressure distribution than the current one. This is done by running the program DES.
- (g) Apply the profile geometry changes to the current aerofoil geometry (*i.e.* update the current aerofoil geometry). The program ADCAMS performs this operation.
- (h) Apply smoothing to the current aerofoil geometry calculated in Step (g). Maintain a record of the differences between the current aerofoil geometry and that of the starting aerofoil, $\Delta\zeta_e \sim \xi$. The program SMTHFOIL performs these operations.
- (i) Return to Step (d) unless a specified number of design cycles has been completed.

3.2.1 Selection of starting aerofoil

The choice of starting aerofoil determines the thickness distribution used in the application of the design method. If the aerofoil characteristics sought are known to be roughly achieved by a particular aerofoil, then this aerofoil might sensibly be selected as the starting aerofoil. The task of obtaining a suitable thickness distribution may be helped by assuming that, at any freestream Mach number, provided any shock waves present are weak, the distribution of the mean of the upper- and lower-surface pressures, $(C_{pL}(\xi) + C_{pU}(\xi))/2$, is determined (approximately) by the thickness distribution and is relatively little affected by the aerofoil camber and incidence (see Ref. 5, for example). Characteristics of a large number of aerofoils are available from various sources, see for example Refs. 7 and 16. Comments on the choice of thickness distribution are given, for example, in Refs. 5 and 8.

3.2.2 Selection of flow conditions at the design point

Aircraft wings have to operate satisfactorily over a range of flow conditions, and consequently their corresponding aerofoils should also have appropriate characteristics for a range of conditions, see, for example, Ref. 5. The choice of a single design point is clearly necessary for an application of the present method, and this choice would probably correspond to the cruise condition in the case of design relevant to a transport aircraft. Comments on the choice of a single design point for different types of aircraft are given in Refs. 5 and 7, for example. Usually, the design point includes specification of the lift coefficient, as well as the flow conditions of Mach number and Reynolds number. In the design method presented here, the lift coefficient may not be specified *a priori*, as has been stated in Section 3.1, since its value, which is determined as one of the outcomes of the design method, depends mainly on the choices of target upper-surface pressure distribution and the thickness distribution of the starting aerofoil. As a result, if the value of the design lift coefficient, C_{Ldes} , obtained is found to be different from that sought, (as it almost certainly would be in the first instance) then it would be necessary to change either the target upper-surface pressure distribution or the thickness distribution, or both, so as to obtain (eventually) a value of C_{Ldes} that was sufficiently close to the value desired. In general terms, raising the values of $|C_{pt}|$ would tend to increase C_{Ldes} (for a given thickness distribution), as would lowering the value of t/c (for a given $C_{pt}(\xi)$).

3.2.3 Parameters defining the target upper-surface pressure distribution and their specification

Two different types of target upper-surface pressure distribution are dealt with, termed respectively (a) shock-free and (b) weak shock, see Sketch 3.1. The only difference between these two types is that the shock-free type has a smooth rounded form for the portion linking the forward and rearward segments, whereas in the weak shock type these segments are linked by a sudden jump corresponding to a pressure rise such as might be produced by a weak shock. (In this context 'weak' refers to a shock which is sufficiently weak not to separate the boundary layer, and which generates only a moderate wave drag, corresponding to a value of C_{D1} of order 0.002.)

Figure 10.10 is a graph showing the shock-free target pressure distribution. The vertical axis is labeled C_p and the horizontal axis is labeled ξ . The origin is marked 0. The horizontal axis has a point ξ_R and a point $\xi_M = (1 + \xi_R) / 2$. The vertical axis has a point C_{p0} . A solid line represents the parabolic curve, and a dashed line represents the shock-free target pressure distribution. Key points A, B, C, D, E, and R are marked. A vertical dashed line at ξ_R indicates the shock location. The distance from the shock to the rear of the body is $\xi_M = (1 + \xi_R) / 2$. The pressure difference ΔC_{pM} is shown as a negative value.

(b) Weak-shock target pressure distribution

Sketch 3.1 Two types of target upper-surface pressure distribution

The common features of the two types of target upper-surface pressure distribution are

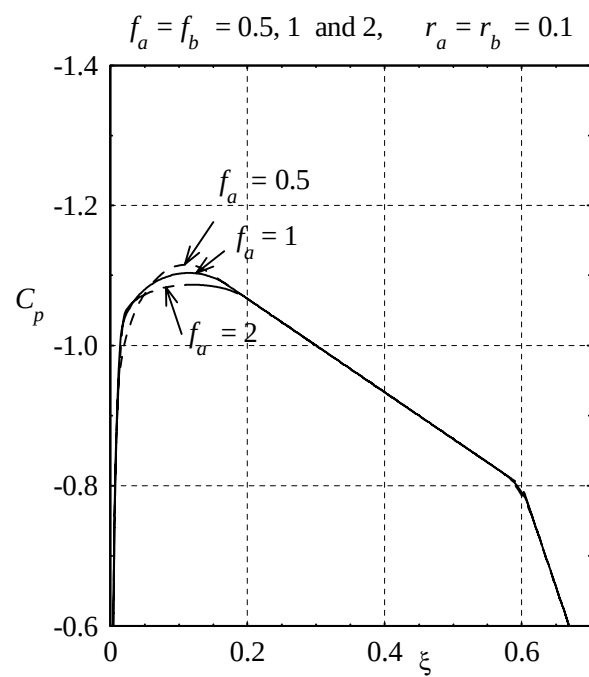
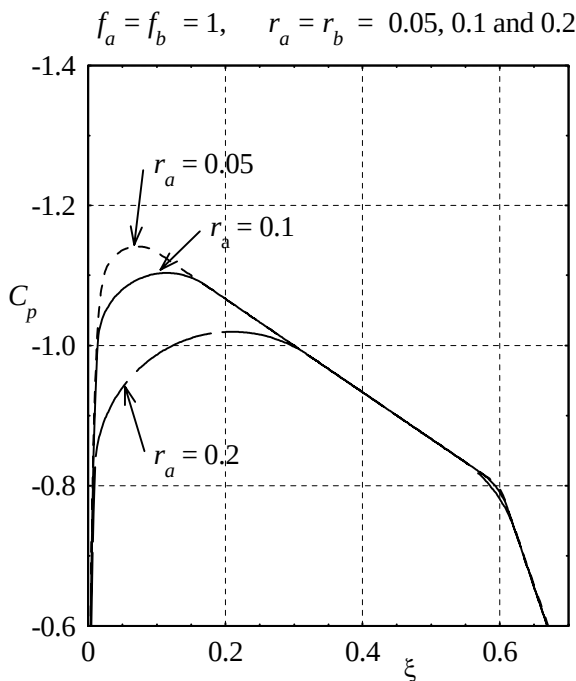
- (i) a region of rapid flow acceleration extending at most to a few percent chord downstream of the leading edge,
- (ii) a smooth rounded portion which links feature (i) to a linear forward segment (iii),
- (iii) a linear forward segment corresponding to a gradual pressure rise in the flow direction, and
- (iv) a rearward segment corresponding to a considerably greater rate of pressure rise in the flow direction than that of the forward segment. This segment may be linear ($\Delta C_{pM} = 0$), or parabolic ($\Delta C_{pM} \neq 0$), see Sketch 3.1.

