

HEAT TRANSFER UNDER CONDITIONS OF FORCED CONVECTION FOR THE SUBSONIC TURBULENT FLOW OF GASES IN SMOOTH STRAIGHT DUCTS OF CONSTANT CROSS SECTION

1. NOTATION AND UNITS

		<i>SI</i>	<i>British</i>
A	cross-sectional area of duct	m^2	ft^2
c_p	specific heat capacity at constant pressure	J/kg K	Chu/slug K
D	duct diameter or hydraulic diameter, $4A/P$	m	ft
h	heat transfer factor (see Sections 2 and 3.4)	$\text{W/m}^2 \text{ K}$	$\text{Chu/ft}^2 \text{ s K}$
k	thermal conductivity	W/m K	Chu/ft s K
Nu	Nusselt number, based on diameter, hD/k		
P	wetted perimeter	m	ft
Pr	Prandtl number, $\mu c_p/k$		
q	rate of heat flux from wall to gas	W/m^2	$\text{Chu/ft}^2 \text{ s}$
R_D	Reynolds number based on diameter, $\rho V D/\mu$		
St	Stanton number, $h/\rho V c_p$		
T	gas static temperature	K	K
T_H	gas total or stagnation temperature	K	K
T_r	recovery temperature	K	K
T_w	wall temperature	K	K
V	velocity	m/s	ft/s
x	distance measured along duct axis	m	ft
μ	viscosity	kg/m s	slug/ft s
ρ	density	kg/m^3	slug/ft^3

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Subscripts

c	appropriate to constant gas properties
f	appropriate to fully developed conditions
m	mean value between station x and $x = 0$

2. GENERAL

This Item gives information on heat transfer, based on the correlation of experimental data, for the subsonic turbulent flow of gases in smooth ducts of constant cross section. The data are applicable only to flow rates and thermal boundary conditions which are independent of time. Furthermore, the thermal boundary conditions are restricted to those of uniform heat flux or constant wall temperature. However, the data may be adapted to conditions of arbitrary heat flux and this extension is treated in Item No. Aero 00.03.26 (Reference 35).

In order to present the information it is usual to consider certain non-dimensional parameters.

Consider the incompressible flow in a circular tube of a fluid with a fully developed temperature profile. Let the heat transfer factor be defined by the expression

$$h = \frac{q}{T_w - T}, \quad (2.1)$$

where q is positive when heat is added from the wall to the gas (negative when heat is extracted from the gas) and T is a suitably defined gas static temperature. The definition of T is considered in the final paragraph of this section and further discussion to generalise the definition of h for compressible flow is contained in Section 3.4.

Dimensional reasoning then leads to the following equivalent expressions:

$$\frac{hD}{k} = f_1\left(\frac{\rho VD}{\mu}, \frac{\mu c_p}{k}\right) \quad (2.2a)$$

and
$$\frac{h}{\rho V c_p} = f_2\left(\frac{\rho VD}{\mu}, \frac{\mu c_p}{k}\right). \quad (2.2b)$$

The variables in Equations (2.2a) and (2.2b) may be written in terms of non-dimensional parameters and are respectively

$$Nu = f_1(R_D, Pr) \quad (2.3a)$$

$$St = f_2(R_D, Pr). \quad (2.3b)$$

Equation (2.3a) is the form in which the non-dimensional parameters are frequently presented, but Equation (2.3b) has certain advantages (Derivation 34) and will be the basis for the presentation of information in the present Item.

The Nusselt, Stanton, Reynolds and Prandtl numbers are related by the expression

$$Nu = St \times Pr \times R_D. \quad (2.4)$$

Figure 1 presents the variation of Stanton number with Reynolds number, for $Pr = 0.72$, for flow with a fully developed temperature profile under conditions of low heat flux. The corrections which should be applied to Figure 1 to obtain the local and mean Stanton numbers in the thermal entrance region (see Section 3.3) are shown in Figures 2 and 3 respectively. Under conditions of high heat flux the variation of gas properties becomes important and Figures 4 and 5 present the influence of wall to total temperature ratio. Where the above effects occur simultaneously the Stanton number is derived by the method illustrated in Examples 1 and 2. Other factors that influence the heat transfer rate are considered in the text.

