

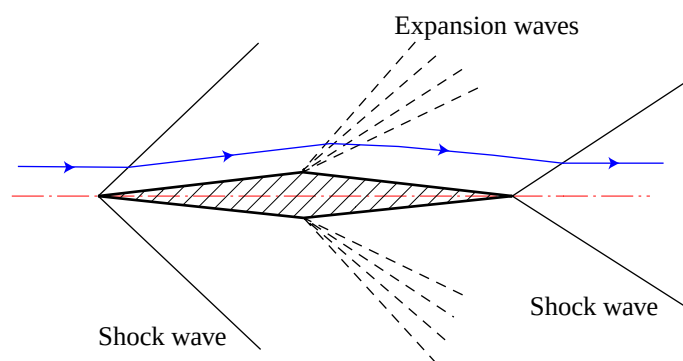
INTRODUCTORY ITEM ON TWO-DIMENSIONAL AEROFOILS AT SUPERSONIC SPEEDS

PART I. INVISCID FLOW

1. THE NATURE OF THE FLOW WITH ATTACHED SHOCK WAVES

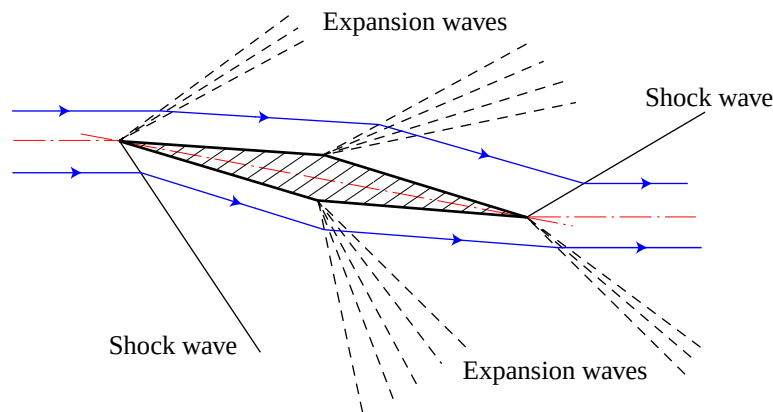
As explained in Item No. Aero S.00.03.06, in steady inviscid two-dimensional supersonic flow, it is possible under certain conditions to derive exact expressions relating the conditions on a surface, *e.g.* speed, pressure, temperature and density, to the initial conditions and the flow deflection from its initial direction. If the flow is deflected in a compressive sense, the deflection occurs discontinuously through a shock wave; if in an expansive sense, it occurs by means of simple-wave flow. Thus, provided that the shock waves are attached and, therefore, that the flow is everywhere supersonic, the flow pattern around an aerofoil may readily be obtained.

As an example, consider the case of the symmetric double-wedge section at zero incidence shown in Sketch 1.1. When the supersonic air stream encounters the leading edge the flow is deflected in a compressive sense and a shock wave is formed. Provided that the flow deflection is not too great for the particular free-stream Mach number, the leading-edge shock wave will be attached and the flow behind it will be supersonic, but at a lower Mach number than the free stream; the static pressure will be higher than the free-stream value. The static pressure will remain constant along the surface until the shoulder of the double-wedge is reached; here there is an expansive deflection which is achieved by means of the centred type of simple-wave flow, sometimes referred to as a Prandtl-Meyer expansion. Through this expansion the static pressure falls to a value below that of the free stream and the Mach number rises to a value greater than that of the free stream. At the trailing edge there is a further compressive deflection through a shock wave which restores the flow to the free-stream direction.



Sketch 1.1

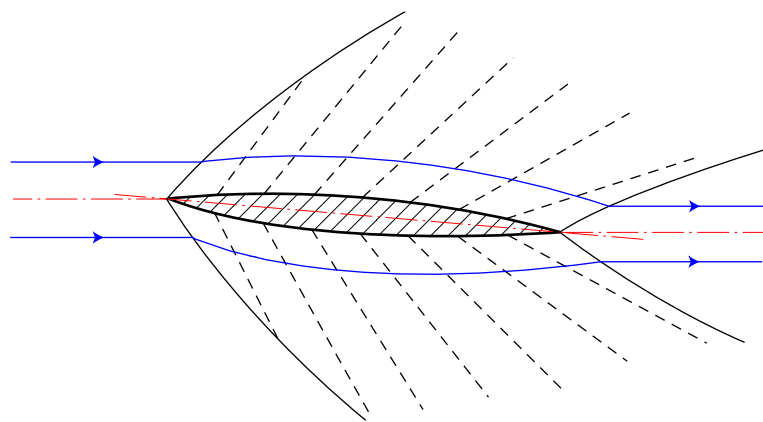
If the incidence of the aerofoil is increased, the strengths of the various systems re-adjust themselves to meet the changed flow deflections until eventually an incidence is reached when there is no deflection necessary at the upper surface leading-edge or the lower surface trailing-edge, and the shock waves at these points disappear. If the incidence is further increased the deflections at these points become expansive, and the shock waves are replaced by centred simple-flows as is shown in Sketch 1.2. At some stage the compressive deflection on the lower surface will become so great that the leading-edge shock wave will become detached. At the trailing edge of an aerofoil at supersonic speeds the flow is deflected so that the air stream very nearly regains its original direction.