In order to make reasonably sure that the aerofoil designed has a realistic geometric form (particularly in the regions near the leading- and trailing-edge), the following restrictions are applied.

- (a) The values of C_{pUt} in the rapid flow acceleration region are equated to those of the **current upper-surface pressure distribution**. The parameter i_G is used to limit the chordwise extent over which this equality is imposed. See Appendix B for details.
- (b) The target value of C_p at the trailing edge, C_{pTE} , is taken to be that of the **current pressure distribution**.

Because of these restrictions, the initial target upper-surface pressure distribution is affected (to a slight extent) by the pressure distribution calculated for the starting aerofoil geometry, and is thus influenced by the value of incidence at the start of the design process, α_i , which must be specified. This influence (of the specified initial incidence) persists to the target upper-surface pressure distribution used in **each** design cycle. To a slight extent, therefore, the geometry of any aerofoil designed by the method is affected by the initial incidence selected for the VGKD calculations. Generally, an increase in this incidence will result in a somewhat more rapid flow acceleration near the leading edge. Also, the trailing-edge pressure may be very slightly changed (generally lowered) by an increase in this incidence.

Each rounded portion that appears in the target pressure distribution (that is, referring to Sketch 3.1, between points B and C for both types and also between points D and E for the shock-free type), is a **circular arc** on the $C_p \sim \xi$ graph (when drawn to a certain scale). This arc is tangent to the segments that it links. Each rounded portion is circular when the C_p against ξ graph is drawn such that equal C_p and ξ intervals correspond to lengths in the ratio of $f:1$, see Sketch 3.2. (If a different value of scale ratio is used the rounding is then part of an ellipse with its major and minor axes parallel to the graph axes.) A rounding is thus defined by the value of the scale factor f , and the radius of curvature, r , of the circular arc (where r is normalised with respect to the length of an interval of ξ of unity). The effects of varying r and f on the target pressure distribution are illustrated in Sketch 3.2. No rounding may be so extensive as to remove completely either of the segments it links.



Sketch 3.2 Illustration of effects of varying r and f on target pressure distribution

In summary, referring to Sketch 3.1, the parameters relating to the target upper-surface pressure distribution that require to be chosen are, for the shock-free type:

$$\alpha_i, C_{p0}, i_G, r_a, f_a, \xi_R, C_{pR}, r_b, f_b, \Delta C_{pM},$$

and for the weak-shock type:

$$\alpha_i, C_{p0}, i_G, r_a, f_a, \xi_R, C_{pS1}, C_{pS2}, \Delta C_{pM}.$$

There are limitations on the choice of these parameters, but it is not possible to be precise about them. For example, the value of α_i chosen should be such as to lead to an acceptable rapid flow acceleration region. In general terms, if C_{p0} corresponds to too high a local Mach number (around 1.4 or more), a smooth isentropic pressure recovery will not be possible. Also, if ξ_R or $|\Delta C_{pM}|$ is too large the rearward pressure gradient will be too high for the boundary layer to remain attached (in which case the VGK aerofoil method will either fail or give misleading results – see Ref. 1). The value of C_{pR} should generally correspond to a local Mach number fairly close to unity. Also, in the case of the weak-shock type of target, the values of C_{pS1} and C_{pS2} should generally correspond respectively to local Mach numbers in the ranges 1.0 to 1.2, and 0.98 to 1.02. C_{pS1} must always be less than C_{pS2} .

3.2.4 Calculation of flow over current aerofoil geometry

The VGK method is used in each design cycle to calculate the flow over the current aerofoil geometry. This is generally a straightforward procedure, but care should be taken to ensure that the calculation is reliable, and for this Parts 1 and 2 (Refs. 1 and 2) should be consulted. The checks needed are to ensure that the VGK calculation has converged well, and that there are no more than a small number of surface grid-point locations at which c_f has been raised artificially, see Part 2. Strictly, these checks need only be made at the final stage of the design process, since it is only the final aerofoil geometry that is of interest. The version of VGK used in the design method (denoted by VGKD) is slightly extended in that it produces an additional file. This is a file consisting of upper-surface values of ξ and C_p , which is used as an input to the program implementing the design algorithm. The file is named S.PDO, and is similar to the .BRF file produced by VGK (see Part 2), except that it contains data relating to only the aerofoil upper-surface.

3.2.5 Termination of design process

Successive cycles of the design method may be examined to see how the upper-surface pressure distribution calculated using VGKD for the current aerofoil geometry compares with the target upper-surface pressure distribution. A simple measure of such a comparison is ΔC_{prms} . A file containing a record of values of ΔC_{prms} is one of the files output by the program DES. Any run of the design method may be terminated when ΔC_{prms} falls to a sufficiently small value. It has been found in trials of the design method that ΔC_{prms} usually falls well below 0.005 after a sensible number of design cycles, and that usually when this occurs the design method may be considered satisfactory. It is recommended that generally IDES be set to 300, as in the examples in Section 4. Values of IDES less than 200 are not recommended. In some cases the design method will not lead to satisfactory results. An example of an unsatisfactory run is given in Section 4.7.

3.2.6 Design algorithm to yield profile geometry change

The design algorithm is the group of expressions and rules that determines the change in aerofoil camber, $\Delta\zeta$, intended to make the upper-surface pressure distribution closer to the target distribution. The algorithm adopted is a development of that proposed by Davis (Ref. 10), and is based on linearised potential theory, with different equations applying depending on whether the local upper-surface flow is subsonic or supersonic. Details are given in Appendix B. The algorithm is implemented in the program DES where it is used to generate $\Delta\zeta$ at ξ values which are multiples of 0.001 and which lie between ξ_s and 1 (inclusively). The parameter ξ_s is chosen by the user. A value of 0.002 has been generally selected in the trials of the design method and has been found to be satisfactory. Two further controlling parameters are also included, the first (RELAX) is the relaxation factor applied to the ζ changes calculated by direct application of the design algorithm, and the second (ISMOOTH) is the number of smoothing cycles to be applied to the calculated ζ changes. Typical values for these parameters are respectively 0.25 and 1. See Section A3 for further details.