The flow within a straight duct is analogous to that in a boundary layer with the velocity, temperature, density and transport properties varying over the cross section, but with the static pressure remaining constant over the cross section. However, the variation along the duct of the average fluid properties at any cross section may be effectively computed by analysing an equivalent one-dimensional flow having the same mass flow and flux of energy. This Item is based on the concept of such an equivalent flow, for which the average flow properties at any section, ρ , V and T are defined by ensuring that they satisfy the equivalent one-dimensional equations corresponding to the flow system. The representative values of the flow properties at a section so defined are equivalent to the “bulk” values frequently encountered in papers on internal heat transfer.

3. VARIABLES WHICH INFLUENCE HEAT TRANSFER RATE

3.1 Fully Developed Temperature Profile

In this section the turbulent flow at low Mach numbers and low, uniform rates of heat flux in ducts of circular cross section is considered. As a further consequence of considering low rates of heat flux it will be assumed that the gas properties are sensibly constant.

Downstream from the commencement of heat transfer the temperature profile develops from the tube wall, with a corresponding reduction of the constant temperature region at the centre of the pipe, until at some stage the constant temperature region is completely eliminated. The subsequent temperature profile and associated flow conditions are referred to as “fully developed”. For constant gas properties the heat transfer factor (and hence Stanton number) in the fully developed region is constant.

Figure 1 is a plot of $St_{f,c}$ against R_D for the fully developed temperature profile for $Pr = 0.72$. The curve is based on results in Derivations 1, 3, 8 and 26, and includes tests on air, helium and carbon dioxide. The curve applies to most common gases including air, oxygen, hydrogen, helium, nitrogen, argon, carbon monoxide and carbon dioxide. This is because all these gases have a Prandtl number close to 0.72 over a very wide temperature range, and the Stanton number is not strongly dependent on Prandtl number in this range. For gases with Prandtl numbers appreciably different from 0.72 (e.g. ammonia and steam) Figure 1 should not be used. Instead, at Reynolds numbers greater than 20000, the formula

$$St_{f,c} = 0.023 R_D^{-0.2} Pr^{-0.6} \quad (3.1)$$

gives good agreement with experiment.

An average scatter band of about ± 10 per cent was associated with the experimental data from which Figure 1 was constructed.

The solid line in Figure 1 corresponds to the extent of available experimental data whereas the dotted line represents an extrapolation to higher Reynolds numbers.

3.2 Thermal Boundary Conditions

It is implicit in the use of the heat transfer factor that it is associated with a thermal boundary condition of a prescribed form. The conditions of uniform heat flux and constant wall temperature give rise in the fully developed region to heat transfer factors of very nearly equal magnitude, which are constant along the duct. These two types of thermal boundary are those most frequently approached in practice and may be considered as standard cases.

The curves in this Item apply, strictly, only to conditions of uniform heat flux or of constant wall temperature. However, the information may be applied directly to other thermal boundary conditions provided that over the entire heat transfer surface, with the exception of the points at commencement and completion of heat transfer, the inequality

$$\frac{1}{q} \frac{dq}{d(x/D)} \ll 1 \quad (3.2)$$

is satisfied. Under these conditions the expression $(1/q)dq/d(x/D)$ is an approximate measure of the error associated with this approach. If, for certain problems, these conditions cannot be satisfied it is then necessary to resort to a more detailed analysis such as that provided in Item No. Aero 00.03.26.

3.3 Thermal Entrance Region

The region following the commencement of heat transfer and prior to the establishment of a fully developed temperature profile is referred to as the thermal entrance region.

Two main types of thermal entrance may be classified. The first is that for which the temperature profile develops when a fully developed velocity profile is already established. Heat transfer measurements for this type of thermal entrance for conditions of uniform heat flux are reported in Derivations 18, 29, 32 and 33 and for conditions approaching constant wall temperature in the fully developed region in Derivation 1. Agreement between all the tests is good and Figure 2 is a curve of the ratio of the local Stanton number to the fully developed value plotted against x/D ; this Stanton number ratio is independent of Reynolds number for $R_D > 20000$. The second type of thermal entrance is that for which the velocity and temperature profiles develop simultaneously. In this case the heat transfer factor is, for the first few diameters, very dependent on the entry configuration. Derivations 1 and 32 show the effect of several different entry shapes encountered in practice such as bends, elbows, etc. besides considering more straightforward designs such as sharp-edged and bellmouth entries. No simple correlation of the experimental data was possible and Derivations 1 and 32 should be referred to for more detailed information. A crude approximation, which in certain instances could yield errors of up to 100 per cent in the first one or two diameters, is to use the curve of Figure 2, since this gives values somewhere between the extremes for the configurations tested.

