

ONE-DIMENSIONAL ISENTROPIC GAS FLOW

1. NOTATION

		<i>SI</i>	<i>British</i>
A	local cross-sectional area of flow	m^2	ft^2
A^*	cross-sectional area of flow at the station at which $M = 1$	m^2	ft^2
a	speed of sound in gas, $\sqrt{(\gamma RT)}$	m/s	ft/s
a_H	speed of sound in gas when brought to rest isentropically, $\sqrt{(\gamma RT_H)}$	m/s	ft/s
M	Mach number, V/a		
m	mass flow rate of gas	kg/s	slug/s
p	static pressure of gas	N/m^2	lbf/ft^2
p_H	pressure of gas when brought to rest isentropically	N/m^2	lbf/ft^2
R	gas constant	J/kg K	ft lbf/slug K
T	static temperature of gas	K	K
T_H	temperature of gas when brought to rest isentropically	K	K
V	velocity of gas	m/s	ft/s
γ	ratio of specific heat capacities		
ρ	density of gas	kg/m^3	slug/ft^3
ρ_H	density of gas when brought to rest isentropically	kg/m^3	slug/ft^3

2. NOTES

In the one-dimensional steady isentropic flow of a perfect gas the following relations apply:

$$p/\rho^\gamma = \text{constant} \quad (2.1)$$

$$\frac{p}{p_H} = \left(\frac{\rho}{\rho_H}\right)^\gamma = \left(\frac{T}{T_H}\right)^{\gamma/(\gamma-1)} \quad (2.2)$$

$$\frac{T}{T_H} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{-1} \quad (2.3)$$

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$$\frac{V}{a_H} = M \sqrt{\left(\frac{T}{T_H}\right)} \quad (2.4)$$

$$\frac{\frac{1}{2}\rho V^2}{p_H} = \frac{\gamma}{2} \frac{p}{p_H} M^2 \quad (2.5)$$

$$\frac{A^*}{A} = M \left(\frac{\gamma + 1}{2 + (\gamma - 1)M^2} \right)^{\frac{\gamma + 1}{2(\gamma - 1)}} \quad (2.6)$$

Equations (2.2) to (2.6) apply more generally to local conditions in non-isentropic flows. In such cases Equations (2.2) to (2.4) are usually the most useful.

The ratios p/p_H , T/T_H , V/a_H and $\frac{1}{2}\rho V^2/p_H$ are plotted in Figures 1a to 4b for a range of values of γ and for values of M up to 4.

Note: $\gamma = 1$ is included in the curves as a limiting value of γ for ease of interpolation.

The density ratio ρ/ρ_H may be obtained from the relationship

$$\frac{\rho}{\rho_H} = \frac{p}{p_H} \frac{T_H}{T} \quad (2.7)$$

The area ratio A^*/A may be obtained from Item No. 81004, using

$$\frac{A^*}{A} = \frac{\frac{m \sqrt{(RT_H)}}{Ap_H}}{\left(\frac{m \sqrt{(RT_H)}}{Ap_H} \right)_{M=1}} \quad (2.8)$$

since in isentropic flow m , T_H and p_H are all constant.

The assumption of a constant value of γ gives rise to increasing errors at high temperatures, where caloric imperfections assume importance. Information on this topic is given in Data Item Nos Aero 00.01.08 and 00.01.09.

Curves for the isentropic expansion of air ($\gamma = 1.4$) are plotted again, but in a different form, in Data Item No. Aero 00.03.05.

3. DERIVATION

The Derivation lists selected sources that have assisted in the preparation of this Item.

1. KEENAN, J.H. *Gas tables*. John Wiley, New York and Chapman and Hall, London, 1948.
KAYE, J.
2. – Unpublished work at Hawker Siddeley Aviation Ltd, Kingston-upon-Thames, 1959.

4. EXAMPLES

4.1 Example 1

Find the flow conditions in the working section of a vacuum wind tunnel using air which has expanded isentropically to a pressure of 420 lbf/ft^2 from a reservoir in which the pressure is 2120 lbf/ft^2 , the density is $0.00238 \text{ slug/ft}^3$ and the temperature 288 K . (For air, $\gamma = 1.4$ and $R = 3090 \text{ ft lbf/slug K}$.)

For $\frac{p}{p_H} = \frac{420}{2120} = 0.198$, from Figure 1b, $M = 1.715$.

From Figure 2a, for $M = 1.715$ and $\gamma = 1.4$, $T/T_H = 0.630$,

$$T = 0.630 \times 288 = 181 \text{ K},$$

$$a_H = \sqrt{(1.4 \times 3090 \times 288)} = 1117 \text{ ft/s}.$$

For $M = 1.715$ and $\gamma = 1.4$:

from Figure 3b, $V/a_H = 1.36$

and, from Figure 4b, $\frac{1}{2}\rho V^2/p_H = 0.408$.

Hence $V = 1.36 \times 1117 = 1520 \text{ ft/s}$

and $\frac{1}{2}\rho V^2 = 2120 \times 0.408 = 865 \text{ lbf/ft}^2$.

Using Equation (2.7), $\rho = \frac{0.00238 \times 0.198}{0.630} = 0.000748 \text{ slug/ft}^3$.

4.2 Example 2

Helium is decelerated isentropically from a Mach number of 0.95 to a Mach number of 0.5. If the initial static pressure and temperature were 1500 lbf/ft^2 and 200 K , respectively, find the final static pressure and temperature; find also the area ratio of the diffuser assuming one-dimensional flow. (For helium take $\gamma = 1.67$.)