3.2.7 Application of geometry changes

The value of $\Delta\zeta$ output from DES corresponding to the leading edge is always zero, but that corresponding to the trailing edge will generally be non-zero. So as to conform to the convention that camber ordinates are zero at both leading and trailing edges, the required displacement of the trailing edge (relative to the leading edge) implied by $\Delta\zeta_{TE}$ being non-zero is generated by a change in incidence, $\Delta\alpha$, given by

$$\Delta\alpha = -\tan^{-1}(\Delta\zeta_{TE}). \quad (3.1)$$

The values of $\Delta\zeta$ are changed to allow for this change of incidence as follows,

$$\Delta\zeta_e = \Delta\zeta + \xi \tan(\Delta\alpha). \quad (3.2)$$

The ordinates of the additional camber, $\Delta\zeta_e$, then conform to the convention of being zero at both the leading and trailing edges.

The values of $\Delta\zeta$ output from DES are interpolated (where necessary) to the ξ coordinate values appearing in the current aerofoil geometry. Corresponding values of $\Delta\zeta_e$ (see Equation (3.2)) are then calculated and these are then added to the ζ values of this geometry, both for the upper- and lower-surface. These operations are performed by the program ADCAMS, which also changes the value of α in the .VIN file, as necessary (see Equation (3.1)). Camber is thus effectively applied by shearing the aerofoil. It has the effect of leaving unchanged the values of the ξ coordinates, so these remain the same as those of the starting aerofoil throughout successive design cycles. Shearing has an advantage over changes applied normal to the surface, since it cannot produce folded surfaces however many changes of camber of whatever magnitude are added to the starting aerofoil.

3.2.8 Smoothing of current aerofoil geometry coordinates

A program named SMTHFOIL serves to smooth the values of the ζ coordinates generated by the program ADCAMS. It also evaluates and outputs the differences between the current aerofoil geometry coordinates and those of the starting aerofoil in a file named DIFFS.DAT. Two forms of smoothing are employed. The first is a form of 'smoothing-by-threes', see Ref. 11 and Section A5, where instead of the three successive coefficients in the smoothing-by-threes each being 1/3 (as is usually the case), the values adopted are 0.01, 0.98 and 0.01. This is a very weak smoothing, but since it is applied at each design cycle, it generally serves to keep the shape of the designed aerofoil reasonably smooth, whilst at the same time allowing the target pressure distribution to be achieved very closely. Used on its own, however, this smoothing may not be completely suitable, especially in cases where relatively large values of additional camber, $\Delta\zeta_e$, are generated. This deficiency is overcome by the introduction of a second smoothing. This second smoothing is applied just once, at the 100th design cycle, and consists of fitting the differences between the current aerofoil geometry coordinates and those of the starting aerofoil by a least-squares polynomial of degree 6. These fitted differences are then applied to the starting aerofoil to generate a new smooth aerofoil, termed the 'intermediate aerofoil', which is written to the file INTFOIL.DAT. (The current aerofoil geometry at this design cycle is replaced by the intermediate aerofoil.) The smoothing-by-threes of subsequent design cycles is then applied to corresponding differences in ζ coordinate values of the current and intermediate aerofoils. An illustration of the effects of applying these forms of smoothing, both in isolation and in combination, is given in Appendix C.

3.2.9 Repetition of design cycle process and its control

The sequence of design cycles is controlled by either a Batch file (for use with MS-DOS or with MS-DOS called up from a Windows operating system) or by a Shell procedure (when a UNIX type of operating system is used). In this respect the design process implementation is very similar to that used with VGK in Ref. 3 (in the estimation of a separation boundary) and with VGKE in Ref. 4 (in the estimation of excrescence drag). Details of the Batch file, named DESSEQ.BAT, and the corresponding Shell procedure, named DESSEQ, are given in Appendix A.

A number of files produced during a design run may be examined with the help of a graph plotting program such as EASYPLOT (Ref. 12) or SIGMAPLOT (Ref. 13). For example the file (named RMSUSE.DAT) containing the record of $\Delta C_{p_{rms}}$ may be examined, as may a file (named CAM1.DAT) which contains the target and current upper-surface pressure distribution C_p values corresponding to the aerofoil grid coordinates (see Part 1). If such an examination is done at various stages through a run of the design method,

it becomes easy to see if the run is likely to be successful, and the number of design cycles that may be needed can be estimated. If desired, the file CPDESI.PAR may be edited to change the value of IDES appearing in it. Termination of the program can be enforced, if desired, by setting the value of IDES appearing in CPDESI.PAR to a number less than or equal to the number of design cycles completed. (This latter number is available, for example, in the file RMSUSE.DAT.)

4. EXAMPLES

In this section a number of results obtained from use of the design method are presented. These examples are selected mainly to show the capabilities of the method, and not to show the aerodynamic benefits or disadvantages associated with the various target pressure distributions and thickness distributions employed.

The graphical presentation of the principal results of each example design run generally comprises the following three parts.

- Part a Plot of ΔC_{prms} against ICYCLE.
- Part b Plot of upper-surface pressure distribution corresponding to final aerofoil geometry (*i.e.* that given as the current geometry at the last complete design cycle), and comparison with (final) target pressure distribution.
- Part c Plot of differences in ζ ordinate between final aerofoil and starting aerofoil. (The initial and final values of α are also noted here.)

In some cases, one or both of the following parts are also given.

- Part d Plot of final aerofoil coordinates.
- Part e Plot of pressure distributions of starting and final aerofoils corresponding to the value of C_{Ldes} applicable to the final design cycle.

It should be noted that, in order to indicate the likely magnitude of a number of effects, values for C_L and C_D (*i.e.* $C_{DV} + C_{D1}$) given by the VGK method are presented to a greater accuracy than would be justified for the method as judged by comparison with experimental results (see Part 1).

4.1 Example of Typical Design Task - Supercritical Shock-free Type of Target Pressure Distribution

This example, which corresponds to a typical design task, is specified as follows.

$$M_{\infty des} = 0.72, R = 20 \times 10^6, \xi_{TU} = 0.05, \xi_{TL} = 0.05$$

Starting aerofoil: RAE 2822 (See, for example, Ref. 15. For this aerofoil $t/c = 0.121$, and $\rho_{LE}/c = 0.00827$. The file RAE2822.DAT (see Ref. 2) provided the ordinates used.)

$$\alpha_i = 2 \text{ deg}$$

$$C_{p0} = -1.2, i_G = 30, r_a = 0.05, f_a = 1.0, \xi_R = 0.6, C_{pR} = -0.8, r_b = 0.2, f_b = 1.0, \Delta C_{pM} = 0, \xi_s = 0.002, \text{RELAX} = 0.25, \text{ISMOOTH} = 1, \text{IDES} = 300.$$

The values of C_{p0} and C_{pR} correspond to values of M of 1.22 and 1.04 respectively.

Results obtained from a run of the design method for this example task are given in Figure 1. It is apparent that the design task has been achieved satisfactorily. The checks on the VGKD calculations referred to in Section 3.2.4 are satisfied. However, C_f had been artificially raised at three grid points near the trailing edge, and it is clear that any significant increase in the adverse pressure gradients specified would result in a design of questionable acceptability.

