THEORETICAL LIFT-DEPENDENT DRAG OF WINGS AT SUPERSONIC SPEEDS

1. NOTATION AND UNITS

		SI	*British*
A	aspect ratio		
b	wing span	m	ft
C_{Di}	lift-dependent drag coefficient		
C_L	lift coefficient		
C_{Lcrit}	value of lift coefficient at which C_{Tmax} is obtained		
C_N	normal force coefficient		
C_T	suction force coefficient, see Equation (2.1)		
C_{Tmax}	maximum value of suction force coefficient		
c	local chord, measured in direction parallel to plane of symmetry	m	ft
c_r	wing centre-line chord	m	ft
c_t	wing tip chord	m	ft
$E(k')$	complete elliptic integral of second kind		
F	ratio of experimental peak suction coefficient to that corresponding to a vacuum, *i.e.* to $2\tau/(\gamma M^2)$		
g	function given by Equation (2.4)		
k'	parameter given by $[1-\beta^2\cot^2\Lambda_0]^{\frac{1}{2}}$		
M	free-stream Mach number		
m	ratio of distance of maximum thickness of wing section aft of leading edge to chord, x_m/c		
R	ratio of average suction of elliptical leading edge to maximum local suction		
t	maximum thickness of wing section	m	ft
x_m	distance of maximum thickness of wing section aft of leading edge, measured in direction parallel to plane of symmetry	m	ft

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α	incidence measured from zero-lift angle	radian	radian
β	Mach number parameter, $(M^2-1)^{\frac{1}{2}}$		
γ	ratio of specific heat capacities, taken as 1.4 for air		
Λ_0	sweepback of wing leading edge	degree	degree
$\Lambda_{\frac{1}{2}}$	sweepback of wing mid-chord line	degree	degree
λ	ratio of tip chord to centre-line chord, c_t/c_r		
τ	thickness/chord ratio, t/c		
τ_n	thickness/chord ratio of ellipse approximating locally to nose profile of wing section measured in plane normal to leading edge, $\tau/(2m\cos\Lambda_0)$		

2. NOTES

The lift-dependent drag coefficient of a wing at supersonic speeds can be written as

$$C_{Di} = \alpha C_L - C_T. \tag{2.1}$$

The so-called suction force coefficient C_T, which is itself dependent on α, is predicted by linearised theory to be zero for wings with sharp supersonic leading edges, *i.e.* if the leading edge of the wing is everywhere swept-back at an angle less than the free-stream Mach angle so that the component of the free-stream Mach number normal to the wing leading edge is supersonic. For a wing with a straight leading edge this condition occurs when $\beta\cot\Lambda_0 > 1.0$. In this case the lift-dependent drag is simply the component of the normal force on the wing resolved along the free-stream direction and is given by

$$\begin{aligned} C_{Di} &= C_N \sin\alpha \\ &\approx C_L\alpha \text{ for small } \alpha. \end{aligned} \tag{2.2}$$

If, however, the local leading-edge sweep is such that the leading edge is wholly or partly subsonic, the flow will take on some of the characteristics of subsonic flow and in particular there will be an interaction between the flow over the upper and lower surfaces. Two features of this interaction will be a movement of the stagnation point with incidence and an associated suction force acting at the leading edge.

According to linearised theory the numerical value of C_T for a wing with a subsonic leading edge is given by

$$C_T = gA\alpha^2 \tag{2.3}$$

where

$$g = \frac{k'}{4\left[E(k')\right]^2}. \tag{2.4}$$

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Figure 1 gives values of g plotted as a function of $\beta\cot\Lambda_0$. Linearised theory tends to become unreliable at low values of βA, but it may be noted that in the special case $\beta A = 0$ slender-wing theory gives the lift-dependent drag as

$$C_{Di} = \tfrac{1}{2}\alpha C_L \tag{2.5}$$

i.e. $C_T/C_L = 0.5$ and $g = \pi/4$.

The suction force coefficient is predicted as being proportional to α but it is clear that C_T cannot exceed the value corresponding to a vacuum over the entire front part of the wing, given by $2\tau/(\gamma M^2)$. In practice the value of C_{Tmax} is a small fraction of this, since such maximum suction as can be achieved will not be uniformly distributed over the forward part of the wing. To arrive at an assessment of C_{Tmax} it has been assumed that the pressure distribution on the leading edge corresponds locally to that on the forward part of an elliptic cylinder and experimental data have been used to provide estimates of the maximum local suction. The elliptic cylinder is chosen having its major axis of length $2mc\cos\Lambda_0$ on the chord line and the minor axis equal to the thickness of the section. The elliptic cylinder therefore approximates locally to the section of the leading edge in a plane normal to the leading edge, and the thickness/chord ratio of the elliptic cylinder is given by

$$\tau_n = \frac{\tau}{2m\cos\Lambda_0}. \tag{2.6}$$

The ratio, R, of the average suction per unit thickness to the maximum local suction on the forward half of an ellipse, is plotted against the thickness/chord ratio of the ellipse, τ_n, in Figure 2. This ratio is obtained from theoretical work by Cane and Collingbourne (Derivation 3). It is assumed that R is unaffected by compressibility while, in addition, theory shows that it is practically independent of incidence. In Figure 3 the factor F, which is the ratio of the maximum local suction coefficient to $2\tau/(\gamma M^2)$, is plotted as a function of Mach number. The maximum suction force coefficient is then given by

$$C_{Tmax} = 2FR\tau/(\gamma M^2). \tag{2.7}$$

The procedure for the estimation of C_{Di} is, therefore, as follows:

(i) The value of $C_T\,(= gA\alpha^2)$ is obtained using Figure 1 and is substituted into Equation (2.1), *i.e.* $C_{Di} = \alpha C_L - C_T$. This relation holds up to the incidence at which C_{Tmax} is achieved.

(ii) The value of C_{Tmax} is obtained from Equation (2.7), *i.e.* $C_{Tmax} = 2FR\tau/(\gamma M^2)$ using Figures 2 and 3. At incidences greater than that at which C_{Tmax} is achieved, the lift-dependent drag is given by

$$C_{Di} = \alpha C_L - C_{Tmax}. \tag{2.8}$$

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