

PROPERTIES OF PLANE STEADY SHOCK WAVES

1. NOTATION AND UNITS

		<i>SI</i>	<i>British</i>
a	speed of sound	m/s	ft/s
M	Mach number, V/a		
p	static pressure	N/m ²	lbf/ft ²
p_H	total pressure	N/m ²	lbf/ft ²
V	velocity	m/s	ft/s
δ	flow deflection angle	deg	deg
γ	ratio of specific heat capacity at constant pressure to specific heat capacity at constant volume, taken as 1.4 for air		
ζ	shock wave inclination to upstream flow direction	deg	deg

Subscripts

1, 2 denote conditions upstream and downstream of shock wave, respectively.

2. NOTES

When a supersonic air stream encounters a sharp-edged obstruction which causes the stream to be deflected a shock wave is formed that is inclined to the flow direction. Through this shock wave there is a rapid compression, decrease in speed of the air and loss in total pressure. This Item presents data on the changes in Mach number, static pressure and total pressure across a plane (two-dimensional) shock wave (Derivation 2).

Figure 1 gives the Mach number M_2 behind the shock and Figure 2 gives the static pressure ratio p_2/p_1 across the shock. In both cases the data are shown as functions of the Mach number M_1 ahead of the shock for constant values of the flow deflection angle, δ (dashed lines). Curves of the shock angle, ζ , are also shown (solid lines). For given values of M_1 and δ there may be two possible values of M_2 , p_2/p_1 and ζ . In practice the higher value of M_2 and the lower value of p_2/p_1 are generally obtained; further details of this are given in Derivation 1.

Figure 3 shows the total pressure ratio p_{H2}/p_{H1} across the shock, plotted against the shock wave angle, ζ , for constant values of M_1 . For a known deflection δ the value of ζ may be obtained from Figure 1 or Figure 2.

As can be seen from Figures 1 and 2, with δ fixed there is a value of M_1 below which the type of flow described by this Item cannot exist. If M_1 is reduced below this minimum, the shock wave will detach from the nose of the obstruction and become curved. For infinite M_1 , the maximum value of δ at which the shock can remain attached is 46.5° .

3. DERIVATION

1. COURANT, R. *Supersonic flow and shock waves.*
FRIEDRICH, K.O. Interscience Publishers Inc., New York, USA, 1948.
2. ROSENHEAD, L. (Ed.) *Compressible airflow: graphs.*
Clarendon Press, 1954.

4. EXAMPLE

Find the Mach number, static pressure and total pressure behind the shock wave formed by a two-dimensional wedge which deflects the air flow 10° . The incident Mach number, static pressure and total pressure are 2.0, $1.0 \times 10^5 \text{ N/m}^2$ and $1.4 \times 10^5 \text{ N/m}^2$, respectively.

From Figure 1 for $M_1 = 2.0$ and $\delta = 10^\circ$, $M_2 = 0.6$ or 1.64 and the corresponding angles of inclination of the shock wave are $\zeta = 84^\circ$ or 39° . The higher Mach number is generally obtained; hence $M_2 = 1.64$ and $\zeta = 39^\circ$.

From Figure 2 for $M_1 = 2.0$ and $\delta = 10^\circ$, $p_2/p_1 = 4.40$ or 1.68 . The lower pressure ratio is generally obtained; hence $p_2/p_1 = 1.68$. Therefore $p_2 = 1.68 \times 1 \times 10^5 = 1.68 \times 10^5 \text{ N/m}^2$.

From Figure 3 for $M_1 = 2.0$ and $\zeta = 39^\circ$, $p_{H2}/p_{H1} = 0.98$. Therefore

$$p_{H2} = 0.98 \times 1.4 \times 10^5 = 1.37 \times 10^5 \text{ N/m}^2.$$