The value of C_{Ldes} determined after 300 design cycles is 0.770. The additional camber obtained has a maximum value for $\Delta\zeta_e$ of about 0.005. The final value of incidence is slightly higher, at 2.150 deg, than the initial value of 2 deg. At the design conditions the aerofoil produced in this example (named AFA41) gives a 20 per cent lower value of drag coefficient (as indicated by $C_{DV} + C_{D1}$, see Ref. 1) than that of the starting aerofoil, see Figure 1e. This is a consequence of the fact that the AFA41 aerofoil has no shock at these conditions. It may be noted that the RAE 2822 aerofoil was designed utilising the analysis method of Ref. 14 (see Ref. 15), corresponding to a subcritical flow and with $M_{\infty des} = 0.66$, $C_{Ldes} = 0.56$. It is not surprising, therefore, that it has higher values of C_D at M_{∞} close to 0.72 than an aerofoil designed specifically for $M_{\infty} = 0.72$.

In Figure 2, a comparison is given of the variation of C_D with M_{∞} at $C_L = 0.770$ for both the starting aerofoil RAE 2822, and the final designed aerofoil, AFA41. Runs of VGK have been made for values of M_{∞} that are multiples of 0.005 and lie within the range from 0.55 to a value at which the drag coefficient has risen to about 1.7 times that at $M_{\infty} = 0.55$. Both aerofoils exhibit the inevitable rise in drag due to shock formation, that of the AFA41 aerofoil being delayed slightly relative to the RAE 2822 aerofoil. On the other hand, the VGK results also indicate a range of Mach numbers around 0.69 for which the drag coefficient of the RAE 2822 aerofoil is less than that of the AFA41 aerofoil.

In practice there are a number of factors that affect the choice of aerofoil for a given task as well as the drag at a single design point. These additional factors include separation margins, pitching moment characteristics and the performance at off-design conditions – all of which incidentally may be investigated by means of the VGK method, see Parts 1, 2 and 3 (Refs 1, 2 and 3). The optimum aerofoil will depend on the relative importance to be given to these various factors and thus further investigation would normally be needed. The present Data Item may be viewed as a means of extending the range of candidate aerofoils to be considered for a given task.

4.2 Example of Effect of Change of Thickness Distribution

In this example the parameters specifying the design are the same as those of Section 4.1, but the thickness distribution is changed to that of the NACA 0012 aerofoil, see Appendix A of Part 2 (Ref. 2) – the file NACA0012.DAT provided the ordinates used. This, of course, is an aerofoil intended for relatively low speed use, and so it would not be expected to be suitable even for providing a thickness distribution for use at supersonic speed. The results of a run of the design method for this example are given in Figure 3. In comparison with the example in Section 4.1, considerably greater values of camber change, $\Delta\zeta_e$, are required. The aerofoil designed is named AFA42. The design method has performed satisfactorily, with the values of ΔC_{prms} achieved after 300 design cycles being comparable to those of Section 4.1. The value of C_{Ldes} found at the end of the design run is 0.710 (i.e. about 10 per cent less than that found in Section 4.1, where the RAE 2822 thickness distribution was used).

In Figure 4 the effect of the change in thickness distribution of the designed aerofoils on the drag characteristics is illustrated for $C_L = 0.710$. Also shown are the corresponding results for the RAE 2822 and NACA 0012 aerofoils.

The fact that in transonic flow the upper-surface pressure distribution is largely determined by the form of

the aerofoil upper surface, (together with its orientation relative to the freestream direction) is illustrated in Figure 5, which shows that the forms of the upper surface of the aerofoils AFA41 and AFA42 are little different, except near the leading edge, once allowance is made for the difference in the values of final incidence, see Figures 1c and 3c. This is indicated by the fact that, apart from the first few per cent of chord, the differences in upper-surface ordinates correspond closely to a simple rotation about the leading edge.

With the upper-surface pressure distribution fixed, and the upper-surface geometry correspondingly largely determined, it is clear that it is the form of the thickness distribution that effectively determines the lower-surface pressure distribution (as may be seen by comparison of Figures 1b and 3b, for example), and consequently in these circumstances it is the thickness distribution that effectively determines the chordwise loading distribution.

In the design of a wing or aerofoil, it is necessary to consider the pitching moment characteristics, partly to avoid large unwanted trim loads and associated drag. Figure 6 presents results calculated using VGK for the C_M characteristics at $C_L = 0.710$ for the same four aerofoils considered in Figure 4. The aerofoils designed by the present method, AFA41 and AFA42, are seen to give nearly equal values of C_M over the Mach number range, and they correspond to centre of lift locations only a few per cent chord aft of those of the RAE 2822 aerofoil. As would be expected, the NACA 0012 aerofoil, which is uncambered, gives values of C_M which correspond to much more forward centre of lift locations – slightly forward of the 25 per cent chord point.

Thus this example, together with that of Section 4.1, shows that, whilst the design method will yield a given target pressure distribution even though starting with widely different thickness distributions, it is important to choose a thickness distribution that is inherently suited to the task in hand.

4.3 Example of Typical Design Task – Weak Shock Type of Target Pressure Distribution

This example is very similar to the one in Section 4.1, except that the target pressure distribution displays a weak shock. The example is specified as follows.

$$M_{\infty des} = 0.72, R = 20 \times 10^6, \xi_{TU} = 0.05, \xi_{TL} = 0.05$$

Starting aerofoil: RAE 2822

$$\alpha_i = 2 \text{ deg},$$

$$C_{p0} = -1.2, i_G = 30, r_a = 0.05, f_a = 1.0, \xi_R = 0.6, C_{pS1} = -0.85, C_{pS2} = -0.75, \Delta C_{pM} = 0,$$

$$\xi_s = 0.002, \text{RELAX} = 0.25, \text{ISMOOTH} = 1, \text{IDES} = 300.$$

The values of C_{p0} , C_{pS1} and C_{pS2} correspond to values of M of 1.22, 1.065 and 1.02 respectively.

Results obtained from the design run are given in Figure 7. Again the design task has been achieved satisfactorily. The aerofoil, named AFA43, gives $C_{Ldes} = 0.777$, marginally higher than the value found in the example of Section 4.1.

In Figure 8, drag coefficient values provided by VGK for the three aerofoils, RAE 2822, AFA41 and AFA43, at $C_L = 0.777$ are shown. Based on this comparison, there appears to be no benefit obtained from designing with a weak shock present; indeed the drag rise of AFA43 is somewhat more rapid than that of AFA41. Whether there might be advantages in designing with a weak shock located elsewhere, or of a different strength, is an open question, which could be resolved by further applications of the design method.

