

CONDITIONS BEHIND NORMAL SHOCK WAVES IN A PERFECT GAS FOR VARIOUS VALUES OF γ

1. NOTATION AND UNITS

		<i>SI</i>	<i>British</i>
a	speed of sound in gas, $(\gamma RT)^{1/2}$	m/s	ft/s
M	Mach number, V/a		
p	static pressure of gas	N/m ²	lbf/ft ²
p_H	pressure of gas when brought to rest isentropically	N/m ²	lbf/ft ²
R	gas constant	J/kg K	lbf ft/slug K
T	static temperature of gas	K	K
T_H	temperature of gas when brought to rest isentropically	K	K
V	velocity of gas	m/s	ft/s
γ	ratio of specific heat capacity at constant pressure to specific heat capacity at constant volume		
ρ	density of gas	kg/m ³	slug/ft ³
Subscripts			
1	denotes conditions upstream of shock waves		
2	denotes conditions downstream of shock waves		

2. NOTES

For the flow of a perfect gas γ is constant and the equation of state is

$$\frac{p}{\rho} = RT. \quad (2.1)$$

The following relations then exist between the flow properties on the two sides of a normal shock:

$$T_{H1} = T_{H2}, \quad (2.2)$$

$$M_2^2 = \frac{(\gamma-1)M_1^2 + 2}{2\gamma M_1^2 - (\gamma-1)}, \quad (2.3)$$

$$\frac{p_2}{p_1} = \frac{2\gamma}{(\gamma+1)} (M_1^2 - 1) + 1, \quad (2.4)$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{2 + (\gamma-1)M_1^2}, \quad (2.5)$$

$$\frac{T_2}{T_1} = \frac{p_2}{p_1} \frac{\rho_1}{\rho_2}, \quad (2.6)$$

and
$$\frac{p_{H2}}{p_{H1}} = \left(\frac{p_H}{p}\right)_2 \frac{p_2}{p_1} \left(\frac{p}{p_H}\right)_1, \quad (2.7)$$

where
$$\frac{p_H}{p} = \left[1 + \frac{\gamma-1}{2} M^2\right]^{\frac{\gamma}{\gamma-1}}. \quad (2.8)$$

Equation (2.8) is plotted in Item No. Aero S.00.03.07.

Elimination of M_1^2 between Equations (2.4) and (2.5) yields

$$\frac{\rho_2}{\rho_1} = \frac{\left(\frac{\gamma+1}{\gamma-1}\right) \frac{p_2}{p_1} + 1}{\frac{p_2}{p_1} + \left(\frac{\gamma+1}{\gamma-1}\right)}. \quad (2.9)$$

Curves for M_2 , T_2/T_1 , p_2/p_1 , p_{H2}/p_{H1} and p_{H2}/p_1 are respectively plotted in Figures 1 to 5 for values of γ from 1 to 1.67 and for values of M_1 up to 4. (Note: $\gamma = 1$ is included in the curves as a limiting value for ease of interpolation.)

The density ratio, ρ_2/ρ_1 , may be calculated from Equation (2.1), using the values of T_2/T_1 and p_2/p_1 from Figures 2 and 3.

The assumption of a constant value of γ gives rise to increasing errors at high temperatures where caloric imperfections assume importance (see Item No. Aero 00.01.08).

3. ACKNOWLEDGMENT

Certain calculations on which this Item is based were made at the Royal Aircraft Establishment, Farnborough in 1963.

4. EXAMPLE

Find the temperature, pressure and density behind a normal shock wave moving at a Mach number of 3 in helium, where $T_1 = 218 \text{ K}$ and $p_1 = 44\,912 \text{ N/m}^2$. Take $\gamma = 1.67$, $R = 2081 \text{ J/kg K}$.

From Figure 2, for $\gamma = 1.67$, $M_1 = 3$,

$$T_2/T_1 = 3.68,$$

so that $T_2 = 3.68 \times 218 = 802 \text{ K}$.

From Figure 3, for $\gamma = 1.67$, $M_1 = 3$,

$$p_2/p_1 = 11.01,$$

so that $p_2 = 11.01 \times 44\,912 = 494\,500 \text{ N/m}^2$.

From Equation (2.1) $\rho_2 = \frac{p_2}{RT_2} = \frac{494\,500}{2081 \times 802} = 0.2963 \text{ kg/m}^3$.