

SPANWISE CENTRE OF PRESSURE OF ISOLATED LIFTING SURFACE AT SUPERSONIC SPEEDS

1. NOTATION AND UNITS (see Sketches on Figures 1 to 6)

		SI	British
A	aspect ratio, h^2/S		
c_r	root chord of lifting surface	m	ft
c_t	tip chord of lifting surface	m	ft
h	span of lifting surface	m	ft
M	free-stream Mach number		
S	area of lifting surface	m ²	ft ²
η	spanwise location of centre of pressure from root, as fraction of span		
$\Lambda_{1/2}$	sweep of mid-chord line, positive for sweepback	degree	degree
λ	taper ratio of lifting surface, c_t/c_r		

2. NOTES

This Item presents the spanwise centre of pressure of an isolated lifting surface with straight edges and streamwise tips at supersonic speeds. The centre of pressure position is fundamental to the estimation of the effectiveness of a control or stabilising surface but requires modification for the effects of wing and body interference. Such interference effects are, however, very dependent upon the particular configuration considered; they are not, therefore, taken into account in this Item.

The surface is regarded as an isolated lifting surface in which the tips can be of unequal length. The longer "tip" is herein referred to as the root.

Figures 1 to 6 give the theoretical spanwise centre of pressure for an isolated lifting surface as a function of Mach number, aspect ratio and sweep for values of taper ratio covering the full range from 0 to 1.

The data are based on linearised supersonic flow theory (Derivations 1 to 3) and refer to thin surfaces at small incidence; they are valid for both positive and negative values of $\Lambda_{1/2}$ provided that both leading and trailing edges are supersonic and that the Mach line from the leading edge of the root chord intersects the trailing edge of the surface, *i.e.*

$$A\sqrt{(M^2 - 1)} \geq 1 + |A \tan \Lambda_{1/2}|. \quad (2.1)$$

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For a swept back wing this implies that the condition of supersonic leading and trailing edges is fulfilled. However, for the case of $\lambda = 0$, the curves for positive values of $\Lambda_{1/2}$ are extended to include surfaces having subsonic leading edges and supersonic trailing edges. Care is required in the interpretations of these results since η is constant and equal to 0.5. This means that at any particular value of $A \tan \Lambda_{1/2}$ the curves do not indicate the lower limit of $A \sqrt{(M^2 - 1)}$ at which the theory is valid. Accordingly it should be noted that in this range the limit of a supersonic trailing edge, *i.e.*

$$A \sqrt{(M^2 - 1)} \geq |A \tan \Lambda_{1/2} - 1| \quad (2.2)$$

must be applied.

3. DERIVATION

The Derivation lists selected sources that have assisted in the preparation of this Item.

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|----|----------------------------------|---|
| 1. | MARTIN, J.C.
MALVESTUTO, F.S. | Theoretical forces and moments due to sideslip of a number of vertical tail configurations at supersonic speeds. NACA tech. Note 2412, 1951. |
| 2. | MALVESTUTO, F.S. | Theoretical supersonic force and moment coefficients on a sideslipping vertical- and horizontal-tail combination with subsonic leading edges and supersonic trailing edges. NACA tech. Note 3071, 1953. |
| 3. | MARGOLIS, K.
BOBBITT, P.J. | Theoretical calculations of the pressures, forces and moments at supersonic speeds due to various lateral motions acting on thin isolated vertical tails. NACA Rep. 1268, 1956. |

4. EXAMPLE

It is required to find the centre of pressure at $M = 4$ for an isolated fin with the following geometry:—

$$h = 6 \text{ ft}, S = 24 \text{ ft}^2, \lambda = 0.2 \text{ and } \Lambda_{1/2} = 30^\circ.$$

It is then required to find the theoretical rolling moment about the root chord for the fin with 1° of sideslip (β) at an altitude of 100000 ft.

The aspect ratio is $A = h^2/S = 6^2/24 = 1.5$.

Therefore $A \sqrt{(M^2 - 1)} = 1.5 \sqrt{(4^2 - 1)} = 5.81$

and $A \tan \Lambda_{1/2} = 1.5 \times \tan 30^\circ = 0.866$.

From Figure 2, for $\lambda = 0.2$, with these values,

$$\eta = 0.434.$$

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From Item No. 77021, for an altitude of 100000 ft, the speed of sound, $a = 991$ ft/s and relative density, $\sigma = 0.01365$.

Thus, the kinetic pressure, $\frac{1}{2}\rho V^2 = \frac{1}{2}\rho_0 \sigma (Ma)^2 = \frac{1}{2} \times 0.002377 \times 0.01365 \times (4 \times 991)^2 = 255$ lbf/ft².

From Item No. Aero A.S.01.03.02, the theoretical lift-curve slope, $a_1 = 0.923$ per rad. Thus, rolling moment about the fin root chord

$$\begin{aligned} &= \frac{1}{2}\rho V^2 \times (a_1 \beta) \times (\eta h) \times S \\ &= 255 \times (0.923 \times 1 \times \pi/180) \times (0.434 \times 6) \times 24 \\ &= 257 \text{ lbf ft.} \end{aligned}$$