4.4 Examples of Effects of Changes in the Rounding Segments of the Target Pressure Distributions

In this Section results from four design runs are given to show the effects of introducing changes to the rounding portions of the target pressure distribution. The example of Section 4.1 is considered as a datum for this purpose. The first results correspond to a **sharper** link between the rapid acceleration and the linear forward pressure recovery segment than in the datum. This sharper link in the target pressure distribution is achieved by specifying decreases in the parameters i_G and r_a . These are changed from $i_G = 30$ and $r_a = 0.05$ (as in Section 4.1), to $i_G = 4$ and $r_a = 0.005$. All the other parameters are left unchanged. The results of the design run with these parameters are given in Figure 9. In comparison with the datum example, according to the VGK method, the sharper link in the target pressure distribution has resulted in a slight increase in C_{Ldes} (of about 1 per cent), as would be expected, and an even smaller relative rise in the corresponding drag coefficients (from $C_{DV} + C_{D1} = 0.00794$ to $C_{DV} + C_{D1} = 0.00795$). A **more rounded** link, achieved by specifying $i_G = 30$ and $r_a = 0.1$, yields the results shown in Figure 10. These show a slight decrease in C_{Ldes} (of about 0.5 per cent) and a relatively smaller decrease in the corresponding drag coefficient, in comparison with the datum example. Comparison of Figures 1c, 9c and 10c (repeated for convenience, in Figure 11) shows the different ordinate changes, $\Delta\zeta_e$, required to achieve the three slightly different target pressure distributions. It is clear that the slightly different targets result in considerably different aerofoil surface slopes near the leading edge.

Changes in the more aft rounding link are obtained by changes in r_b (and/or f_b). Effects of changing r_b from 0.2 (as in the datum example) to 0.001 are shown in Figure 12, and to 0.4 in Figure 13. In each case $f_b = 1.0$. As with changes in the forward link, small changes in C_{Ldes} and $C_{DV} + C_{D1}$ are produced, the values being shown in the figures.

Figure 14 compares the values of $\Delta\zeta_e$ given in Figures 1c, 12c and 13c. The effects on the ordinates of the designed aerofoils resulting from changes in the more aft rounding are much less pronounced than in the case of changes in the more forward rounding. This is seen by comparison of Figures 11 and 14.

4.5 Examples of Effects of Changes in the Rearward Segment of the Target Pressure Distribution

In this Section, examples are presented to show the effects of changes from the linear form in the rearward segment of the target pressure distribution. As was done in Section 4.4, the example presented in Section 4.1 is taken as a datum for comparison purposes. For all the examples in this section, the starting aerofoil used is AFA41, *i.e.* that obtained in the design run of Section 4.1, which utilised the RAE 2822 as the starting aerofoil geometry.

With $\Delta C_{pM} = 0.1$, and the starting incidence, $\alpha_i = 2.150$ deg (*i.e.* the *final* value of incidence obtained in the design run of Section 4.1) and all the other design parameters kept the same as for the datum example, the results given in Figure 15 are obtained. It is seen that, in comparison with the datum example, the change in the rearward segment has decreased C_{Ldes} by about 7 per cent, but $C_{DV} + C_{D1}$ has decreased by only 1 per cent, approximately. Scrutiny of the relevant .FUL file reveals that no artificial raising of C_f has been imposed in this case, whereas it was for the datum design. Thus the design of Figure 15 should be somewhat more reliable (in the sense of its design condition performance being more reliably predicted by the VGK calculations), than the datum example.

Increasing ΔC_{pM} to 0.2 (keeping all other parameters unchanged) provides the results shown in Figure 16. Compared with the datum example, C_{Ldes} has decreased by about 13 per cent, whilst the value of $C_{DV} + C_{D1}$ has hardly changed. Again the relevant .FUL file reveals that no artificial raising of C_f has been imposed. For this example Figure 16b shows that the upper- and lower-surface pressure distributions cross over near the trailing-edge. It is clear that the thickness distribution employed (which is that of the RAE 2822 aerofoil) is unlikely to be considered suitable for this particular form of target pressure

distribution. A thickness distribution with a greater degree of concavity over the rearward region might be more suitable and avoid the cross-over shown in Figure 16b, whilst yielding a higher value of C_{Ldes} .

Use of a negative value for ΔC_{pM} would generally be expected to result in a more adverse pressure distribution for the upper-surface boundary-layer development (in overall terms) than is obtained with either zero or small positive values. It is found in the present case that only small negative ΔC_{pM} values may be employed, otherwise the number of positions where artificial raising of C_f takes place in the VGK calculations becomes large enough to indicate that the calculations are highly unreliable. Indeed, setting $\Delta C_{pM} = -0.1$ results in C_f being artificially raised at 4 grid points close to the trailing edge, and so only limited reliance can be placed on the VGK predictions obtained. Results of the design run corresponding to $\Delta C_{pM} = -0.1$ are shown in Figure 17.

4.6 Example of Design with Subsonic Flow

The parameters in this example are the same as for the example in Section 4.1, except for $M_{\infty des}$, which is 0.60 (rather than 0.72) and i_G , which is set to 6, rather than 30. The value of C_{p0} corresponds to $M = 0.97$, and so the flow corresponding to the target pressure distribution is fully subsonic.

Results obtained from the design run are given in Figure 18. The design task may be considered to have been achieved satisfactorily, although, as seen in Figure 18b, the target pressures are not very well achieved near the leading-edge. Further design cycles might improve the match with the target pressure distribution. In Figure 18a, the kink at ICYCLE = 200 is associated with a change in one of the pre-set parameters in the design algorithm (*i.e.* ζ_f) which occurs at this value (see Appendix B).

As seen in Figure 18c and 18d the added camber required to meet the design objective is considerably greater than in Section 4.1. The designed aerofoil, named AFA44, gives $C_{Ldes} = 0.875$. Figure 18b shows a comparison of the pressure distributions of AFA44 and of RAE 2822 (the starting aerofoil) at $C_L = 0.875$. It is seen that the flow around the RAE 2822 aerofoil has a strong shock at these conditions, and that the camber added to change this aerofoil into AFA44 has served to eliminate the shock and thus to decrease the drag. One further result of the added camber is to change the value of C_M at $C_L = 0.875$ from -0.0728 (for RAE 2822) to -0.1189 (for AFA44), corresponding to an aft movement of the centre of lift.

4.7 Example of Failure of Design Method

It is known that if the maximum value specified for M is too high, by virtue of having values of C_{pt} too low, then either the specified shock-free flow will be unattainable or, if attainable, it may be unsuitable in some other way. For $M > 1.58$, an isentropic flow is unstable to unsteady disturbances (see Ref. 17), but even Mach numbers above 1.4 are generally held to be unlikely to be of practical use, in that, even if achieved, the resultant aerofoil may generate unacceptably strong shocks at off-design conditions. In this example the design method is run to try to achieve a target pressure distribution with $C_{p0} = -1.5$. With i_G set to 10 and the other parameters the same as in Section 4.1, this gives a maximum Mach number in the upper-surface flow of about 1.32.

