ONE-DIMENSIONAL ISENTROPIC FLOW OF A THERMALLY PERFECT, CALORICALLY IMPERFECT DIATOMIC GAS

1. NOTATION AND UNITS

		SI	*British*
A	local cross-sectional area of flow	m^2	ft^2
A^*	cross-sectional area of flow at the station at which $M = 1$	m^2	ft^2
a	local speed of sound	m/s	ft/s
a_H	speed of sound in gas when brought to rest isentropically	m/s	ft/s
c_p	specific heat capacity at constant pressure	J/kg K	ft lbf/slug K
$(c_p)_0$	specific heat capacity at constant pressure of calorically perfect diatomic gas ($7R/2$)	J/kg K	ft lbf/slug K
M	Mach number, V/a		
p	static pressure	N/m^2	lbf/ft^2
p_H	pressure of gas when brought to rest isentropically	N/m^2	lbf/ft^2
R	gas constant	J/kg K	ft lbf/slug K
T	static temperature	K	K
T_H	temperature of gas when brought to rest isentropically	K	K
V	velocity of gas	m/s	ft/s
γ	ratio of the specific heat capacities		
γ_0	ratio of the specific heat capacities of calorically perfect diatomic gas (7/5)		
θ	characteristic vibrational temperature	K	K
ρ	density of gas	kg/m^3	$slug/ft^3$
ρ_H	density of gas when brought to rest isentropically	kg/m^3	$slug/ft^3$

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2. NOTES

2.1 General

The ratios p/p_H, T/T_H, V/a_H, $\frac{1}{2}\rho V^2/p_H$ and A/A^* are plotted in Figures 1 to 5b as functions of Mach number for a range of the parameter T_H/θ. The curves for $T_H/\theta = 0$ correspond to the case of a calorically perfect gas.

These curves have been obtained by assuming that the gas is thermally perfect and that the variation of γ and c_p may be expressed by the relations

$$\gamma = 1 + \frac{\gamma_0 - 1}{1 + (\gamma_0 - 1)\left(\frac{\theta}{T}\right)^2 \frac{e^{\theta/T}}{(e^{\theta/T} - 1)^2}} \tag{2.1}$$

and

$$c_p = (c_p)_0 \left\{ 1 + \frac{(\gamma_0 - 1)}{\gamma_0} \left(\frac{\theta}{T}\right)^2 \frac{e^{\theta/T}}{(e^{\theta/T} - 1)^2} \right\}. \tag{2.2}$$

These expressions correspond to the assumption that the molecules behave like a simple harmonic vibrator.

Equation (2.1) closely predicts the variation of γ with temperature for several common diatomic gases over a wide temperature range, provided that a suitable value of θ is chosen (see Data Item No. 00.01.09). Appropriate values of θ and the corresponding temperature range for which Equation (2.1) is then applicable are given in Table 2.1.

TABLE 2.1

Gas	θ K	*Temperature Range* K
Air	3050	250 – 1200
Nitrogen	3250	250 – 2000 +
Oxygen	2150	250 – 1230
Carbon Monoxide	3050	250 – 2000 +

Should a flow process extend marginally outside the given temperature range, the errors in p/p_H and T/T_H will generally be negligible, but errors in V/a_H, $\frac{1}{2}\rho V^2/p_H$ and A/A^* may be appreciable. To reduce these errors to a minimum these values should be evaluated directly from Equations (2.10) to (2.12), using values of γ given in Data Item No. 00.01.08 (see Example).

2.2 Isentropic Flow Equations

The equation of state for a thermally perfect gas is

$$\frac{p}{\rho} = RT. \tag{2.3}$$

The momentum equation is

$$\frac{1}{\rho} dp + V dV = 0 \tag{2.4}$$

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and the energy equation for a thermally perfect, calorically imperfect gas may be written as

$$c_p \mathrm{d}T + V\mathrm{d}V = 0\,. \tag{2.5}$$

The speed of sound is given by

$$a^2 = \gamma RT \tag{2.6}$$

and the Mach number is defined by

$$M = V/a\,. \tag{2.7}$$

From these basic equations, together with Equations (2.1) and (2.2), the required parameters are obtained.

