

ESDU Controls 04.01.06

FULL-SPAN CONTROL HINGE-MOMENT COEFFICIENT DERIVATIVES FOR UNSWEPT WINGS IN INCOMPRESSIBLE FLOW WITH ALLOWANCE FOR SPANWISE VARIATION OF SECTIONAL PROPERTIES

This Item is for use when there is a large spanwise variation in wing and control section properties. The simpler method of Item No. 89009 (Reference 9) assumes constant section properties and includes part-span effects.

1. NOTATION AND UNITS

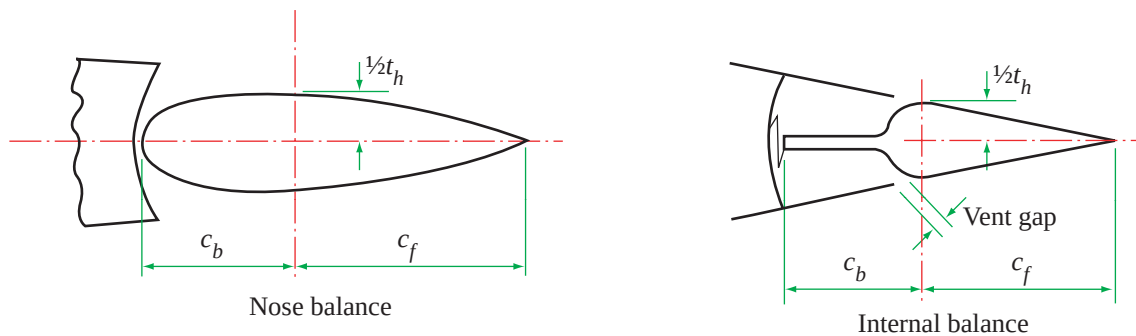
		SI	British
A	aspect ratio, b^2/S		
$(a_1)_0$	rate of change of lift coefficient with angle of attack for two-dimensional section in incompressible flow	rad^{-1}	rad^{-1}
$(a_2)_0$	rate of change of lift coefficient with control deflection for two-dimensional section in incompressible flow	rad^{-1}	rad^{-1}
b	wing span	m	ft
b_1	rate of change of control hinge-moment coefficient with angle of attack, $\partial C_H / \partial \alpha$	rad^{-1}	rad^{-1}
$(b_1)_0$	rate of change of control hinge-moment coefficient with angle of attack for two-dimensional section, $\partial C_{H0} / \partial \alpha$	rad^{-1}	rad^{-1}
b_2	rate of change of control hinge-moment coefficient with control deflection, $\partial C_H / \partial \delta$	rad^{-1}	rad^{-1}
$(b_2)_0$	rate of change of control hinge-moment coefficient with control deflection for two-dimensional section, $\partial C_{H0} / \partial \delta$	rad^{-1}	rad^{-1}
C_H	control hinge-moment coefficient, $H / \frac{1}{2} \rho V^2 \bar{c}_f^2 s_f$		
C_{H0}	control hinge-moment coefficient for two-dimensional section in incompressible flow, $H_0 / \frac{1}{2} \rho V^2 c_f^2$		
c	wing chord	m	ft
c_b	control chord forward of hinge line	m	ft
c_f	control chord aft of hinge line	m	ft
$\bar{c}_f^2$	aerodynamic mean control chord aft of hinge line	m	ft
c_r	wing root chord	m	ft
c_t	wing tip chord	m	ft

Issued December 1949

With Amendments A and B, August 1989 – 13 pages

ESDU Controls 04.01.06

F	α_i / δ		
F_1	induced angle of attack contribution in Equation (2.1), value of F when $c_f/c = 1$		
G	streamline curvature contribution in Equation (2.2)		
G_1	streamline curvature contribution in Equation (2.1), value of G when $c_f/c = 1$		
G_B	factor on G and G_1 allowing for nose or internal balance (see Sketch 1.1)		
H	control hinge moment	N m	lb ft
H_0	control hinge moment per unit span for two-dimensional section in incompressible flow	N m/m	lbf ft/ft
K_1	ratio of induced angle of attack at any section to induced angle of attack of elliptic wing, from lifting-line theory		
S	wing area	m ²	ft ²
s	semispan of wing	m	ft
s_f	control span, $s_f = s$ in this Item	m	ft
t	maximum section thickness	m	ft
t_h	section thickness at hinge line	m	ft
V	free-stream velocity	m/s	ft/s
α	angle of attack	rad	rad
α_i	mean induced angle of attack	rad	rad
δ	control deflection angle	rad	rad
η	distance from wing centre-line as fraction of semispan		
λ	wing taper ratio, c_t/c_r		
ρ	free-stream density	kg/m ³	slug/ft ³
τ	section trailing-edge angle	deg	deg
Superscript			
*	as in $(a_1)_0^*$ denotes properties for a 'standard' section for which $\tau = 2 \tan^{-1}(t/c)$		
Subscript			
T	as in $(a_1)_{0T}$ denotes theoretical value		