The results, which are shown in Figure 19, indicate that the design objective has not been achieved. In Figure 19a the irregular nature of the successive design cycles is apparent, and Figure 19b reveals that this is due to shock waves forming in the upper-surface flow, which move to different locations in successive cycles, but do not weaken – as would be required for the objective to be achieved. Unfortunately it cannot be stated that no successful design could be achieved, only that this method does not achieve it with the parameter values employed. It is probable that the reason for this is that the target pressure distribution is unsuitable, but this is not proven.

4.8 Example of Design with Inviscid Flow Assumptions

An example is given in this section of the design method used with inviscid flow assumptions. Such use would be of interest if the changes in attainable designs attributable to boundary layers and viscous wakes were to be investigated, for example.

The example parameters are the same as those in Section 4.1, but the VGK calculations employed are inviscid, rather than viscous. The principal results are presented in Figure 20. The design run is seen to be successful. In Figure 20b, it is seen that the upper- and lower-surface pressure distributions cross over near the trailing edge. This is a consequence of the imposition of the linear pressure recovery form for the rearward segment of the target upper-surface pressure distribution, coupled with the relatively high value of C_{pTE} (of about 0.35) associated with inviscid flow. (The value of C_{pTE} for the viscous flow example of Section 4.1 is about 0.23, see Figure 1b, and there is no cross-over.) The differences in target upper-surface pressure distribution between the viscous and inviscid examples also largely account for the difference between the values of C_{Ldes} in the two cases, namely 0.770 in the viscous example, but only 0.728 in the inviscid case.

It is concluded that the design method can operate satisfactorily with inviscid as well as with viscous-flow assumptions.

5. LIMITATIONS AND PRECAUTIONS

The design method is limited principally by the use of the VGK method for the prediction of the flow around each aerofoil geometry. Any designed aerofoil obtained which has a flow at the design conditions which is not reliably predicted by the VGK method should be treated with scepticism. A full description of the VGK method and comparisons of VGK results with experiment are presented in Part 1 (Ref. 1). Generally, doubts about reliability arise when the boundary layer on either the upper or lower aerofoil surface is close to separation. It may be noted that Part 3 (Ref. 3) provides a means of utilising VGK results to estimate a separation boundary.

A further limitation is associated with cases where the design method fails to converge. It is not then possible to distinguish the cause of such failure. It may either be due to inadequacy of the design algorithm or the target pressure distribution might correspond to an impossible or impractical flow.

So far as precautions are concerned, these simply relate to taking care that any design run undertaken does very nearly produce the desired upper-surface flow conditions and also ensuring that the geometry of any designed aerofoil is sufficiently smooth. Minor differences between the achieved and specified pressure distributions are not likely to have any significant effect on the aerodynamic characteristics of the designed aerofoils, but lack of smoothness would be very detrimental. It is recommended that each starting aerofoil be checked for smoothness, and also the file DIFFS.DAT (containing $\Delta\zeta$ values) should be similarly scrutinised.

6. REFERENCES

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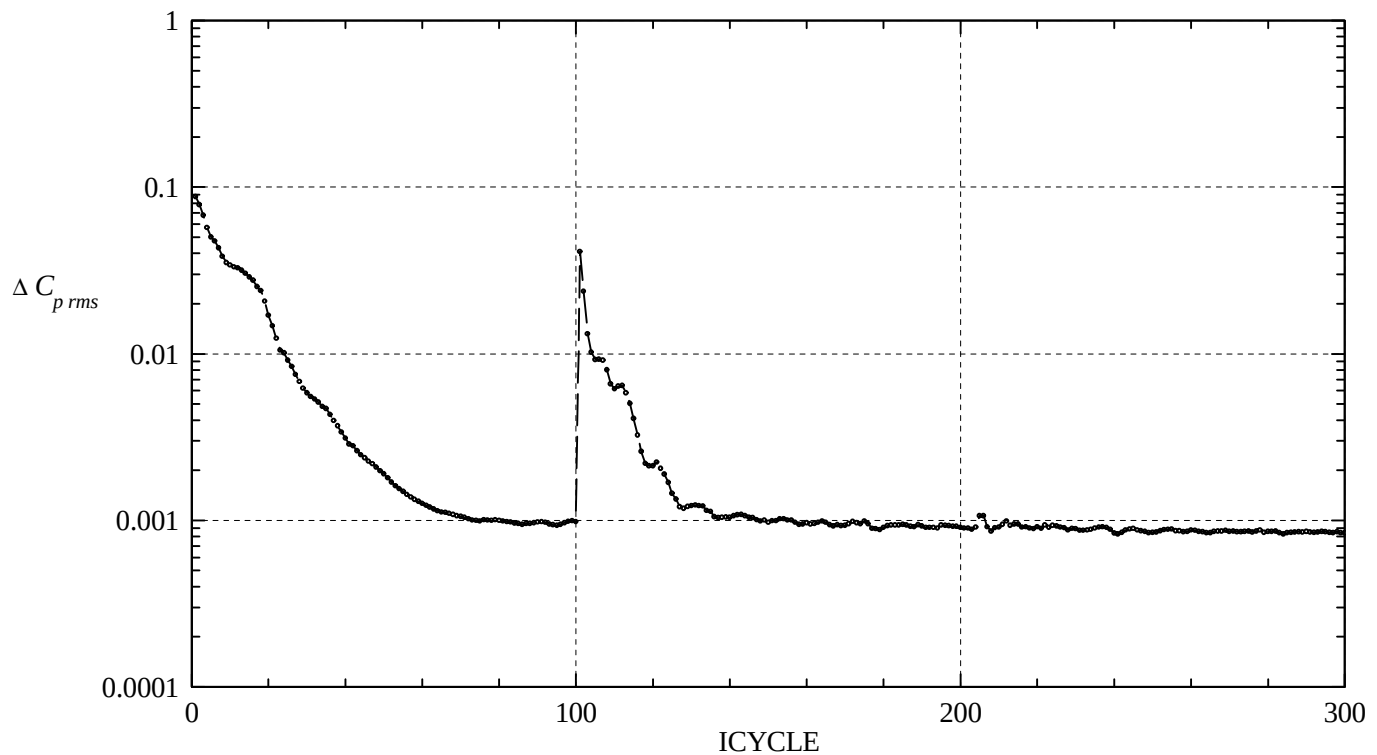


FIGURE 1a Principal results of design run described in Section 4.1:
Variation of root-mean-square difference between target and current
pressure distribution with number of design cycles completed

