

STABILITY DERIVATIVE $(L_v)_\Gamma$ CONTRIBUTION OF PART-SPAN DIHEDRAL TO ROLLING MOMENT DUE TO SIDESLIP

1. NOTATION AND UNITS

The derivative notation used is that proposed in ARC R&M 3562 (Hopkin, 1970) and described in Item No. 86021. Coefficients and aeronormalised derivatives are evaluated in aerodynamic body axes with origin at the aircraft centre of gravity, assumed to coincide with the aerodynamic centre of the wing, and with the wing span as the characteristic length. The derivative L_v is often written as $\partial C_l / \partial \beta$ or $C_{l\beta}$ in other systems of notation, but attention must be paid to the reference dimensions used and it is to be noted that a constant datum value of V is employed in the Hopkin system.

		SI	British
A	aspect ratio, b^2/S		
$(a_{10})_M$	two-dimensional lift-curve slope of wing section at Mach number M	radian ⁻¹	radian ⁻¹
b	wing span	m	ft
b'	span of part of wing containing dihedral angle (see sketch on Figure 1)	m	ft
C_l	rolling moment coefficient, $\mathcal{L} / \frac{1}{2}\rho V^2 S b$		
c_r	wing root chord	m	ft
c_t	wing tip chord	m	ft
$\mathcal{L}$	rolling moment	N m	lbf ft
L_v	aeronormalised rolling moment derivative due to sideslip, $L_v = (\partial \mathcal{L} / \partial v) / \frac{1}{2}\rho V S b$		
$(L_v)_\Gamma$	that part of rolling moment derivative due to dihedral		
M	Mach number		
S	gross wing area	m ²	ft ²
V	velocity of aircraft relative to air	m/s	ft/s
v	sideslip velocity	m/s	ft/s
β	sideslip angle, $\sin^{-1}(v/V) \approx v/V$	radian	radian
Γ	dihedral angle of wing (see sketch on Figure 1)	degree	degree
δ_t	deflection of wing tip due to load	m	ft
κ	parameter $(a_{10})_M (1 - M^2)^{1/2} / 2\pi$		

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$\Lambda_{1/4}$	sweepback of wing quarter-chord line	degree	degree
Λ_E	equivalent sweepback at Mach number M , $\tan^{-1}[(\tan \Lambda_{1/4})/(1 - M^2)^{1/2}]$	degree	degree
η	ratio of b' to b		
λ	wing taper ratio, c_t/c_r		
ρ	density of air	kg/m ³	slug/ft ³

2. NOTES

The variation of the parameter $-(L_v)_\Gamma(1 - M^2)^{1/2}/\kappa\Gamma$ with η for $\Lambda_E = -40^\circ, 0^\circ, 40^\circ$ and 60° and for $A(1 - M^2)^{1/2}/\kappa = 2, 4$ and 8 is given in Figures 1 to 2 at $\lambda = 1, 0.5$ and 0 respectively. Cross-plotting may be used to find $(L_v)_\Gamma$ at intermediate values of Λ_E , $A(1 - M^2)^{1/2}/\kappa$ and λ . The contribution of dihedral to the rolling moment due to sideslip varies linearly with dihedral and is independent of the lift coefficient, provided that the latter is below about 60 per cent of its stalling value.

As plotted, Figures 1 to 2 apply directly to a gull wing arrangement in which the dihedral starts at the wing root and extends outboard to the spanwise stations $\pm \eta$. For a wing in which the centre section is flat and the dihedral starts at some outboard station, $(L_v)_\Gamma$ is found from the difference between the rolling moment derivatives for a wing with full span dihedral and for a wing with dihedral extending from the wing root to the spanwise stations $\pm \eta$. A similar approach is used for compound dihedral.

The Figures have been obtained by the use of Weissinger's extended lifting-line theory as described in Derivation 2, and have been found to give values of $(L_v)_\Gamma$ which generally agree with experimental results to within ± 10 per cent for a wide range of wing planforms. Weissinger's theory normally applies only to thin wings in inviscid, incompressible flow, but by the use of the subsonic, linearised-theory similarity laws and the parameter κ it has been extended to cover the case of wings of finite thickness in viscous, compressible flow, provided that there is no boundary-layer separation from the wing. It has been assumed that there is no significant spanwise variation in κ . In cases where κ does vary significantly, a mean value can be taken as suggested in Derivation 2.

For non-rigid wings, allowance must be made for the additional dihedral due to the wing deflection under load. This dihedral is not constant over the span and it is suggested by Derivation 1 that the equivalent full-span dihedral angle corresponding to a tip deflection δ_t should be taken as $67.6\delta_t/(b/2)$ degrees (see Item No. Aero A.06.01.03).

3. DERIVATION

The Derivation lists selected sources that have assisted in the preparation of this Item.

1. LEVACIC, I. Rolling moment due to sideslip. Part I. The effect of dihedral. RAE Rep. Aero. 2028, 1945.
2. DeYOUNG, J. Theoretical antisymmetric span loading for wings of arbitrary plan form at subsonic speeds. NACA Rep. 1056, 1951.

4. EXAMPLE

Find the contribution of dihedral to the rolling moment derivative due to sideslip at a Mach number of 0.4 for a wing with $A = 3.0$, $\Lambda_{1/4} = 30^\circ$, $\lambda = 0.5$, $(a_{10})_M = 5.6 \text{ radian}^{-1}$, and a dihedral angle of five degrees between 40 per cent of each semi-span and the wing tips.

The value of $(L_v)_\Gamma$ for the given wing is obtained by subtracting the values of $(L_v)_\Gamma$ for two corresponding gull wings with $\eta = 1$ (full-span dihedral) and $\eta = 0.4$ (dihedral from wing root to 40 per cent semi-span).

The equivalent sweepback of the wing is

$$\Lambda_E = \tan^{-1} \left(\frac{\tan \Lambda_{1/4}}{(1-M^2)^{1/2}} \right) = \tan^{-1} \left(\frac{\tan 30^\circ}{(1-0.4^2)^{1/2}} \right) = 32.2^\circ,$$

the parameter κ is given by

$$\kappa = \frac{(a_{10})_M (1-M^2)^{1/2}}{2\pi} = \frac{5.6(1-0.4^2)^{1/2}}{2\pi} = 0.817,$$

and

$$\frac{A(1-M^2)^{1/2}}{\kappa} = \frac{3(1-0.4^2)^{1/2}}{0.817} = 3.37.$$

The values of $-(L_v)_\Gamma (1-M^2)^{1/2} / \kappa \Gamma$ for the two gull wings are found from Figure 2 by cross-plotting and interpolating at $\Lambda_E = 32.2^\circ$ and $A(1-M^2)^{1/2} / \kappa = 3.37$, as shown in Sketch 4.1. The values obtained are 0.0082 for $\eta = 1$ and 0.0019 for $\eta = 0.4$. (For the wing being studied $\lambda = 0.5$ so it is only necessary to use Figure 2; in general, for wings with taper ratios other than 1, 0.5 or 0 it is necessary to interpolate in λ , between the results obtained from Figures 1, 2 and 2.)

The required stability derivative is therefore given by

$$\begin{aligned} \frac{-(L_v)_\Gamma (1-M^2)^{1/2}}{\kappa \Gamma} &= 0.0082 - 0.0019 \\ &= 0.0063 \text{ degree}^{-1}, \end{aligned}$$