$$M^2 = \frac{2T_H}{\gamma T}\left[\frac{\gamma_0}{\gamma_0 - 1}\left(1 - \frac{T}{T_H}\right) + \frac{\theta}{T_H}\left(\frac{1}{\mathrm{e}^{\theta/T_H} - 1} - \frac{1}{\mathrm{e}^{\theta/T} - 1}\right)\right] \tag{2.8}$$

$$\frac{p}{p_H} = \left(\frac{T}{T_H}\right)^{\gamma_0/(\gamma_0 - 1)}\left(\frac{\mathrm{e}^{\theta/T_H} - 1}{\mathrm{e}^{\theta/T} - 1}\right)\exp\left[\left(\frac{\theta}{T}\right)\frac{\mathrm{e}^{\theta/T}}{\mathrm{e}^{\theta/T} - 1} - \left(\frac{\theta}{T_H}\right)\frac{\mathrm{e}^{\theta/T_H}}{\mathrm{e}^{\theta/T_H} - 1}\right] \tag{2.9}$$

$$\left(\frac{V}{a_H}\right)^2 = \frac{\gamma}{\gamma_H} M^2 \left(\frac{T}{T_H}\right) \tag{2.10}$$

$$\frac{\tfrac{1}{2}\rho V^2}{p_H} = \frac{\gamma}{2} M^2 \frac{p}{p_H} \tag{2.11}$$

$$\frac{A}{A^*} = \frac{\left(\dfrac{\rho}{\rho_H}\right)_{M=1}\left(\dfrac{V}{a_H}\right)_{M=1}}{\left(\dfrac{\rho}{\rho_H}\right)\left(\dfrac{V}{a_H}\right)}, \tag{2.12}$$

where

$$\frac{\rho}{\rho_H} = \frac{p}{p_H}\frac{T_H}{T}\,. \tag{2.13}$$

3. DERIVATION

The Derivation lists selected sources that have assisted in the preparation of this Item.

1. EGGERS, A.J., Jr — One-dimensional flows of an imperfect diatomic gas. NACA Report 959, 1950.

2. AMES RESEARCH STAFF — Equations, tables and charts for compressible flow. NACA Report 1135, 1953.

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4. EXAMPLE

Find the flow conditions in the working section of a blow-down wind tunnel using air which has expanded isentropically to a Mach number of 5 from a reservoir in which the pressure is 600 lbf/in^2 (86400 lbf/ft^2) and the temperature 750 K. (For air, $\theta = 3050$ K and $R = 3090$ ft lbf/slug K).

$$T_H/\theta = 750/3050 = 0.246 .$$

From Figure 1, $p/p_H = 1.83\times10^{-3}$ and so $p = 86400\times0.00183 = 158$ lbf/ft^2 .

From Figure 2, $T/T_H = 0.170$ and so $T = 750\times0.170 = 128$ K.

This value for T is outside the range for which Equation (2.1) is applicable (Table 2.1) and hence V/a_H, $\frac{1}{2}\rho V^2$ and A/A^* should be calculated using Equations (2.10) to (2.12) (see Section 2.2).

Referring to Data Item No. 00.01.08, Figure 1a, the accurate value for γ is given.

At $T = 128$ K, $\gamma = 1.415$.
At $T_H = 750$ K, $\gamma_H = 1.359$.

Equation (2.10) is

$$\left(\frac{V}{a_H}\right)^2 = \frac{\gamma}{\gamma_H}M^2\left(\frac{T}{T_H}\right)$$

and, substituting appropriate values,

$$\left(\frac{V}{a_H}\right)^2 = \frac{1.415}{1.359}\times25\times0.170 = 4.42 \quad \text{and so} \quad \frac{V}{a_H} = 2.105 ,$$

$$a_H = \sqrt{(\gamma_H R T_H)} = \sqrt{(1.359\times3090\times750)} = 1775 \text{ ft/s}$$

and $V = 2.105\times1775 = 3740$ ft/s.

Using Figure 3, V is found to be 3728 ft/s.

Equation (2.11) is

$$\frac{\frac{1}{2}\rho V^2}{p_H} = \frac{\gamma}{2}M^2\frac{p}{p_H}$$

i.e. $\frac{1}{2}\rho V^2 = \frac{1.415}{2}\times25\times0.00183\times86400 = 2800$ lbf/ft^2 .

Using Figure 4b, $\frac{1}{2}\rho V^2$ is found to be 2785 lbf/ft^2 .

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