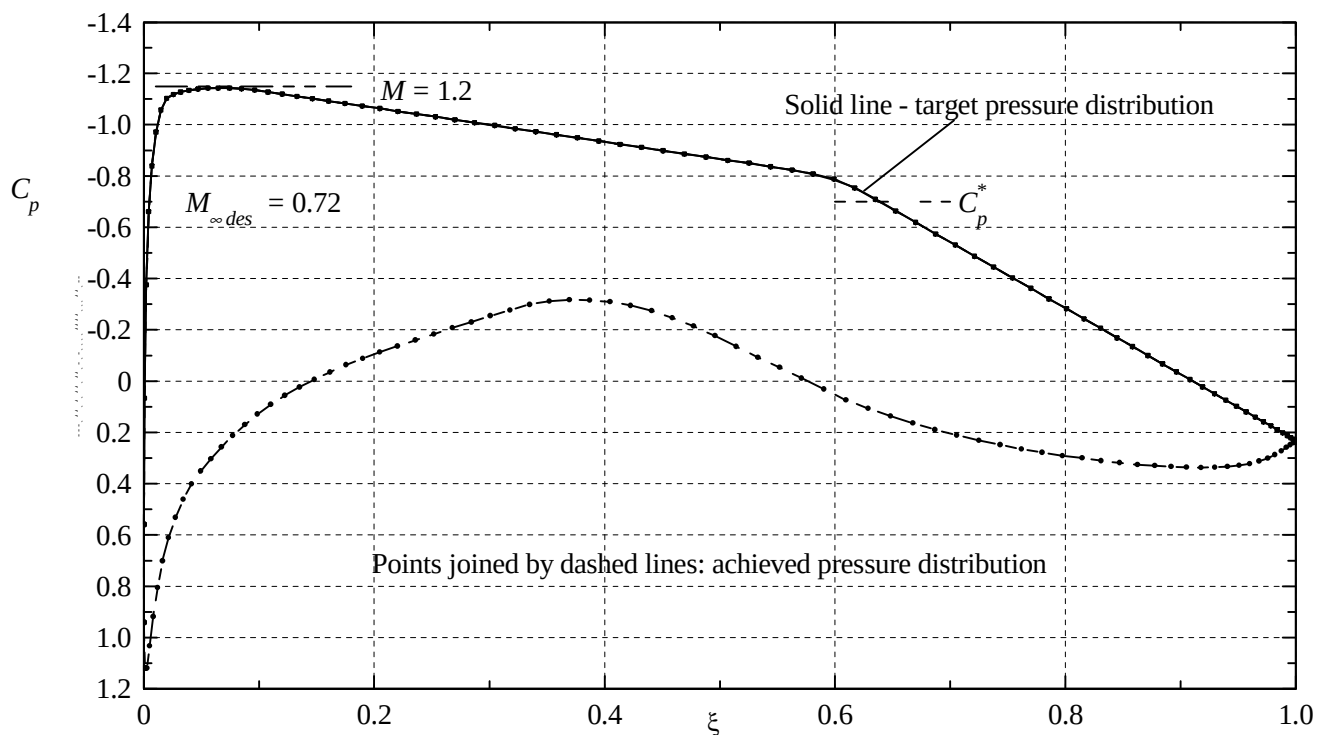


FIGURE 1b Principal results of design run described in Section 4.1:
Pressure distribution about designed aerofoil at end of run, and target upper-surface pressure distribution

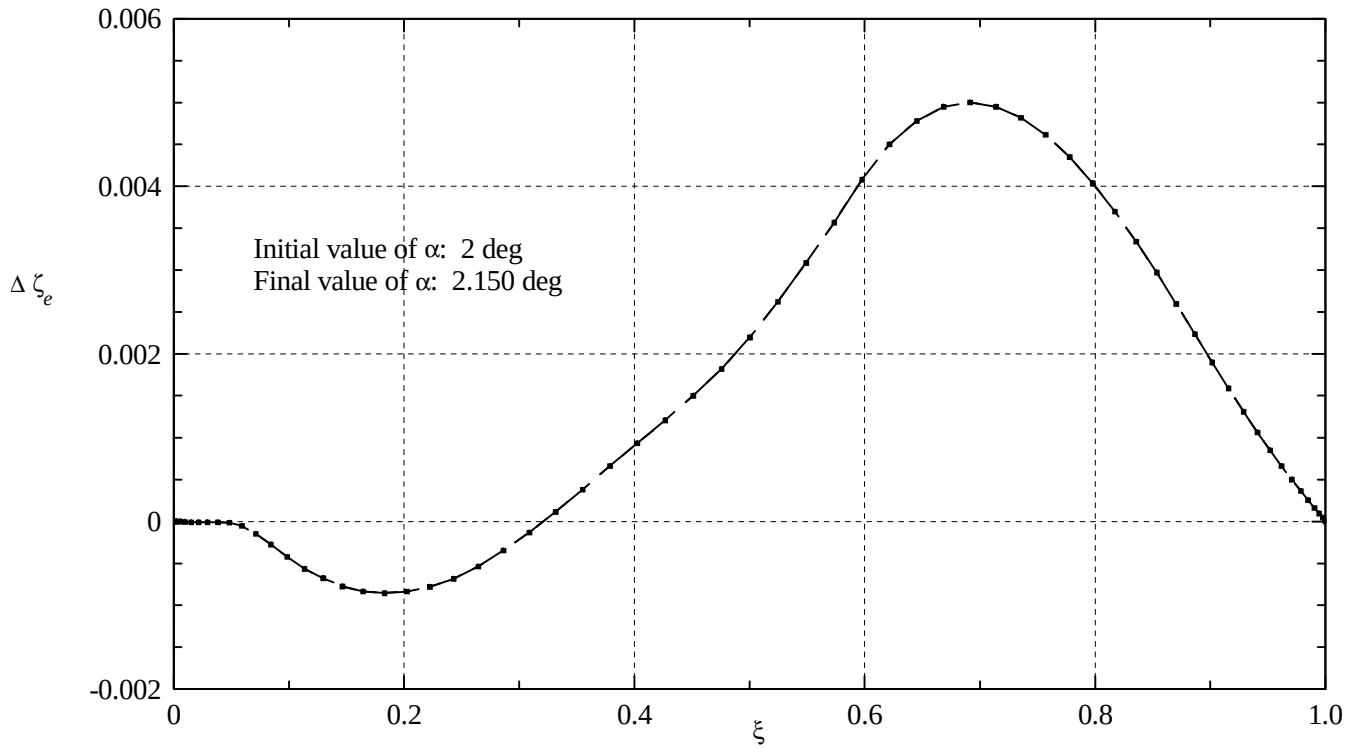


FIGURE 1c Principal results of design run described in Section 4.1:
Distribution of difference in ζ ordinates between final (designed) aerofoil and starting aerofoil

