

CONDITIONS BEHIND NORMAL SHOCK WAVES FOR AIR IN DISSOCIATION EQUILIBRIUM

1. NOTATION AND UNITS

		<i>SI</i>	<i>British</i>
<i>a</i>	speed of sound	m/s	ft/s
<i>F</i>	function defined in Section 2		
<i>G</i>	function defined in Section 2		
<i>i</i>	specific enthalpy	J/kg	ft lbf/slug
<i>M</i>	Mach number		
<i>p</i>	static pressure	N/m ²	lbf/ft ²
<i>R</i>	gas constant, taken equal to 287.1 J/kg K, 3090 ft lbf/slug K	J/kg K	ft lbf/slug K
<i>T</i>	temperature	K	K
<i>Z</i>	real gas factor, $p/\rho RT$		
β	dimensionless enthalpy parameter, $i/(p/\rho)$		
γ	ratio of specific heat capacity at constant pressure to specific heat capacity at constant volume, taken as 1.4 for air		
ρ	density	kg/m ³	slug/ft ³
Subscript			
1	denotes conditions in air ahead of shock		
2	denotes conditions in air in dissociation equilibrium behind shock		

2. NOTES

Derivation 2 gives the following approximate relations between the flow conditions behind the shock (after dissociation equilibrium is reached) and conditions ahead of the shock:–

$$\frac{i_2}{R} \approx T_1 M_1^2 F(M_1), \tag{2.1}$$

$$p_2 \approx p_1 M_1^2 F(M_1) \left[G(M_1) - \frac{1}{\beta_2} \right] \quad (2.2)$$

$$\text{and} \quad \frac{\rho_2}{\rho_1} \approx \beta_2 G(M_1) - 1. \quad (2.3)$$

For a perfect gas ($\beta_2 = \gamma/(\gamma - 1) = 3.5$), these relations hold exactly; thus the functions F and G shown in Figure 1 may be derived directly from the perfect gas normal shock relations.

To obtain the temperature and pressure behind the shock, first read off $F(M_1)$ and $G(M_1)$ from Figure 1 of this Item and then derive i_2/R and p_2 from Equations (2.1) and (2.2); for this preliminary estimate of p_2 , substitute in Equation (2.2) any value of β_2 consistent with the range shown in Figure 2 of Item No. Aero 00.01.06. Use Figures 1 and 2 of Item No. Aero 00.01.06 to find T_2 and β_2 . Hence obtain a more precise value of p_2 from Equation (2.2), using this derived value of β_2 . No further iteration should be necessary.

The density behind the shock follows from

$$\rho_2 = \frac{p_2 \beta_2}{i_2}, \quad (2.4)$$

and the real gas factor Z_2 is given by

$$Z_2 = \frac{i_2}{R \beta_2 T_2}. \quad (2.5)$$

To obtain the Mach number behind the shock, determine ρ_2/ρ_1 by substituting β_2 for and $G(M_1)$ in Equation (2.3), and read off the speed of sound from Figure 3 of Item No. Aero 00.01.06, whence M_2 is given by

$$M_2 = M_1 \times \frac{\rho_1}{\rho_2} \times \frac{(\gamma R T_1)^{1/2}}{a_2}. \quad (2.6)$$

Equations (2.1), (2.2) and (2.3) are approximate forms of the exact relations of Derivation 1; the use of these approximations introduces only small errors (*e.g.* less than ½ per cent and 2½ per cent in ρ_2/ρ_1 and T_2/T_1 , respectively) and has the advantage of providing a quick direct method with no lengthy iterations. It is assumed throughout that the air ahead of the shock wave is undissociated and acts as a perfect gas with $\gamma = 1.4$; this implies that $T_1 < 500$ K.

3. DERIVATION

1. EVANS, J.S. Method for calculating effects of dissociation on flow variables in the relaxation zone behind normal shock waves.
NACA tech. Note 3860, 1956.
2. WEAVER, A.K.
DOMMETT, R.L. Unpublished work