

WALL AND AVERAGE GAS TEMPERATURES FOR SUBSONIC TURBULENT FLOW WITH HEAT TRANSFER IN DUCTS OF CONSTANT CIRCULAR CROSS SECTION

1. NOTATION AND UNITS

		<i>SI</i>	<i>British</i>
A, B	constants in Equation (5.6)		
c_p	specific heat capacity at constant pressure	J/kg K	Chu/slug K
D	duct diameter	m	ft
h	heat transfer factor, $q/(T_w - T_H)$	W/m ² K	Chu/ft ² s K
L	length of heat transfer surface	m	ft
m	mass flow	kg/s	slug/s
q	rate of heat flux from wall to gas	W/m ²	Chu/ft ² s
St	stanton number, $h/\rho V c_p$		
T_H	average gas total, or stagnation, temperature	K	K
T_w	wall temperature	K	K
V	velocity	m/s	ft/s
x	distance, measured along axis of duct, from commencement of heat transfer	m	ft
z	x/D		
α, β	constants in Equation (5.6)		
ρ	density	kg/m ³	slug/ft ³
ϕ^*	non-dimensional parameter defined by Equation (5.4)		
ψ	dummy variable of integration		
Suffixes			
f	denotes fully-developed		
m	denotes mean value over length L		
0	corresponding to $x = 0$		
Superfix			
$*$	appropriate to uniform heat flux conditions		

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2. GENERAL

This Item enables the variation of wall and gas temperatures along the length of a pipe to be determined for the subsonic turbulent flow of a perfect gas under conditions of forced convection heat transfer. It is assumed that the rates of heat flux are sufficiently low for the gas properties to be sensibly constant.

Simple calculations, based on information in Item No. Aero 00.02.05, are presented for the thermal boundary conditions of uniform heat flux and constant wall temperature.

It is necessary, in the general case of arbitrary heat flux distribution, to resort to a more comprehensive analysis as described in detail below.

The flow in a straight duct is analogous to that in a boundary layer, with the velocity, temperature, density and gas properties varying over the cross section, but with the static pressure remaining constant over the cross section. The variation of the wall and average gas temperatures along the duct may be effectively computed, however, by analysing an equivalent one-dimensional flow having the same mass flow and flux of energy. This Item is based on the concept of such an equivalent flow.

3. HEAT TRANSFER CALCULATIONS AND ENERGY BALANCE

The equations are set out for subsonic flow with constant gas properties. (Where velocities are sufficiently low for the flow to be considered incompressible, stagnation conditions may be replaced by static conditions.)

From an energy balance, found by equating the change in stagnation enthalpy with the heat transferred to the gas,

$$\frac{dT_H}{dx} = q \frac{\pi D}{mc_p}, \quad (3.1)$$

where the mass flow

$$m = \frac{\pi D^2}{4} \rho V. \quad (3.2)$$

Equation (3.1) may be written as

$$\frac{dT_H}{dx} = \frac{4}{D} (T_w - T_H) \frac{q}{(T_w - T_H) \rho V c_p}, \quad (3.3)$$

or
$$\frac{dT_H}{dx} = \frac{4}{D} (T_w - T_H) St. \quad (3.4)$$

In Equation (3.4)

$$St = \frac{q}{(T_w - T_H) \rho V c_p} = \frac{h}{\rho V c_p}. \quad (3.5)$$

St is the non-dimensional parameter known as the Stanton number, and is considered in detail in Item No. Aero 00.02.05.

4. GAS TEMPERATURE DISTRIBUTION ALONG A DUCT

4.1 Thermal Boundary of Uniform Heat Flux

Equation (3.1) may be integrated directly to give

$$T_H^* - T_{H0}^* = \frac{q^* \pi D}{mc_p} x = \frac{4q^*}{\rho V c_p} z, \quad (4.1)$$

where $x = 0$ corresponds to the commencement of heat transfer and T_{H0} is the average gas stagnation temperature at $x = 0$.

4.2 Thermal Boundary of Constant Wall Temperature

Equation (3.4) may be integrated to yield

$$\log_e \frac{\frac{T_w}{T_{H0}} - 1}{\frac{T_w}{T_{H0}} - \frac{T_H}{T_{H0}}} = \frac{4}{D} \int_0^x St \, dx = \frac{4x}{D} St_m, \quad (4.2)$$

where St_m is the mean Stanton number. The variation of St_m with x/D is given in Item No. Aero 00.02.05. Figures 1a and 1b give T_H/T_{H0} for a range of values of $4St_m x/D$ and T_w/T_{H0} .

Further discussion of this boundary condition will be found under Section 5.2.2.

4.3 Thermal Boundary of Specified Arbitrary Heat Flux Distribution

The variation of the average gas stagnation temperature along the duct may be determined directly by integration of Equation (3.1) and is

$$T_H - T_{H0} = \frac{\pi D}{mc_p} \int_0^x q \, dx = \frac{4}{\rho V c_p} \int_0^z q \, dz. \quad (4.3)$$

5. WALL TEMPERATURE DISTRIBUTION ALONG A DUCT

The general procedure for evaluating the wall temperature is to consider the identity

$$T_w = (T_w - T_H) + T_H. \quad (5.1)$$

The method for determining $T_H(x)$ has been considered in Section 4. This section will be concerned with the variation of $(T_w - T_H)$ with x . Equation (5.1) then gives T_w since $(T_w - T_H)$ and T_H are known separately.

5.1 Thermal Boundary of Uniform Heat Flux

The value of $(T_w - T_H)$ can be found from the identity

$$(T_w - T_H)^* = \frac{q^*}{\rho V c_p} \times \frac{(T_w - T_H)^* \rho V c_p}{q^*} = \frac{q^*}{\rho V c_p St^*(x)} \quad (5.2)$$

where $St^*(x)$ is a function of x/D in the thermal entrance region and is a constant, St_f^* , when the temperature profile is fully-developed.

It is convenient, therefore, to write Equation (5.2) in the form

$$(T_w - T_H)^* = \frac{q^*}{\rho V c_p St_f^*} \phi^*(z) \quad (5.3)$$

$$\text{where} \quad \phi^*(z) = \frac{St_f^*}{St^*(x)} = \frac{(T_w - T_H)^*}{(T_w - T_H)_f^*}. \quad (5.4)$$

It is shown in Item No. Aero 00.02.05 that $\phi^*(z)$ is independent of Reynolds number[†] when the Reynolds number is greater than 20000 and where heat transfer commences in the region of a fully-developed velocity profile. In this case $\phi^*(z)$ may be approximated very closely by the expressions

$$\left. \begin{array}{ll} 0 < z < \alpha & \phi^*(z) = 0 \\ \alpha < z < \beta & \phi^*(z) = A \log_e z + B \\ \beta < z & \phi^*(z) = 1 \end{array} \right\}, \quad (5.5)$$

$$\text{where} \quad A \log_e \alpha + \beta = 0 \quad \text{and} \quad A \log_e \beta + B = 1. \quad (5.6)$$

The recommended values of A, B, α, β as obtained from Figure 2 of Item No. Aero 00.02.05 are

$$A = 0.1, \quad B = 0.729, \quad \alpha = 0.00068, \quad \beta = 15.$$

The value of β effectively defines the thermal entrance length, whereas α may be considered as a small displacement which shifts the origin of heat transfer by an insignificant amount but at the same time permits a tractable analysis to be performed.

[†] $\phi^*(z)$ is a function of Prandtl number and hence, although the method is appropriate, the present results do not apply to viscous liquids or liquid metals. Also, it is interesting to note that in laminar flow $\phi^*(z)$ is a strong function of Reynolds number.