Sometimes the overall effect of the variation of the Stanton number in the thermal entrance region, rather than the local variation is required. Figure 3 is based on the same experimental data as Figure 2 and gives $St_{m,c}$ plotted against x/D , where

$$St_{m,c} = \frac{1}{x} \int_0^x St_c dx . \quad (3.3)$$

For $x/D > 20$, the mean Stanton number is given closely by the expression

$$\frac{St_{m,c}}{St_{f,c}} = 1 + \frac{2}{x/D}. \quad (3.4)$$

3.4 Influence of Mach Number

For flow at higher Mach numbers the gas temperature on which the heat transfer factor is based is, to some extent, open to a choice between T , T_H or T_r . However, it is clear that when $T_w = T_r$ then $q = 0$. This leads logically to the choice of T_r and it has been found experimentally (Derivation 2) that, at subsonic speeds, the influence of Mach number on the Stanton number may be eliminated by defining the heat transfer factor in terms of this temperature. In many flow problems the temperature difference $(T_w - T_r)$ is very nearly equal to $(T_w - T_H)$ (e.g. at high rates of heat flux and/or low subsonic speeds) and hence in general it is more convenient to redefine the heat transfer factor in compressible flows by the relation

$$h = \frac{q}{T_w - T_H}. \quad (3.5)$$

With this definition established it is advantageous to reconsider the definition of h , given by Equation (2.1), for the incompressible flow of a gas. Since at low speeds T and T_H are very nearly equal, and consequently $(T_w - T_H)$ and $(T_w - T)$ are also very nearly equal, the definition of the heat transfer factor given by Equation (3.5) may be generalised to apply over the complete subsonic speed regime. Errors associated with this revised definition of the heat transfer factor are only significant when high subsonic speeds and small temperature differences $(T_w - T_H)$ occur simultaneously.

3.5 Variable Gas Properties

So far in this Item it has been assumed that the properties of the gas have not been changed as a result of the heat transfer process. This assumption is adequate at low rates of heat transfer. High rates of heat flux, at constant mass flow, correspond to large differences in the wall and gas temperatures. Thus, if the actual flow is considered (instead of the equivalent one-dimensional model) it is apparent that there are large radial temperature variations. Furthermore, the high rates of heat flux produce large temperature changes in the axial direction. The properties of gases are strongly dependent on temperature and it has been established experimentally that the local Stanton number is modified as a result of the high rates of heat flux. In general, where the effects of variable gas properties become appreciable there no longer exists a region where the heat transfer factor is constant.

Heat transfer measurements at high, uniform rates of heat flux are reported in Derivations 4, 6, 8, 16 to 21, 23, 24, 26 and 27 and include data on air, hydrogen, helium and carbon dioxide. The tests covered x/D in the range $14 < x/D < 120$ and both local and mean values of the heat transfer factor were measured. In addition high subsonic speeds were achieved in some of the tests. In Figures 4 and 5 separate curves are presented for the influence of wall to total temperature ratio on the local and mean Stanton numbers respectively. Where applicable, in the derivation of these curves corrections for the thermal entrance region effects were made from Figures 2 and 3.

Sometimes, e.g. when the wall temperature is not known, it is necessary to resort to iterative methods to extract the required information from Figure 4 or 5. For the mean Stanton number data under conditions of uniform heat flux the value of T_{Hm} should be taken equal to the value of T_H at the point midway along the heat transfer surface and gas properties should be evaluated at T_{Hm} . The integrated mean value of the wall temperature along the heat transfer surface is T_{wm} but for general purposes may be taken equal to the value of the T_w at the point midway along the heat transfer surface.