

EXAMPLES IN THE USE OF THE MOLLIER CHART (ISENTROPIC FLOW AND PRANDTL-MEYER EXPANSION FOR AIR IN DISSOCIATION EQUILIBRIUM)

1. NOTATION AND UNITS

		<i>SI</i>	<i>British</i>
A	local cross-sectional area of flow	m^2	ft^2
a	speed of sound	m/s	ft/s
i	specific enthalpy	J/kg	ft lbf/slug
M	Mach number		
m	rate of mass flow	kg/s	slug/s
p	static pressure	N/m^2	lbf/ft^2
R_0	gas constant for air (taken as 288.3 J/kg K, 3103 ft lbf/slug K)	J/kg K	ft lbf/slug K
s	specific entropy	J/kg K	ft lbf/slug K
T	temperature	K	K
V	(resultant) flow velocity	m/s	ft/s
V_{max}	constant with dimensions of velocity, defined by Equation (4.1)	m/s	ft/s
Z	real gas factor, $p/\rho RT$		
γ	ratio of specific heat capacity at constant pressure to specific heat capacity at constant volume		
η	parameter defined by Equation (4.2)		
θ	angular co-ordinate, see Sketch 4.1	deg	deg
ρ	density	kg/m^3	slug/ft^3
ψ	angle defined by Equation (4.3)	deg	deg
ω	angle through which flow has turned, see Sketch 4.1	deg	deg

Subscripts

H	denotes stagnation conditions
r	denotes component along radius vector

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θ	denotes component perpendicular to radius vector
0	denotes constant value used in Item Nos Aero 00.01.11 and 12
∞	denotes value along Mach line on which Prandtl-Meyer expansion begins

2. INTRODUCTION

Item Nos Aero 00.01.11 and 12 give thermodynamic properties of air in dissociation equilibrium in the form of Mollier charts. The present Item gives examples in the use of these charts.

It should be noted that the charts are plotted in non-dimensional form and may therefore be used in conjunction with any consistent set of units.

3. ISENTROPIC EXPANSION

3.1 Use of Mollier Chart

From the energy equation (see Item No. Aero S.00.03.06) in isentropic flow

$$\frac{V^2}{2} + i = i_H = \text{constant} . \quad (3.1)$$

Any two flow properties define the position on the Mollier chart of the stagnation conditions. Thus, stagnation entropy, s_H , and enthalpy, i_H , are known. A vertical line (constant entropy) may then be drawn on the chart to the point at which it is required to terminate the expansion. At this point enthalpy, density, pressure, speed of sound, temperature and real gas factor are known; flow velocity, and thence Mach number, may be determined from Equation (3.1).

If the ratio of cross-sectional areas at two points in the flow is required, the equation

$$m = \rho AV \quad (3.2)$$

may be used, where m is a constant.

3.2 Numerical Example of Isentropic Expansion

Air, initially at a stagnation pressure, p_H , of 0.1 atmospheres and temperature, T_H , of 6000 K, is expanded to a pressure, p , of 0.001 atmospheres. (i) Find the temperature, T , real gas factor, Z , density, ρ , speed of sound, a , flow velocity, V , and Mach number, M , when the expansion has taken place. (ii) Determine the ratio of the cross-sectional area of the flow at this condition to the cross-sectional area when the air has been expanded to a pressure of 0.01 atmospheres.

- (i) Initially $p_H = 0.1$ atmospheres, so $\log_{10} p/p_0 = -1$. From the Mollier chart in Item No. Aero 00.01.11 at $T_H = 6000$ K and $\log_{10} p/p_0 = -1$,

$$s/R_0 = s_H/R_0 = 49.65$$

$$\text{and } i/(R_0 T_0) = i_H/(R_0 T_0) = 281.5.$$

After expansion $p = 0.001$ atmospheres, so $\log_{10} p/p_0 = -3$. As the expansion is isentropic, s/R_0 is constant and from the Mollier chart at $s/R_0 = 49.65$ and $\log_{10} p/p_0 = -3$, the following values are obtained (cross-plotting at constant s/R_0 where necessary),

$$T = 4395 \text{ K},$$

$$Z = 1.310,$$

$$\log_{10} \rho/\rho_0 = -4.320,$$

$$a/a_0 = 4.090$$

$$\text{and } i/(R_0 T_0) = 158.5.$$

The temperature and real gas factor are thus found directly from the Mollier chart. The remaining properties that are required are calculated as follows.

For the density,

$$\log_{10} \rho/\rho_0 = -4.320,$$

$$\text{so } \rho/\rho_0 = 4.786 \times 10^{-5},$$

$$\text{and } \rho = 4.786 \times 10^{-5} \times 1.288 = 61.64 \times 10^{-6} \text{ kg/m}^3.$$

For the speed of sound

$$a/a_0 = 4.090,$$

$$\text{so that } a = 4.090 \times 332.0 = 1358 \text{ m/s}.$$

The flow velocity is calculated by using Equation (3.1),

$$\begin{aligned} V &= [2(i_H - i)]^{1/2} \\ &= [2(281.5 - 158.5)R_0 T_0]^{1/2} \\ &= [2(281.5 - 158.5)288.3 \times 273.16]^{1/2} \\ &= 4401 \text{ m/s}, \end{aligned}$$

and therefore the associated Mach number is

$$M = \frac{V}{a} = \frac{4401}{1358} = 3.241.$$

- (ii) When the air has expanded to a pressure of 0.01 atmospheres (*i.e.* at $s/R_0 = 49.65$ and $\log_{10} p/p_0 = -2$) the Mollier chart gives

$$\log_{10} \rho/\rho_0 = -3.425$$

$$\text{and } i/(R_0 T_0) = 212.5.$$

Therefore

$$\begin{aligned} \rho &= 3.758 \times 10^{-4} \rho_0 \\ &= 3.758 \times 10^{-4} \times 1.288 = 484.0 \times 10^{-6} \text{ kg/m}^3, \end{aligned}$$

and, from Equation (3.1),

$$\begin{aligned} V &= [2(i_H - i)]^{1/2} \\ &= [2(281.5 - 212.5)R_0 T_0]^{1/2} \\ &= [2(281.5 - 212.5)288.3 \times 273.16]^{1/2} \\ &= 3297 \text{ m/s}. \end{aligned}$$

The required area ratio is then obtained by using Equation (3.2),

$$\rho A V = \text{constant}.$$

Therefore

$$61.64 \times 10^{-6} A(\text{at } p/p_0 = 0.001) 4401 = 484.0 \times 10^{-6} A(\text{at } p/p_0 = 0.01) 3297$$

and so

$$\frac{A(\text{at } p/p_0 = 0.001)}{A(\text{at } p/p_0 = 0.01)} = 5.882.$$

4. PRANDTL-MEYER EXPANSION

4.1 Use of Mollier Chart

This section outlines the method of calculating a Prandtl-Meyer expansion; for brevity, only the equations necessary for computation are given. If a more detailed explanation of the derivation of these equations is required reference may be made to Section 4.2 and to Derivations 1 and 2.

The initial conditions of the flow define a position on the Mollier chart, so that entropy, s , enthalpy, i , and speed of sound, a , may be determined.