

ESDU Bodies 02.03.02

EXTERNAL WAVE DRAG AT ZERO INCIDENCE IN INVISCID FLOW OF SIMPLE DUCTED FOREBODIES AND TRUNCATED AFTERBODIES

1. NOTATION AND UNITS

		<i>SI</i>	<i>British</i>
A	length of semi-major axis for body of elliptic cross section (see sketch on Figures 5 and 6)	m	ft
B	length of semi-minor axis for body of elliptic cross section (see sketch on Figures 5 and 6)	m	ft
C_{Dwa}	wave drag coefficient for afterbody of circular cross section, based on maximum cross-sectional area πR_1^2		
C_{Dwf}	wave drag coefficient for forebody of circular cross section, based on maximum cross-sectional area πR_1^2		
l	length of forebody or afterbody for use in Figures 5 and 6	m	ft
l_a	length of afterbody	m	ft
l_f	length of forebody	m	ft
M	free-stream Mach number		
R_0	radius of cross section of forebody at inlet or afterbody at base	m	ft
R_1	radius of forebody or afterbody cross section at junction with midbody	m	ft
S_0	inlet area of forebody or base area of afterbody	m ²	ft ²
S_1	area of forebody or afterbody cross section at junction with midbody	m ²	ft ²
ΔC_{Dw}	decrease in wave drag coefficient, based on S_1 , from value for forebody or afterbody of circular cross section to that for body of elliptic cross section having the same values of S_0/S_1 and l^2/S_1		

2. FOREBODIES OF CIRCULAR CROSS SECTION

Figure 1 presents the variation of wave drag coefficient with free-stream Mach number and body geometry for ducted conical forebodies at zero incidence in inviscid supersonic flow. Figure 2 provides corresponding data for ducted parabolic forebodies of circular cross section. These are bodies of revolution generated by parabolic arcs which are tangential to the midbody.

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The data in Figures 1 and 2 were obtained (Derivation 4) by the application of similarity laws to the results of various calculations on the wave drag of forebodies. It was assumed in these calculations that the flow was undisturbed ahead of the nose and that there was no spillage. The results include exact solutions for non-ducted (pointed, $R_0 = 0$) forebodies together with slender body and quasi-cylinder solutions for ducted forebodies. The errors involved in the use of similarity laws for ducted bodies have not been assessed but the data for non-ducted bodies have been shown to agree with exact solutions to within ± 5 per cent provided the surface slope of the body is less than about 20 degrees. Data obtained from exact solutions for pointed conical forebodies are given in Item Nos Aero S.00.03.13 (Reference 5) and 70008 to 70010 (References 7 to 9).

To the order of accuracy of the theory used the data in Figure 2 may also be applied to circular arc forebodies.

3. AFTERBODIES OF CIRCULAR CROSS SECTION

Figures 3 and 4 present the variation of wave drag coefficient with free-stream Mach number and body geometry for truncated afterbodies having circular cross sections and linear and parabolic profiles, respectively, at zero incidence in inviscid supersonic flow.

The data in Figure 3 and 4 were obtained (Derivations 1 and 4) by the application of similarity laws to the results of calculations by the method of characteristics and quasi-cylinder theory. In the calculations it was assumed that the flow conditions upstream of the afterbody were uniform and that the flow was parallel to the axis of the afterbody. It was also assumed that the midbody was long enough to avoid interference effects between the afterbody and any forebody that might be fitted; see Item No. Aero B.S.02.03.08 (Reference 6, incorporating Item Nos Aero B.S.02.03.09 to 11) for interference effects.

Slender-body theory gives the wave drag coefficient for pointed conical afterbodies as (see inset on Figure 3)

$$C_{Dwa} = \frac{1}{(l_a/R_1)^2} \left\{ \left[2 \log_e \frac{2(l_a/R_1)}{(M^2 - 1)^{1/2}} \right] - 1 \right\} \quad (3.1)$$

and for pointed parabolic afterbodies as

$$C_{Dwa} = \frac{14}{3} \frac{1}{(l_a/R_1)^2}, \quad (3.2)$$

which are valid only for small surface slopes. For an error of less than ± 5 per cent the use of Equation (3.1) should be restricted to the values of $(M^2 - 1)^{1/2} / (l_a/R_1) < 0.1$.

To the order of accuracy of the theory the data in Figure 4 may also be applied to circular arc afterbodies.

No allowance has been made for the pressure acting on the base. Base pressure gives rise to an additional increment in drag which may be estimated using Item No. 79022 (Reference 12) for conical afterbodies at supersonic speeds.

At subsonic and transonic speeds pressure drag data are available in Item Nos 77020 (Reference 10) and 78041 (Reference 11), respectively, for conical afterbodies and Item No. 96012 (Reference 13) for circular arc afterbodies.

