

ISENTROPIC EXPANSION OF AIR FROM REST

1. NOTATION AND UNITS

		<i>SI</i>	<i>British</i>
a	speed of sound in air at velocity V , $(\gamma p/\rho)^{1/2}$	m/s	ft/s
a_H	speed of sound in air at rest, $(\gamma p_H/\rho_H)^{1/2}$	m/s	ft/s
M	Mach number of air at velocity V , V/a		
p	static pressure of air at velocity V	N/m ²	lbf/ft ²
p_H	pressure of air at rest	N/m ²	lbf/ft ²
T	temperature of air at velocity V	K	K
T_H	temperature of air at rest	K	K
V	velocity of air after expansion	m/s	ft/s
γ	ratio of specific heat capacity of air at constant pressure to specific heat capacity at constant volume, taken as 1.4		
ρ	density of air at velocity V	kg/m ³	slug/ft ³
ρ_H	density of air at rest	kg/m ³	slug/ft ³

2. NOTES

The ratios T/T_H , ρ/ρ_H , V/a and V/a_H are plotted against p/p_H . The boundary between subsonic and supersonic flow, occurring at $p/p_H = 0.528$, is indicated on Figure 1.

It is assumed that air behaves as a perfect gas and that the expansion from pressure p_H to pressure p takes place isentropically according to the law $p/\rho^\gamma = \text{constant}$ with $\gamma = 1.4$. The curves are derived from the following equations:

$$\frac{T}{T_H} = \left(\frac{p}{p_H}\right)^{(\gamma-1)/\gamma} = \left(\frac{p}{p_H}\right)^{2/7}, \quad (2.1)$$

$$\frac{\rho}{\rho_H} = \left(\frac{p}{p_H}\right)^{1/\gamma} = \left(\frac{p}{p_H}\right)^{5/7}, \quad (2.2)$$

$$\frac{V}{a} = M = \left\{ \frac{2}{\gamma-1} \left[\left(\frac{p}{p_H}\right)^{-(\gamma-1)/\gamma} - 1 \right] \right\}^{1/2} = \left\{ 5 \left[\left(\frac{p}{p_H}\right)^{-2/7} - 1 \right] \right\}^{1/2}, \quad (2.3)$$

$$\frac{V}{a_H} = \left\{ \frac{2}{\gamma - 1} \left[1 - \left(\frac{p}{p_H} \right)^{(\gamma - 1)/\gamma} \right] \right\}^{1/2} = \left\{ 5 \left[1 - \left(\frac{p}{p_H} \right)^{2/7} \right] \right\}^{1/2}. \quad (2.4)$$

3. DERIVATION

1. ROSENHEAD, L. *Compressible airflow: tables.*
Clarendon Press, 1952.
2. – *Equations, tables and charts for compressible flow.*
NACA Rep. 1135, 1953.
3. LIEPMANN, H.W. *Elements of gasdynamics.* Ch. 1 and 2.
ROSHKO, A. John Wiley & Sons, Inc., New York and London, 1957.

4. EXAMPLE

Find the Mach number, the density and temperature of air that has expanded isentropically to a pressure of 43 500 N/m², assuming it was initially at rest in a reservoir where the pressure was 101 400 N/m², the density 1.225 kg/m³ and the temperature 288 K.

For $p/p_H = 43\,500/101\,400 = 0.429$, from Figure 1,

$$\begin{aligned} T/T_H &= 0.785, \\ \rho/\rho_H &= 0.545, \\ V/a &= 1.17. \end{aligned}$$

Hence,

$$\begin{aligned} M &= 1.17, \\ \rho &= 0.545 \times 1.225 = 0.67 \text{ kg/m}^3, \\ T &= 0.785 \times 288 = 266 \text{ K}. \end{aligned}$$