Sketch 1.2

In supersonic flow there is drag even in the absence of viscosity. This drag force may be related to the rate at which energy is expended to maintain the wave system and is, therefore, termed wave drag. It may, however, be readily observed as a resultant of the pressure distribution on the surface of the aerofoil caused by the shock wave and simple-wave system. It may be noted that there is a mutually weakening interaction between the shock wave and the simple-wave system and the drag thus remains finite. This mutual interaction generally has negligible effect on the surface pressure distribution except perhaps at very high Mach numbers.

In the foregoing description of the flow pattern around an aerofoil a double-wedge section was considered. This simplifies the description since the expansion at the shoulder occurs at a single point through the centred simple-wave flow and close to the aerofoil the shock waves are straight because they do not interact with the expansion waves. If the surfaces of the aerofoil are curved as for example in a biconvex section aerofoil, the expansions take place continuously along them. The shock wave systems at the leading and trailing edges follow the same form as for the double wedge section except that the interaction between the shock waves and the expansion waves commences from the surface of the aerofoil causing the shock waves to take up a slightly curved form as is shown in Sketch 1.3. It has already been noted that this interaction and the consequent curvature of the shock waves can usually be neglected when calculating the conditions on the surface.



Sketch 1.3

2. METHODS OF CALCULATING INVISCID FLOW PAST AN AEROFOIL

There are three basic methods of calculating the inviscid supersonic flow past an aerofoil with attached shock waves. These methods are the nearly exact or shock-expansion method, the simple-wave flow method, and the second- and first-order theory methods. Calculations making use of these methods may be made with the assistance of the appropriate Items. Where greater accuracy is required use may be made of tables such as those of References 4, 8 and 13.

2.1 Shock-expansion Method

As indicated earlier, the changes in the conditions around the surface of an aerofoil are brought about by a combination of shock waves and simple-wave type flow. These types of flow are described in detail in Item Nos Aero S.00.03.08 and 09, and the changes in pressure and velocity on the surface of an aerofoil can be readily obtained from those Items. When calculating lift and drag coefficients for an aerofoil, however, a high degree of accuracy is required in the pressure distribution, as these coefficients depend on small differences in pressure on the upper and lower surfaces which are caused by relatively small flow deflections. Item Nos Aero S.00.03.08 to 11 cover a fairly large range of deflections, and do not generally give a sufficiently accurate result for use in connection with aerofoils. Item No. Aero. W.S.00.03.05 contains curves giving similar data plotted in a form more suitable for calculations of this type and also indicates the manner in which such calculations should be made.

2.2 Simple-wave Flow Method

Provided that the flow deflection is not too large the changes in flow conditions through a compressive deflection can be calculated on the assumption that the equations of simple-wave flow apply, and thus data on simple-wave flow (Item Nos Aero S.00.03.08 and W.S.00.03.05) can be used over the entire flow field. This assumption implies that the flow is isentropic, and, for an aerofoil with an attached shock wave at low incidence, gives fairly accurate results. The method is most useful when working from data in tabular form as shock wave data are generally cumbersome to use. The method is not recommended when suitable graphs are available as the curves for the shock-expansion method are as simple to use as would be the corresponding curves for the simple-wave flow method.

2.3 Second- and First-order Theory Method

Item No. Aero S.00.03.14 describes a theory in which the local pressure coefficient is obtained in a power series in the local flow deflection measured from the free-stream direction. For most purposes this power series yields sufficiently accurate results if the first two terms only are used, when it is referred to as second-order theory. If the second term is also omitted the method is referred to as first-order theory.

The use of this theory makes it a relatively simple matter to obtain analytical expressions for the various characteristics of aerofoils, and a convenient method by which these equations may be derived is given in Reference 1. Item No. Aero W.S.00.03.04 gives the results of applying the theory to three types of supersonic aerofoils, and it may be seen that the equations considerably simplify the calculation of the variation of the aerodynamic characteristics with Mach number, incidence or geometry of the section.

3. ACCURACY OF THE METHODS

The methods have been described in their order of accuracy. The shock-expansion method is almost exact, the interaction of shock waves and expansion waves away from the surface having been neglected, and this, as has been stated earlier, has negligible effect. In the simple-wave method the loss of total pressure through the shock wave system is also neglected, whilst in both second and first-order theory methods there