From Figures 1a and 2a, for $\gamma = 1.67$:

$$\text{at } M_1 = 0.95, \quad \frac{p_1}{p_H} = 0.518, \quad \frac{T_1}{T_H} = 0.768,$$

$$\text{and at } M_2 = 0.5, \quad \frac{p_2}{p_H} = 0.818, \quad \frac{T_2}{T_H} = 0.9225.$$

In isentropic flow p_H and T_H are constant,

hence
$$\frac{p_1}{p_2} = \frac{0.518}{0.818} = 0.634; \quad p_2 = \frac{1500}{0.634} = 2370 \text{ lbf/ft}^2,$$

and
$$\frac{T_1}{T_2} = \frac{0.768}{0.9225} = 0.833; \quad T_2 = \frac{200}{0.833} = 240 \text{ K}.$$

From Item No. 81004, Figure 1a, for $\gamma = 1.67$:

at $M_1 = 0.95$,
$$\frac{m\sqrt{(RT_H)}}{A_1 p_h} = 0.725$$

and at $M_2 = 0.5$,
$$\frac{m\sqrt{(RT_H)}}{A_2 p_h} = 0.550.$$

Since m and R are constant,
$$\frac{A_1}{A_2} = \frac{0.550}{0.725} = 0.760.$$

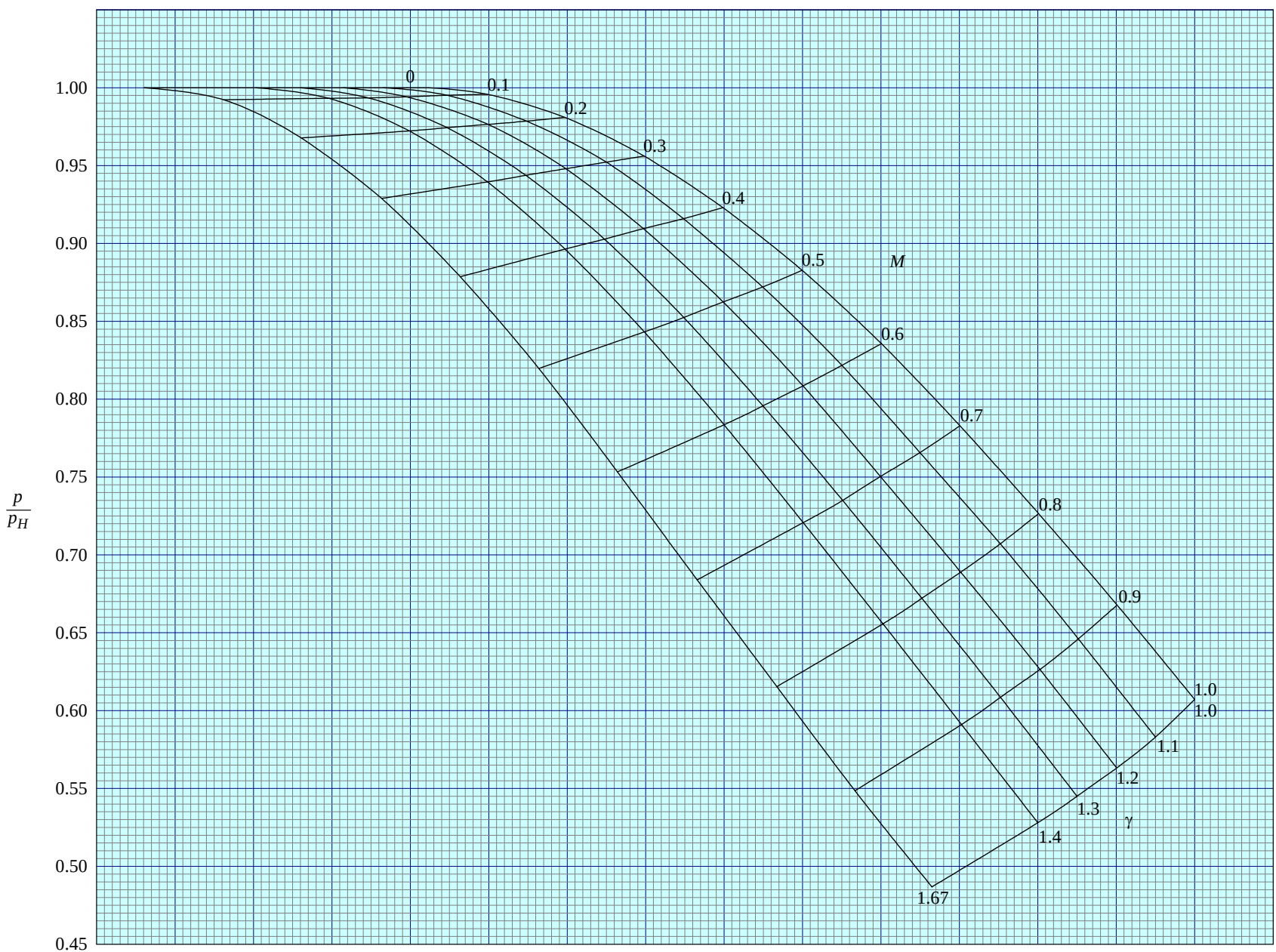


FIGURE 1a