

SOLUTIONS TO THE OBLIQUE SHOCK WAVE RELATIONS FOR A RANGE OF SPECIFIC HEAT RATIOS: PRESSURE COEFFICIENT AND DETACHMENT ANGLE

1. NOTATION

		SI	British
a_1	speed of sound upstream of shock wave	m/s	ft/s
C_p	pressure coefficient = $(p_2 - p_1)/\frac{1}{2}\gamma p_1 M_1^2$		
K_δ	flow deflection parameter = $(\gamma + 1)(M_1^2/\beta)\sin\delta$		
M_1	Mach number upstream of shock wave = V_1/a_1		
p_1, p_2	static pressures upstream and downstream of shock wave, respectively	N/m ²	lbf/ft ²
V_1	velocity upstream of shock wave	m/s	ft/s
β	compressibility factor = $(M_1^2 - 1)^{1/2}$		
γ	ratio of specific heat capacity at constant pressure to specific heat capacity at constant volume		
δ	flow deflection	deg	deg
δ_d	flow deflection for shock detachment	deg	deg
ε	error limits on pressure coefficient (per cent)		

2. NOTES

Figure 1, which is based on an approximate solution to the oblique shock wave relations^{1, 2}, presents a pressure coefficient parameter, $\beta C_p/\sin\delta$, as a function of a flow deflection parameter, $(\gamma + 1)(M_1^2/\beta)\sin\delta$. Using this one curve it is thus possible to estimate the pressure coefficient knowing only the flow deflection angle, upstream Mach number, and ratio of specific heat capacities. It will be noted that this Item provides approximate pressure coefficients for any perfect gas and any Mach number and, as such, supplements Item No. Aero S.00.03.09 which presents static pressure ratios using exact theory for air only and a restricted range of Mach numbers.

The formula, on which the curve in Figure 1 is based, is

$$\frac{\beta C_p}{2\sin\delta} \approx \frac{K_\delta}{4} + \left[1 + \left(\frac{K_\delta}{4} \right)^2 \right]^{1/2}.$$

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For values of K_δ outside the range presented, use may be made of

$$\frac{\beta C_p}{2 \sin \delta} \approx \frac{K_\delta}{2}, \quad \text{for large } K_\delta,$$

and
$$\frac{\beta C_p}{2 \sin \delta} \approx 1 + \frac{K_\delta}{4}, \quad \text{for small } K_\delta.$$

The presentation of Figure 1 has the advantages of speed and simplicity, with only a slight loss in accuracy, when compared with the use of the exact oblique shock wave relations. Approximate error boundaries using Figure 1 are shown in Figure 2 for errors of ± 1 per cent, ± 2 per cent and ± 3 per cent. It can be seen that, as the flow deflection angle tends to the shock detachment value, *i.e.* $\delta/\delta_d \rightarrow 1.0$ (for a given M_1 and γ), the accuracy obtainable deteriorates. For low Mach numbers the percentage errors increase, as would be expected from a method based on the hypersonic similarity laws, and below a Mach number of about 1.5 the accuracy deteriorates rapidly. The closed error boundaries are caused by the appearance of positive errors, which exceed the specified positive limit over a certain range of Mach number. The open boundaries, on the other hand, are governed by the specified negative limit.

In general, accurate estimates of pressure coefficient can be obtained using Figure 1 for low values of stream deflection angle (away from shock detachment), for high Mach numbers and low ratios of specific heat capacities.

Figure 1 also provides a very quick and simple means of converting the pressure coefficient obtained in one medium to that in another, *e.g.* conversion from helium ($\gamma = 1.667$) to air ($\gamma = 1.4$) for a given Mach number and flow deflection angle.

To determine the shock detachment angle for a given Mach number and ratio of specific heat capacities, see Figure 3.

3. DERIVATION

1. BERTRAM, M.H. The correlation of oblique shock parameters for ratios of specific heats
COOK, B.S. from 1 to 5/3 with application to real gas flows.
 NASA tech. Rep. R-171, 1963.
2. COLLINGBOURNE, J.R. Unpublished work .
 Royal Aircraft Establishment, Farnborough, 1964.