

SIMPLE-WAVE FLOW

1. NOTATION AND UNITS

		SI	British
a	local speed of sound	m/s	ft/s
M	Mach number, V/a		
p	static pressure of gas	N/m ²	lbf/ft ²
p_H	pressure of gas brought to rest isentropically	N/m ²	lbf/ft ²
V	velocity of gas	m/s	ft/s
γ	ratio of specific heat capacity at constant pressure to specific heat capacity at constant volume		
μ	Mach angle, $\sin^{-1} 1/M$	deg	deg
ω	deflection angle relative to flow direction when $M = 1$	deg	deg

2. NOTES

The special case of two-dimensional isentropic supersonic flow in which one family of Mach waves or characteristics is straight is referred to as simple-wave flow. It occurs in regions of flow that are subject to the influence of either an upper or lower boundary only, provided that the flow upstream is uniform.

Figure 1a gives the Mach angle, μ , plotted against local Mach number. Figure 1b gives values of the flow deflection angle, ω , as a function of Mach number for a range of values of the ratio of specific heat capacities, γ . (Note that Figure 1b includes the limiting case of $\gamma = 1$ for ease of interpolation.)

The changes in flow direction and magnitude are produced entirely by the family of straight Mach waves that originate from the single boundary; thus the flow is made to conform to the latter and the local Mach number is a function of the local direction of flow only. An important feature of simple-wave flow is that the flow velocity and direction are constant along any member of the family of straight Mach waves.

Regions of simple-wave flow are to be found downstream of regions of uniform flow and the flow near a thin aerofoil with attached shock waves can be very closely approximated by simple-wave flow.

A particular case of simple-wave flow occurs when a two-dimensional uniform and otherwise unbounded supersonic stream is expanded round a sharp corner. Such a flow is generally referred to as a Prandtl-Meyer expansion; it is in effect a centred simple-wave flow with the family of straight Mach waves originating from the corner.

The maximum possible flow deflection from $M = 1$ occurs when the flow expands to zero pressure and infinite Mach number and is given by

$$\omega = \left[\left(\frac{\gamma+1}{\gamma-1} \right)^{1/2} - 1 \right] \frac{\pi}{2} \text{ rad.} \quad (2.1)$$

Item No. Aero S.00.03.27 gives an example of the solution of problems of simple-wave flow when γ is not constant.

The computations on which this Item is based were obtained from Derivation 1.

3. DERIVATION

1. DOMMETT, R.L. Unpublished work .
Royal Aircraft Establishment, Farnborough, 1962.

4. EXAMPLE

Find the final Mach number, Mach angle and static pressure when a uniform two-dimensional supersonic stream of helium at a Mach number of 1.9 and a static pressure of 50 000 N/m² is deflected 15° round a convex surface. (For helium, take $\gamma = 1.67$.)

From Figure 1b, when $M = 1.9$ and $\gamma = 1.67$, $\omega = 19.5^\circ$, i.e. to expand a flow from $M = 1.0$ to $M = 1.9$ requires a deflection of 19.5°.

As the surface is convex, the flow is an expansion and the final deflection is

$$\omega = 19.5 + 15 = 34.5^\circ.$$

Hence, from Figure 1b, the final Mach number is 2.71, and from Figure 1a the Mach angle is $\mu = 21.7^\circ$.

From Item No. Aero S.00.03.07

at $M = 1.9$, $p/p_H = 0.138$

and at $M = 2.71$, $p/p_H = 0.0465$.

Since the flow is isentropic, p_H is constant and therefore the static pressure after the expansion is

$$p = \frac{0.0465}{0.138} \times 50\,000 = 16\,850 \text{ N/m}^2.$$