

ESDU Aircraft S.02.03.02

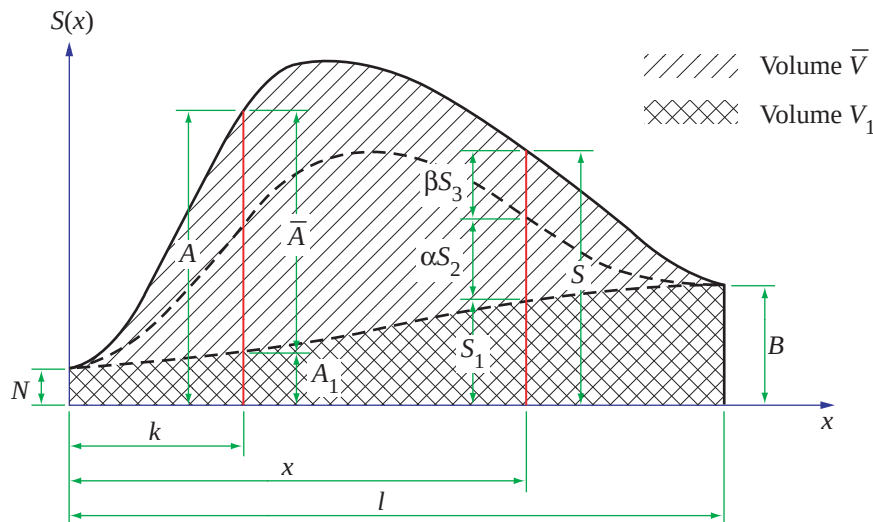
OPTIMUM AREA DISTRIBUTION AND ASSOCIATED THEORETICAL TRANSONIC DRAG-RISE FOR AIRCRAFT AT ZERO LIFT

1. NOTATION AND UNITS (see Sketch 1.1)

		SI	British
A	cross-sectional area specified at axial distance k , made dimensionless with respect to l^2		
B	cross-sectional area at base, made dimensionless with respect to l^2		
k	axial distance of area A from nose, made dimensionless with respect to l		
l	length	m	ft
M	free-stream Mach number		
N	cross-sectional area at nose, made dimensionless with respect to l^2		
q	free-stream kinetic pressure, $\frac{1}{2}\rho U^2$	N/m ²	lbf/ft ²
$S(x)$	cross-sectional area at x , made dimensionless with respect to l^2		
U	free-stream velocity	m/s	ft/s
V	volume, made dimensionless with respect to l^3		
x	axial distance from nose, made dimensionless with respect to l		
ΔC_D	theoretical transonic drag-rise at zero lift, in coefficient form, $\Delta D/q l^2$		
ΔD	theoretical transonic drag-rise at zero lift	N	lbf
ρ	free-stream density	kg/m ³	slug/ft ³

NB: other symbols are defined in the text as they occur.

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Sketch 1.1 Area distributions

2. NOTES

This Item, based on Derivation 1, gives the transonic drag-rise, as defined below, for configurations of optimum area distribution, *i.e.* having minimum transonic drag-rise, where B , N , k and l and minimum values of A and V are specified. These configurations are restricted to wing-body combinations which have two mutually perpendicular planes of symmetry, the intersection of which defines the longitudinal axis of the configuration.

The wave drag of a smooth body in inviscid flow is, subject to the assumptions of linearised theory (see Item No. Aero A.S.02.03.01), equal to zero as M approaches unity from below, but the formula for the wave drag tends to a non-zero value as M tends to unity from above; the transonic drag-rise is here defined as the theoretical wave drag as M tends to unity from above. In this context a body is said to be smooth when the area distribution is such that $S(x)$ and $dS(x)/dx$ are continuous for $0 < x < 1$ and $dS(x)/dx = 0$ at $x = 0$ and $x = 1$. For bodies with discontinuities the linearised theory predicts that the wave drag approaches infinity as the Mach number tends to unity from above.

The linearised theory has in fact no validity as M tends to unity; nevertheless, on experimental evidence, it can be argued that it offers an indication of the actual drag-rise at $M = 1$ and acts as a guide to the relative merits of differing configurations in the region of $M = 1$.

This Item can be used for aircraft having nose intakes (*i.e.* $N \neq 0$) provided there is no spillage. For cases where intake and exit are not at the nose and tail respectively it is suggested that an area equal to the intake area be added to the area distribution forward of the intake, and a corresponding addition, equal to the exit area, be made aft of the exit; again the condition of no spillage must be applied.

For any given configuration the theoretical wave drag at $M = 1$, which appears as a pressure drag and is associated with the wave system, may be determined within the assumptions of slender-body theory in terms of the cross-sectional area distribution $S(x)$ only. This implies that the drag is otherwise independent of the shape of the configuration, whether body alone or wing-body combination. It is shown in Derivation 1 that by representing the cross-sectional area of an optimum body as the sum of three basic distributions, suitably weighted, the overall drag-rise can be similarly represented as the sum of three drag-rises correspondingly weighted.

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The process of determining the cross-sectional area distribution is carried out in two stages (see Sketch 1.1); first an area distribution $S_1(x)$ beginning with N and ending with B is obtained according to the formula

$$S_1(x) = N + (B - N)F(x) \quad (2.1)$$

where $F(x)$ is plotted in Figure 1. The remaining area distribution is made up of two terms according to the formula

$$S(x) - S_1(x) = \alpha S_2(x) + \beta S_3(x), \quad (2.2)$$

$$\text{where} \quad S_2(x) = \frac{16}{3\pi} \bar{V} G(x) \quad (2.3)$$

$$\text{and} \quad S_3(x) = \frac{\bar{A} H(x, k)}{2[k(1 - k)]^{1/2}}. \quad (2.4)$$

Functions $G(x)$ and $H(x, k)$ are plotted in Figures 1 and 2 respectively. The volume $\bar{V}$ and area $\bar{A}$ are defined as

$$\bar{V} = V - \frac{1}{2}(N + B) \quad (2.5)$$

$$\text{and} \quad \bar{A} = A - [N + (B - N)F(k)] \quad (2.6)$$

where $F(k)$ is given in Figure 1.

If $N > B$, N and B must be interchanged and k becomes $(1 - k)$. The parameters α and β are defined in terms of two additional functions χ and σ ,

$$\text{where} \quad \chi = 4k(1 - k) \quad (2.7)$$

$$\text{and} \quad \sigma = \frac{6\bar{V}}{\pi\bar{A}} G(k) \quad (2.8)$$

where χ and $G(k)$ are given in Figure 1.

$$\text{Then, if} \quad \sigma \geq \frac{9}{8}, \alpha = 1 \text{ and } \beta = 0; \quad (2.9)$$

in this instance the optimum value for A is greater than the minimum specified value,

$$\text{if} \quad \sigma \leq \chi, \alpha = 0 \text{ and } \beta = 1; \quad (2.10)$$

in this instance the optimum value for V is greater than the minimum specified value,

$$\text{and if} \quad \chi \leq \sigma \leq \frac{9}{8}, \alpha = \frac{\frac{9}{8}(\sigma - \chi)}{\sigma(\frac{9}{8} - \chi)} \text{ and } \beta = \frac{\frac{9}{8} - \sigma}{\frac{9}{8} - \chi}. \quad (2.11)$$

When these parameters have been determined, $S(x)$ can be determined from Equation (2.2).