

SOLUTIONS TO THE PRANDTL-MEYER EXPANSION RELATIONS FOR A RANGE OF SPECIFIC HEAT RATIOS: PRESSURE COEFFICIENT AND MAXIMUM FLOW DEFLECTION ANGLE

1. NOTATION

		SI	British
a	speed of sound	m/s	ft/s
C_p	pressure coefficient = $(p_2 - p_1)/\frac{1}{2}\gamma p_1 M_1^2$		
M	Mach number = V/a		
p	static pressure	N/m ²	lbf/ft ²
V	flow velocity	m/s	ft/s
β	compressibility factor = $(M_1^2 - 1)^{1/2}$		
γ	ratio of specific heat capacity at constant pressure to specific heat capacity at constant volume		
δ	flow deflection (considered negative for expansion)	deg	deg
ω	Prandtl-Meyer angle (angle through which supersonic stream is turned to expand from $M_2 = 1$ to $M_2 > 1$)	deg	deg

Suffixes

1	conditions upstream of expansion
2	conditions downstream of expansion
max	conditions for $M_2 = \infty$

2. NOTES

Figure 1, which is based on a modified hypersonic similarity solution to the Prandtl-Meyer expansion relations (see Derivation 1), presents a pressure coefficient parameter, $M_1^2 C_p$, as a function of a flow deflection parameter, $(M_1^2/\beta)\sin\delta$, and the ratio of the specific heat capacities, γ . Using this one carpet it is thus possible to estimate the pressure coefficient, knowing only the flow deflection angle, upstream Mach number and ratio of the specific heat capacities. It will be noted that this Item provides approximate pressure coefficients for any perfect gas (within the range $1.1 \leq \gamma \leq 1.667$) and any Mach number and, as such, supplements Item No. Aero S.00.03.08, which presents Prandtl-Meyer angle and Mach angle for an isentropic expansion over a restricted Mach number range, together with Item No. Aero S.00.03.07 giving pressure ratio.

The formula on which the curves in Figure 1 are based is

$$M_1^2 C_p \approx \frac{2}{\gamma} \left\{ \left[1 + \frac{(\gamma-1)}{2} \frac{M_1^2}{\beta} \sin \delta \right]^{2\gamma/(\gamma-1)} - 1 \right\}. \quad (2.1)$$

Figure 2 gives approximate error boundaries, using Figure 1, for errors of ± 1 per cent, ± 2 per cent, ± 3 per cent and ± 4 per cent in C_p , for the three γ values 1.2, 1.4 and 1.667. It can be seen that the larger the ratio of the specific heat capacities the greater the usable range of Mach number and flow deflection angle for a given error. The accuracy at Mach numbers near unity for small flow deflections (*i.e.* $\delta/\delta_{max} \rightarrow 0$) is poor, as would be expected from a method based on hypersonic similarity parameters. For Mach numbers greater than about 5.0, Figure 1 will give pressure coefficients to within 1 per cent of the exact values over the total flow deflection range for the γ range $1.2 \leq \gamma \leq 1.667$.

To determine the maximum flow deflection angle for a given Mach number and ratio of the specific heat capacities see Figure 3, which is based on the exact formula

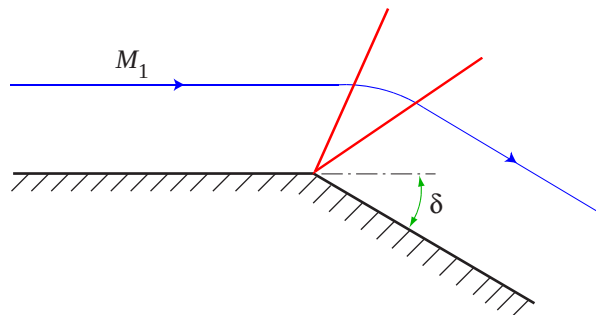
$$\begin{aligned} \delta_{max} &= -(\omega_{max} - \omega_1), \\ &= - \left\{ 90 \left[\sqrt{\frac{\gamma+1}{\gamma-1}} - 1 \right] - \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \left[\beta \sqrt{\frac{\gamma-1}{\gamma+1}} + \tan^{-1} \beta \right] \right\}. \end{aligned} \quad (2.2)$$

3. DERIVATION

1. BERTRAM, M.H.
COOK, B.S. The correlation of oblique shock parameters for ratios of specific heats from 1 to 5/3 with application to real gas flows.
NASA tech. Rep. R-171, 1963.

4. EXAMPLE

A uniform two-dimensional supersonic stream of air (assume $\gamma = 1.4$) at a Mach number of 3.0 is deflected 10° round a corner (see sketch). Estimate the error limits involved in using this Item to calculate the pressure coefficient rather than the exact Prandtl-Meyer expansion relations, and find the pressure coefficient.



From Figure 3, for $M_1 = 3.0$ and $\gamma = 1.4$, $\delta_{max} = -81^\circ$.

Hence, $\delta/\delta_{max} = \frac{10}{81} \approx 0.12$.