4. FOREBODIES OR AFTERBODIES OF ELLIPTIC CROSS SECTION

The wave drag coefficient for a forebody or afterbody of elliptic cross section is less than that for the corresponding body of circular cross section and the same cross-sectional area distribution and the decrement in wave drag coefficient is ΔC_{Dw} .

Figures 5 and 6 present the variation in ΔC_{Dw} with body geometry for forebodies or afterbodies of elliptic cross section and, respectively, linear and parabolic profiles. The data in Figures 5 and 6 were obtained by the use of slender-body theory (Derivations 2 and 3).

It is assumed that the afterbody is preceded by a long midbody. The interference drag due to the forebody when the midbody is short may be taken as being equal to that for a body of circular cross section and of the same cross-sectional area distribution and may be estimated from Item No. Aero B.S.02.03.08 (incorporating Item Nos Aero B.S.02.03.09 to 11). It is further assumed that in the case of forebodies, the nose of the forebody is open (or pointed when closed, $S_0 = 0$) and that the external flow over the body is everywhere supersonic.

5. DERIVATION AND REFERENCES

5.1 Derivation

The Derivation lists selected sources that have assisted in the preparation of this Item.

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| 1. | VAN DYKE, M.D | Practical calculation of second-order supersonic flow past non-lifting bodies of revolution. NACA tech. Note 2744, 1952. |
| 2. | FRAENKEL, L.E. | Supersonic flow past slender bodies of elliptic cross section. RAE Rep. Aero 2466, 1952. |
| 3. | KAHANE, A.
SOLARSKI, A. | Supersonic flow about slender bodies of elliptic cross section. <i>J. aero. Sci.</i> , Vol. 20, No. 8, August 1953. |
| 4. | FRAENKEL, L.E. | Curves for estimating the wave drag of some bodies of revolution, based on exact and approximate theories. ARC CP 136, 1953. |

5.2 References

The References given are recommended sources of information supplementary to that in this Item.

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| 5. | ESDU | Inviscid supersonic flow past cones at zero incidence: (ii) surface pressure coefficient and shock angle for circular cones.
ESDU, Item No. Aero S.00.03.13, 1954. |
| 6. | ESDU | Forebody-afterbody interference wave drag of simple pointed or ducted body shapes with short midbodies.
ESDU, Item No. Aero B.S.02.03.08, 1954. |
| 7. | ESDU | Inviscid supersonic flow past cones at zero incidence: (i) shock angle.
ESDU, Item No. 70008, 1970. |
| 8. | ESDU | Inviscid supersonic flow past cones at zero incidence: (iii) flow deflection in the conical flow field.
ESDU, Item No. 70009, 1970. |

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| 9. | ESDU | Invisid supersonic flow past cones at zero incidence: (iv) Mach number in the conical flow field.
ESDU, Item No. 70010, 1970. |
| 10. | ESDU | Subsonic pressure drag of conical boat-tails.
ESDU, Item No. 77020, 1977. |
| 11. | ESDU | Transonic base and boat-tail pressure drag of cylindrical bodies with conical boat-tails.
ESDU, Item No. 78041, 1978. |
| 12. | ESDU | Supersonic base drag of cylindrical bodies with conical boat-tails.
ESDU, Item No. 79022, 1979. |
| 13. | ESDU | Subsonic and transonic base and boat-tail pressure drag of cylindrical bodies with circular-arc boat-tails.
ESDU, Item No. 96012, 1996. |

6. EXAMPLES

6.1 Example 1

Find the wave drag coefficient at a Mach number of 2.0 and zero incidence of a ducted parabolic forebody of circular section having the dimensions $l_f = 1.0$ m, $R_0 = 0.2$ m and $R_1 = 0.4$ m.

$$\frac{(M^2 - 1)^{1/2}}{l_f/R_1} = \frac{(2^2 - 1)^{1/2}}{1/0.4} = 0.693 \text{ and } \left(\frac{R_0}{R_1}\right)^2 = \left(\frac{0.2}{0.4}\right)^2 = 0.25 .$$

Figure 2, entered with these values, gives

$$C_{Dwf} \left(\frac{l_f}{R_1}\right)^2 = 1.21 .$$

Thus
$$C_{Dwf} = 1.21 \times \left(\frac{0.4}{1}\right)^2 = 0.194 .$$

6.2 Example 2

Find the wave drag coefficient at a Mach number of 2.0 and zero incidence of a truncated conical afterbody attached to a long midbody. The afterbody dimensions are $l_a = 2$ m, $R_0 = 0.3$ m and $R_1 = 0.4$ m.

$$\frac{(M^2 - 1)^{1/2}}{l_a/R_1} = \frac{(2^2 - 1)^{1/2}}{2/0.4} = 0.346 \text{ and } \left(\frac{R_0}{R_1}\right)^2 = \left(\frac{0.3}{0.4}\right)^2 = 0.5625 .$$