

ESDU Wings S.00.03.04

THEORETICAL PROPERTIES OF TWO-DIMENSIONAL SUPERSONIC AEROFOILS OF SIMPLE SECTION

1. NOTATION AND UNITS

		SI	British
C_1, C_2	first- and second-order Busemann coefficients, described in Item No. Aero S.00.03.14		
C_{DWL}	wave drag coefficient due to lift, (drag due to lift per unit span)/($\frac{1}{2}\rho_0 V_0^2 c$)		
C_{DW}	total wave drag coefficient, (total wave drag per unit span)/($\frac{1}{2}\rho_0 V_0^2 c$)		
C_{DW0}	wave drag coefficient at zero lift, (wave drag at zero lift per unit span)/($\frac{1}{2}\rho_0 V_0^2 c$)		
C_L	lift coefficient, (lift per unit span)/($\frac{1}{2}\rho_0 V_0^2 c$)		
C_m	pitching moment coefficient about leading edge, (pitching moment per unit span)/($\frac{1}{2}\rho_0 V_0^2 c^2$)		
c	chord of aerofoil	m	ft
d	chordwise distance from leading edge expressed as fraction of chord		
h_p	distance of centre of pressure aft of leading edge expressed as fraction of chord, $-C_m/C_L$		
h_0	distance of aerodynamic centre aft of leading edge expressed as fraction of chord, $-dC_m/dC_L$		
M_0	free-stream Mach number		
t	thickness of aerofoil expressed as fraction of chord, see Table 2.1 (Type 2)		
t_u, t_l	thickness of upper, lower part of aerofoil expressed as fraction of chord, see Table 2.1 (Types 1 and 3)		
V_0	free-stream velocity	m/s	ft/s
α	incidence	radian	radian
γ	ratio of specific heat capacities, assumed equal to 1.4 for air		
ρ_0	free-stream density	kg/m ³	slug/ft ³

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2. NOTES

This Item gives the results obtained by applying the first- and second-order theories of Derivation 1 to some two-dimensional supersonic aerofoils of simple geometric section.

The expressions for the lift, drag and pitching moment coefficients and aerodynamic centre position are given in Table 2.1 in terms of the dimensions of the aerofoil and the first- and second-order Busemann coefficients C_1 and C_2 . These latter coefficients are functions of Mach number and are given by

$$C_1 = \frac{2}{(M_0^2 - 1)^{1/2}} \quad (2.1)$$

and

$$C_2 = \frac{(\gamma + 1)M_0^4 - 4(M_0^2 - 1)}{2(M_0^2 - 1)^2} \quad (2.2)$$

Item No. Aero S.00.03.14 tabulates values of C_1 and C_2 and C_2/C_1 for values of M_0 from 1.2 to 4. These expressions apply only if the flow is everywhere supersonic. When the deflection at the leading edge of the section, due to the combination of nose angle and incidence, is compressive and greater than the shock detachment angle for the free-stream Mach number, the nose shock will detach and the flow will become partly subsonic and the expressions will no longer apply. The limiting angle for this condition can be determined from Item No. Aero S.00.03.16. This Item does not include the effects of viscosity.

The first-order (linearised theory) approximations to the lift, drag and pitching moment coefficients are obtained by omitting the terms in C_2 from the expressions given in Table 2.1.

Most practical supersonic sections are symmetrical, or nearly so, and for such sections the following conclusions can be drawn.

- (i) The lift-curve slope is independent of section shape and is given by

$$\frac{dC_L}{d\alpha} = 2C_1 \quad (2.3)$$

- (ii) The wave drag coefficient in inviscid flow may be split into two parts, the wave drag coefficient at zero lift, C_{DW0} , which is a function of section shape, and the wave drag coefficient due to lift, C_{DWL} , which is a function of lift alone. Thus for a symmetrical section of Type 2 (see Table 2.1), to first order,

$$C_{DW0} = C_1 \frac{t^2}{2} \left(\frac{1}{d_1} + \frac{1}{1 - d_2} \right), \quad (2.4)$$

and

$$C_{DWL} = C_L \alpha = 2C_1 \alpha^2 \quad (2.5)$$

For a section of this type and for a given thickness/chord, t , the minimum drag occurs when $d_1 = d_2 = 0.5$, i.e. a doubly-symmetrical wedge section. This type of section has the minimum wave drag of all sharp-edged sections for a given thickness/chord. The effect of variation of the geometry of aerofoils of Type 2 is illustrated in Sketch 2.1, which shows the ratio of the wave drag coefficient at zero lift of a symmetrical section with particular values of d_1 and d_2 to that of the minimum drag section of the same thickness/chord.

