

THERMODYNAMIC PROPERTIES OF AIR IN DISSOCIATION EQUILIBRIUM

1. NOTATION AND UNITS

		<i>SI</i>	<i>British</i>
a	speed of sound	m/s	ft/s
e	specific internal energy	N m/kg	ft lbf/slug
i	specific enthalpy	N m/kg	ft lbf/slug
m_0	standard molecular mass for air taken as 29 kg/kmol (slug/slug-mol)	kg/kmol	slug/slug-mol
m_{av}	average molecular mass	kg/kmol	slug/slug-mol
p	static pressure	N/m ²	lbf/ft ²
p_0	datum pressure, taken as one atmosphere (1.01325 × 10 ⁵ N/m ² , 2.11622 × 10 ³ lbf/ft ²)	N/m ²	lbf/ft ²
R_0	gas constant, taken as 287.1 N m/kg K, 3090 ft lbf/slug K	N m/kg K	ft lbf/slug K
s	specific entropy	N m/kg K	ft lbf/slug K
T	temperature	K	K
Z	real gas factor, $p/\rho R_0 T$		
β	dimensionless enthalpy parameter, $i/(p/\rho)$		
γ^*	modified specific heat capacity ratio, see Equation (2.1)		
ρ	density	kg/m ³	slug/ft ³

2. NOTES

Figures 1 to 5 give, respectively, the temperature, T , the enthalpy parameter, β , the speed of sound, a , the entropy parameter, s/R_0 , and the modified specific heat capacity ratio, γ^* , as functions of pressure ratio, p/p_0 , and i/R_0 for argon-free air. It is assumed that the gas is in dissociation equilibrium and that the perfect gas laws apply to each component.

The modified specific heat capacity ratio, γ^* , is defined by

$$\gamma^* = \frac{\rho}{p} \left[\frac{\partial p}{\partial \rho} \right]_s = \frac{i}{e} + e \left[\frac{\partial(i/e)}{\partial e} \right]_s \quad (2.1)$$

where $[\]_s$ indicates that the derivative is taken at constant specific entropy. Thus, for small changes at constant entropy, p/ρ^{γ^*} is a constant and, since $e[\partial(i/e)/\partial e]_s$ is usually less than 7 per cent,

$$e \approx \frac{i}{\gamma^*}. \quad (2.2)$$

Since the speed of sound, a , is the speed of propagation of pressure disturbances of infinitesimal magnitude

$$a^2 = \left[\frac{\partial p}{\partial \rho} \right]_s = \frac{\gamma^* p}{\rho}. \quad (2.3)$$

The real gas factor, Z , is given by

$$Z = \frac{p}{\rho R_0 T} = \frac{i}{R_0 T \beta} \quad (2.4)$$

and, since $p/\rho R_0 T$ is inversely proportional to the molecular mass, it follows that the average molecular mass, m_{av} , is given by

$$m_{av} = \frac{m_0}{Z}. \quad (2.5)$$

The dashed portions of the curves giving a and γ^* have been obtained from Derivation 7 in which the air is considered to be a mixture of nitrogen, oxygen, and argon in the ratio 0.781:0.209:0.01. All other data were obtained from Derivations 1, 3 and 5, in which the air is considered to be a mixture of nitrogen and oxygen in the ratio 0.788 to 0.212, the normal ratio of nitrogen to oxygen being increased to make up for small amounts of carbon dioxide and argon which are omitted. That these omissions give rise to negligible errors is shown by comparison with the results of Derivation 2, which include the effects of argon, and those of Derivation 8, which include the effects of carbon dioxide.

3. DERIVATION

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|----|---|---|
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4. EXAMPLE

Find the density, entropy, internal energy, speed of sound and average molecular mass for air at 4000 K and 101.3 N/m².

From the given information

$$T = 4000 \text{ K and } \log_{10}(p/p_0) = -3,$$

so that, from Figure 1, $i/R = 33\,000 \text{ K}$

$$\begin{aligned} i &= 33\,000 \times 287.1 \\ &= 9.47 \times 10^6 \text{ N m/kg.} \end{aligned}$$

From Figure 2, with $i/R = 33\,000$,

$$\beta = 6.68$$

$$\begin{aligned} \text{so that } \rho &= \frac{\beta p}{i} = \frac{6.68 \times 101.3}{9.47 \times 10^6} \\ &= 7.15 \times 10^{-5} \text{ kg/m}^3. \end{aligned}$$

From Figure 5, $\gamma^* = 1.14$,

$$\begin{aligned} \text{whence } e &\approx \frac{i}{\gamma^*} = \frac{9.47 \times 10^6}{1.14} \\ &= 8.31 \times 10^6 \text{ N m/kg.} \end{aligned}$$

From Figure 4, $s/R = 47.2$,

$$\begin{aligned} \text{so that } s &= 47.2 \times 287.1 \\ &= 1.36 \times 10^4 \text{ N m/kg K.} \end{aligned}$$

From Equation (2.4)

$$\begin{aligned} Z &= \frac{33\,000}{4000 \times 6.68} \\ &= 1.235, \end{aligned}$$