

EXPLANATORY NOTES ON TRANSONIC AND SUPERSONIC AREA RULES

1. INTRODUCTION

To estimate the drag of a slender or quasi-cylindrical body in a uniform inviscid stream, the flow may be represented, according to linearised theory, by a line distribution of sources and multisources. Similarly, the flow past a thin wing with zero loading may be represented by a surface system of sources. For any particular wing-body configuration, the overall distribution of sources (or multisources) is made up of the contributions from the source distributions which represent the wing and body separately, together with “interference” sources which are necessary in order to represent correctly the boundary conditions of the wing-body combination.

In the particular case of a body of revolution at zero incidence (when the body is both smooth (see Section 2.1) and slender), the distribution of sources or multisources reduces to a line distribution of sources along its axis which is proportional to the derivative of the body cross-sectional area with respect to distance along the axis. In the case of a wing of symmetrical section with sharp leading and trailing edges at zero incidence the source strength is proportional to the streamwise derivative of the wing thickness.

Thus, subject to certain limitations, given in the next paragraph, the equivalent source distribution for a wing-body combination at zero lift can be represented by the streamwise derivative of the cross-sectional area of the combination; it therefore follows that the drag of such a wing-body combination may be found by consideration of the streamwise derivative of its cross-sectional area. This fact is the basis of what is known as the area rule and the limitations on its use for drag evaluation are demonstrated and summarised in Reference 15, to which reference should be made for a more detailed discussion of the foregoing.

These limitations are as follows:

- (i) The body is smooth and slender.
- (ii) The wing is thin and has sharp edges.
- (iii) The drag due to interference sources is negligible. This condition is usually satisfied if condition (i) is satisfied (Reference 13).

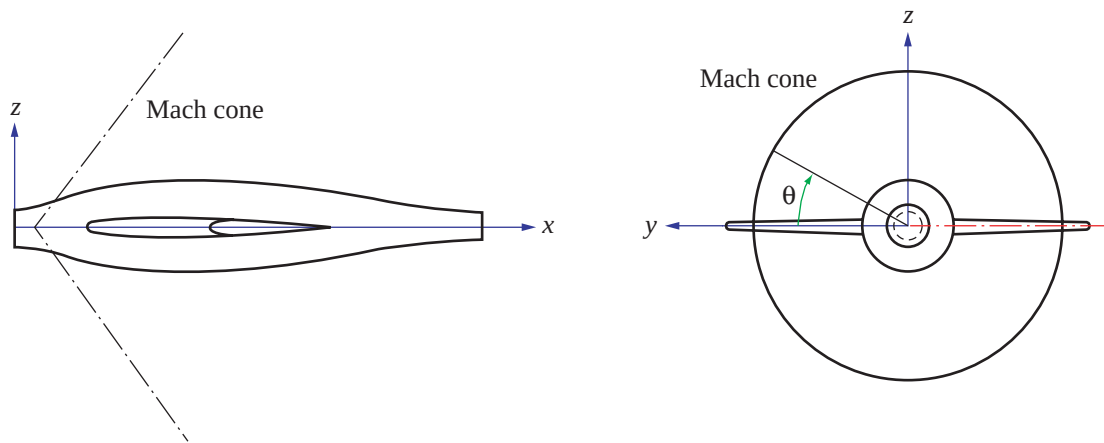
That these limitations also apply to the transfer rule (see Section 3) is also demonstrated in Reference 15, where the relationship between the area and transfer rules is illustrated both theoretically and by means of a numerical example.

2. SUPERSONIC AND TRANSONIC AREA RULES

2.1 Theoretical Basis

A theory of drag calculation, the supersonic area rule, is given in References 4 and 10. This states that the wave drag of a wing-body combination at Mach number M is given by

$$\frac{D}{q} = \frac{-1}{2\pi} \int_0^{2\pi} \frac{1}{2\pi} \left[\int_0^l \int_0^l S''(x_1, \theta, M) S''(x_2, \theta, M) \log_e |x_1 - x_2| dx_1 dx_2 \right] d\theta. \quad (2.1)$$



Sketch 2.1

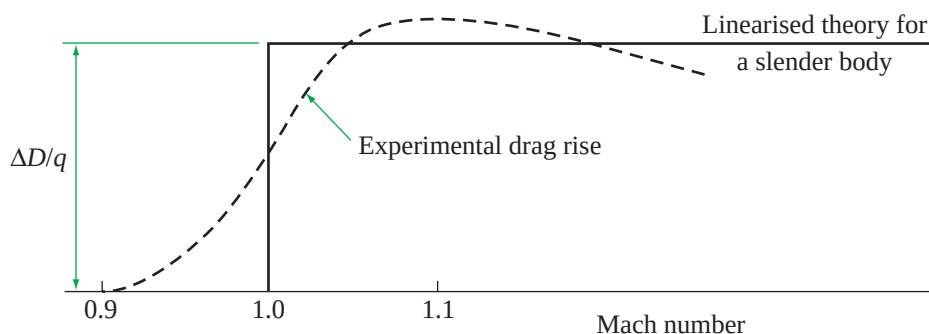
In this equation $S(x, \theta, M)$ is the “elemental” area distribution given by projecting on the plane $x = 0$ the area of the configuration cut by planes tangential to the Mach cone from x and touching along a generator whose projection makes an angle θ with the x, y plane, as shown in Sketch 2.1. The symbol q denotes the kinetic head and l the length of the elemental area distribution, while S'' represents $\partial^2 S / \partial x^2$.

As M tends to unity, $S(x, \theta, M)$ tends to $S(x)$, the cross-sectional area distribution, and Equation (2.1) then reduces to

$$\frac{\Delta D}{q} = \frac{-1}{2\pi} \int_0^l \int_0^l S''(x_1) S''(x_2) \log_e |x_1 - x_2| dx_1 dx_2. \quad (2.2)$$

ΔD is referred to as the theoretical transonic drag rise.

The linearised theory has in fact no validity as M tends to unity; nevertheless, on experimental evidence, it can be argued that it offers an indication of the actual drag rise at $M = 1$ and acts as a guide to the relative merits of differing configurations in the region of $M = 1$.



Sketch 2.2

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Equations (2.1) and (2.2) apply to configurations with smooth elemental area distributions (*i.e.* such that $S(x, \theta, M)$ and $S'(x, \theta, M)$ are continuous functions of x). This means that, for example, the equations will not apply to configurations which have straight wing edges or ridge lines swept at the Mach angle and neither will they apply to bodies for which $S'(x)$ has a value other than zero at $x = 0$ or l .

2.2 Methods of Calculation

Provided that the area distribution of a configuration is sufficiently smooth to be represented by 19 points, Reference 7 gives a relatively simple method, using Fourier series, to calculate the drag rise from Equation (2.2). A more direct application of Fourier series to this calculation is also found in Reference 3 and simplified in Reference 9. In Reference 16 a method is given for calculating the optimum area distribution under certain fixed geometric conditions as M tends to unity and this method also yields the corresponding wave drag given by Equation (2.2), *i.e.* the theoretical transonic drag rise. This work is the basis of Data Item No. Aero A.02.03.02.