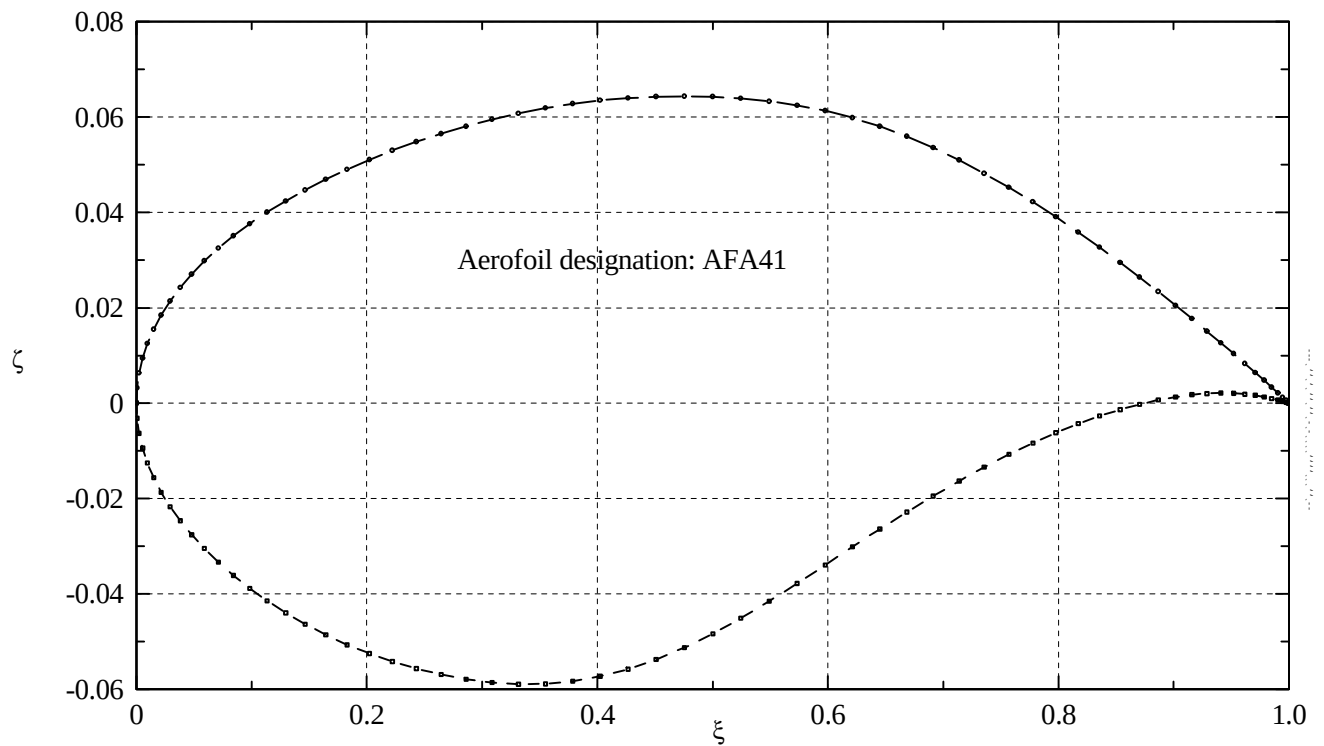


FIGURE 1d Principal results of design run described in Section 4.1:
Aerofoil profile at end of design run

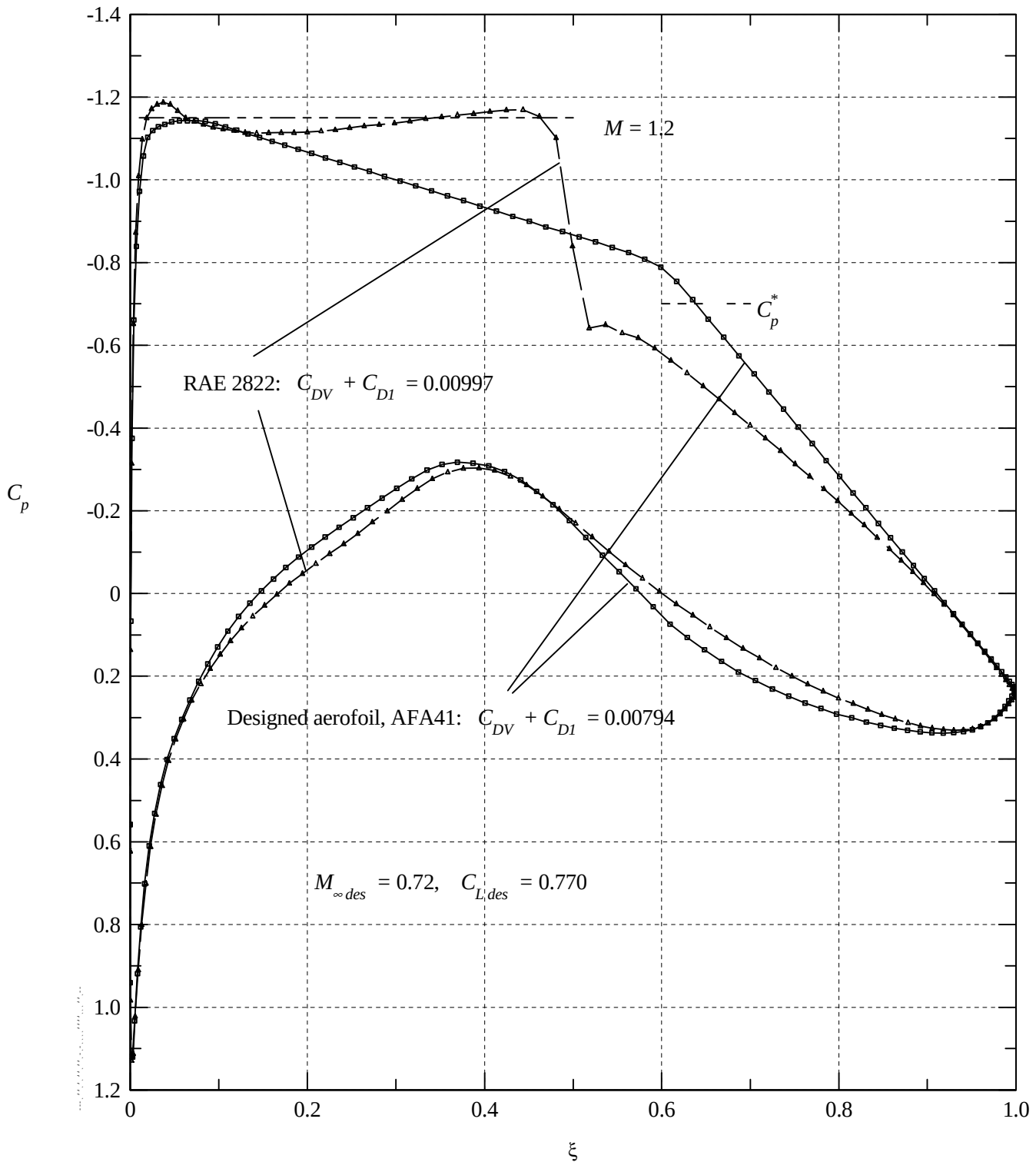


**FIGURE 1e Principal results of design run described in Section 4.1:
Pressure distribution about designed aerofoil and starting aerofoil at
design Mach number and resultant lift coefficient**