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The wave drag at zero lift of the biconvex section of Type 3 with $t_u = t_l$ is 1.33 times that of the minimum drag of doubly symmetrical wedge section of the same thickness.

- (iii) In first-order theory, *i.e.* for a thin aerofoil, the aerodynamic centre of any symmetrical section is at the mid-chord point, since

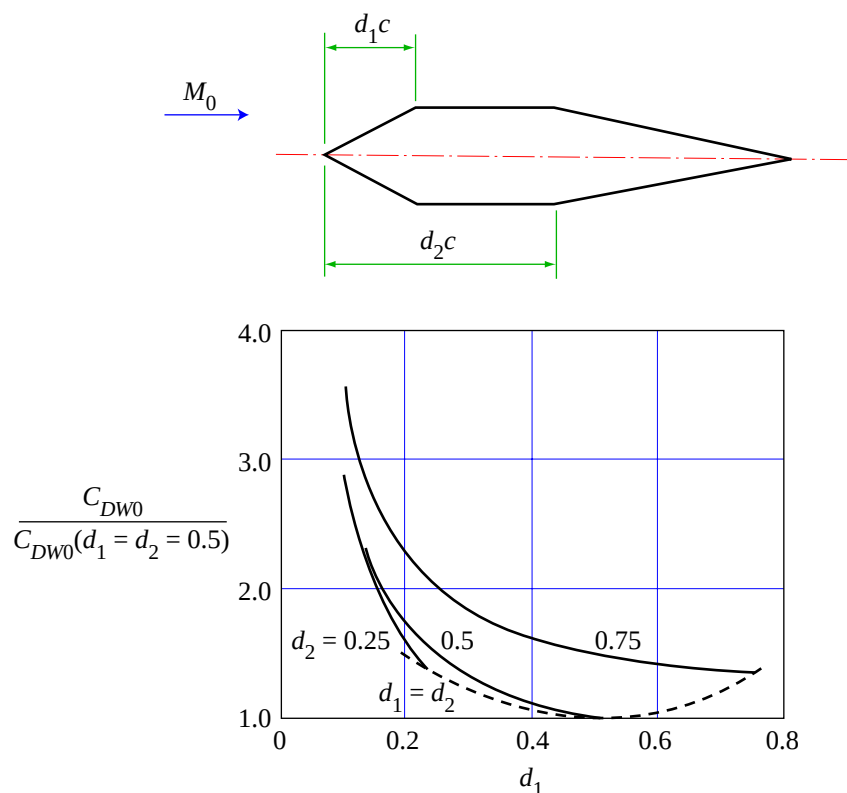
$$\begin{aligned} h_0 &= -\left(\frac{dC_m}{dC_L}\right) = -\left(\frac{dC_m}{d\alpha}\right)\left(\frac{d\alpha}{dC_L}\right) \\ &= -(-\frac{1}{2} \times 2C_1)\left(\frac{1}{2C_1}\right) = \frac{1}{2}. \end{aligned} \quad (2.6)$$

For a doubly-symmetrical aerofoil of Type 2, second-order theory gives the aerodynamic centre position as

$$h_0 = \frac{1}{2}\left(1 - \frac{C_2}{C_1}t\right). \quad (2.7)$$

The variation of h_0 with Mach number for various thickness/chord ratios is illustrated in Sketch 2.2.

The theoretical properties of a section with curved surfaces and variable maximum thickness position are given in Derivation 3.



Sketch 2.1 Wave drag coefficient at zero lift for symmetrical double-wedge sections (first-order theory)