Sketch 1.1

2. NOTES

For an unswept wing, tailplane or fin with a spanwise variation of sectional properties Derivation 2 gives the hinge-moment coefficient derivative due to angle of attack for a full-span control with nose or internal balance as

$$b_1 = \int_0^1 (b_1)_0 [1 - F_1 K_1] (c_f / \bar{c}_f)^2 d\eta + G_B G_1 (a_1)_0. \quad (2.1)$$

The hinge-moment coefficient derivative due to control deflection is given as

$$b_2 = \int_0^1 (b_2)_0 (c_f / \bar{c}_f)^2 d\eta - F \int_0^1 (b_1)_0 K_1 (c_f / \bar{c}_f)^2 d\eta + G_B G(a_2)_0. \quad (2.2)$$

Figure 1 gives F and F_1 , Figure 2 gives G and G_1 , and Figures 3a and 3b give G_B for nose and internal balances respectively. Those parameters have been obtained from Derivation 2 in which calculations were made using an extended lifting-line theory for unswept wings with elliptical loading and constant section properties. In addition, F is presented assuming theoretical thin-aerofoil values for $(a_2)_0 / (a_1)_0$. Values of $(a_1)_0$ and $(a_2)_0$ may be obtained from Item Nos Aero W.01.01.05 (Reference 5) and C.01.01.03 (Reference 6), respectively. The values of $(b_1)_0$ and $(b_2)_0$ for plain controls may be obtained from Item Nos Aero C.04.01.01 and 02 (References 7 and 8), and must be corrected for nose or internal balance by the method of Item No. Aero C.04.01.03 or 04 (Reference 3 or 4). If experimental values of the two-dimensional properties are known they may be used instead of the theoretical estimates.

The integral expressions in Equations (2.1) and (2.2) allow for a spanwise variation in $(b_1)_0$, $(b_2)_0$ and c_f / c , and give improved estimates when the variations are in excess of 10%. Values for other parameters should be taken for a mean section geometry.

The function K_1 allows for wing loadings other than elliptical and is the ratio of the induced angle of attack at any section to the induced angle of attack for an elliptical wing of the same aspect ratio. It was evaluated in Derivation 2 on the basis of modified lifting-line theory by the method of Derivation 1. For an elliptical wing $K_1 = 1$. Figure 4a gives values of K_1 as a function of η for straight-tapered wings, and Figure 4b gives corresponding curves for wings with tips that are elliptical over the outboard 15% of the span.

ESDU Controls 04.01.06

3. DERIVATION AND REFERENCES

3.1 Derivation

The Derivation lists selected sources that have assisted in the preparation of this Item.

1. HILDEBRAND, F.B. A least-squares procedure for the solution of the lifting-line integral equation. NACA tech. Note 925, 1944.
2. SWANSON, R.S.
CRANDALL, S.M. Lifting-surface-theory aspect-ratio corrections to the lift and hinge-moment parameters for full-span elevators on horizontal tail surfaces. NACA tech. Note 1175, 1946.

3.2 References

The References are sources of information supplementary to that in this Item.

3. ESDU Effect of nose balance on two-dimensional control hinge-moment coefficients. Item No. Aero C.04.01.03. ESDU International, 1949.
4. ESDU Effect of Irving internal balance on hinge-moment coefficient in two-dimensional flow. Item No. Aero C.04.01.04. ESDU International, 1949.
5. ESDU Slope of lift curve for two-dimensional flow. Item No. Aero W.01.01.05. ESDU International, 1955.
6. ESDU Rate of change of lift coefficient with control deflection in incompressible two-dimensional flow, $(a_2)_0$. Item No. Aero C.01.01.03. ESDU International, 1956.
7. ESDU Rate of change of hinge-moment coefficient with incidence for a plain control in incompressible two-dimensional flow $(b_1)_0$. Item No. C.04.01.01. ESDU International, 1956.
8. ESDU Rate of change of hinge-moment coefficient with control deflection for a plain control in incompressible two-dimensional flow, $(b_2)_0$. Item No. Aero C.04.01.02. ESDU International, 1956.
9. ESDU Hinge moment coefficient derivatives for trailing-edge controls on wings at subsonic speeds. Item No. 89009. ESDU International, 1989.