

ESDU Bodies S.02.03.01

INTRODUCTORY NOTES ON THE DRAG AT ZERO INCIDENCE OF BODIES AT SUPERSONIC SPEEDS

1. NOTATION AND UNITS

		<i>SI</i>	<i>British</i>
C_D	total drag coefficient		
C_{Db}	base drag coefficient		
C_{Df}	skin friction coefficient		
C_{Dw}	total wave drag coefficient in inviscid flow		
C_{Dwa}	afterbody wave drag coefficient in inviscid flow		
C_{Dwf}	forebody wave drag coefficient in inviscid flow		
C_{Dwi}	forebody-afterbody interference wave drag coefficient in inviscid flow		
C_{Dwv}	viscous form drag coefficient		
l	length of body	m	ft
M_0	free-stream Mach number		
R_0	minimum radius of cross-section of body, <i>i.e.</i> radius at nose of forebody or at base of afterbody	m	ft
R_1	maximum radius of cross-section of body, <i>i.e.</i> radius of midbody	m	ft
γ	ratio of specific heat capacity at constant pressure to specific heat capacity at constant volume, assumed equal to 1.4 for air		
η	maximum slope of body profile in axial plane		

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2. GENERAL

A typical body is considered to consist of a *forebody*, a *midbody* and an *afterbody*, and for the purpose of this Item the midbody will be that portion of the body which is of cylindrical shape and maximum cross-sectional area.

The drag of a body at supersonic speeds at zero incidence may be divided arbitrarily into four main parts.

1. *The wave drag in inviscid flow*, which is the drag that the surface ahead of the base would have in an inviscid fluid, and which can conveniently be taken as the sum of the forebody, afterbody and interference wave drags; in coefficient form this is

$$C_{Dw} = C_{Dwf} + C_{Dwa} + C_{Dwi} \quad (2.1)$$

2. *The skin friction drag*, which is due to the viscous shear stress at the surface of the body; in coefficient form C_{Df} .
3. *The viscous form drag*, which is due to the effect of the boundary layer in changing the pressure on the body surface ahead of the base; in coefficient form C_{Dwv} .
4. *The base drag*, which is due to the pressure acting on the base of the body in a region of separated flow; in coefficient form C_{Db} .

Thus the total drag coefficient is given by

$$C_D = C_{Dw} + C_{Df} + C_{Dwv} + C_{Db} \quad (2.2)$$

where it is understood that all coefficients are based on the same reference area.

Experience suggests (see, for example, Reference 18) that for a given transition position, a rough estimate of the skin friction drag may be obtained by assuming that the total skin friction coefficient is the same as that for a flat plate at the same free-stream Mach number and Reynolds number. Reference 13 or Item No. Aero W.S.02.04.12 may be used to obtain this skin friction coefficient. A more accurate method is given in Reference 20.

Viscous form drag may be calculated by assuming that a good approximation to the pressure distribution on a body in viscous flow is obtained by considering the inviscid flow about the shape which results from adding the boundary-layer displacement area to the original body cross-sectional area. Along forebodies and midbodies the effect of the growth of the boundary layer is small. On afterbodies, however, the boundary layer thickens more rapidly and this can have an appreciable effect on the external pressure distribution. The afterbody viscous form drag is usually negative and to a greater or lesser extent counterbalances the positive forebody viscous form drag. Results for a particular family of bodies are given in Reference 18.

The base drag depends on the characteristics of the boundary layer over the body, on the Mach number of the air stream outside the boundary layer immediately ahead of the base and on the geometry of the afterbody. It has been found that for cylindrical bodies, when the boundary layer over the body is laminar, the position of transition in the wake is important in determining base drag. However, for the high Reynolds numbers normally encountered in practice the flow over the body is turbulent and the effects of Reynolds number and boundary-layer thickness are insignificant in terms of the scatter on experimental data. Base pressure is given as a function of Mach number for slender bodies of revolution without boat-tailing and with laminar or turbulent boundary layers in Item No. Aero B.S.02.03.12. Item Nos 78041 and 79022 give the base drag of bodies with conical boat-tails and turbulent boundary layers.

Item Nos Aero B.S.02.03.02 (including 03 to 05 and 07) and 08 (including 09 to 11) are concerned with the estimation of wave drag, for which solutions of various degrees of accuracy exist. For the particular problem of cones, there are exact solutions (Reference 19) which have been tabulated for a wide range of Mach number and cone angle (References 4, 21 and 24 to 26) and are summarised in Item Nos Aero S.00.03.13 and 70008 to 10. For more general problems three principal theories are used.

1. Linearised theory (References 1, 2 and 7).
2. Van Dyke's second-order theory (References 9 and 15).
3. The method of characteristics (Reference 5).

As listed they are in the order of increasing accuracy and also of increasing complication for generalised calculations. A convenient parameter for defining the range of validity of each of these theories is provided by $\eta(M_0^2 - 1)^{1/2}$, where η is the maximum slope of the body profile and M_0 is the free-stream Mach number.

If an error of up to about 5 per cent is acceptable, linearised theory may be used in the range $\eta(M_0^2 - 1)^{1/2} < 0.1$, second-order theory in the range $\eta(M_0^2 - 1)^{1/2} < 0.5$, and, in the range where characteristics theory may be used, entropy-gradient or vorticity effects may be neglected for $\eta(M_0^2 - 1)^{1/2} < 0.9$. These remarks are based on the work of References 10, 14 and 16. For a given value of $\eta(M_0^2 - 1)^{1/2}$ the error in the value given by linearised theory for the drag of an afterbody appears to be about 1.5 times the error for the corresponding forebody; it seems probable that this effect may persist in the second-order theory.

A semi-empirical but rapid method of obtaining the pressure distribution on a forebody of arbitrary shape at zero incidence has been developed by Zienkiewicz (References 22 and 23). This method is based on the most accurate available data and can be used reliably over a wide range of thickness ratios and Mach numbers.

All these theories tend to fail as $M_0 \rightarrow 1$. An essential criterion for their applicability is that the flow must be supersonic everywhere in the field ahead of the base. This can be investigated by the use of shock data such as are given in Item Nos Aero S.00.03.16 and 70008 to 10.

3. LINEARISED THEORY

For bodies there are three principal types of solution of the linearised equation of supersonic flow, "exact-linearised", "slender-body" and "quasi-cylinder". It is assumed in all of them that the slope of the profile is small and that $\eta(M_0^2 - 1)^{1/2}$ is less than about 0.1. Exact linearised solutions (References 1 and 2) satisfy the linearised equation for the potential function and satisfy boundary conditions on the body surface. These solutions can in general be obtained only by the numerical solution of an integral equation. For slender-body solutions (Reference 7) the additional assumption is made that the body is so slender that changes parallel to the main stream flow direction have a negligible effect on the flow pattern normal to the body. The linearised equation is satisfied only approximately and the boundary condition is satisfied in a certain sense on the axis. Analytic answers can be readily obtained for profiles defined by reasonably simple equations.

The quasi-cylinder solution (Reference 6) refers to bodies of nearly constant radius ($0.75 < R_0/R_1 < 1.0$, say). This solution satisfies the linearised equation exactly but only satisfies the boundary condition at a mean radius. Analytic answers can again be obtained for reasonably simple profiles.