

ESDU Wings S.08.03.02

PITCHING VELOCITY DERIVATIVES FOR WINGS AT SUPERSONIC SPEEDS Z_q AND M_q

1. NOTATION AND UNITS (see Sketch 1.1)

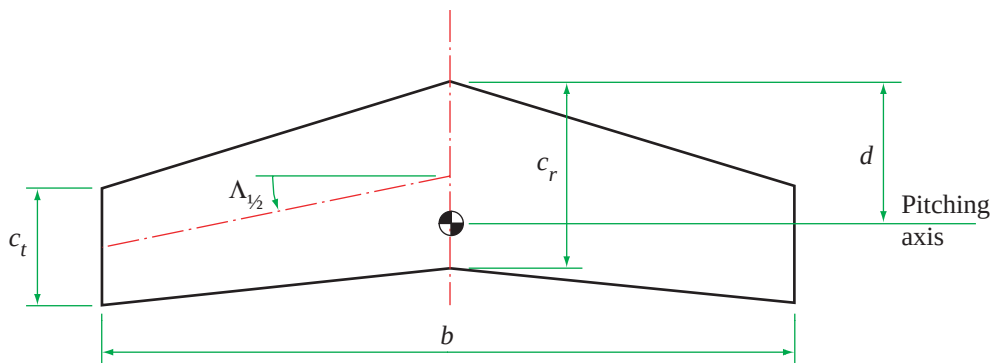
The derivative notation used is that proposed in ARC R&M 3562 (Hopkin, 1970) and described in Item No. 86021. Coefficients and aerodynamical derivatives are evaluated in aerodynamic body axes with origin on the body centre-line chord and with the aerodynamic mean chord of the wing as the characteristic length. The derivatives M_q and Z_q are often written as C_{mq} and C_{zq} in other systems of notation, but attention must be paid to the reference dimensions used. In particular, in forming C_{mq} and C_{zq} differentiation may be carried out with respect to $\bar{q}\bar{c}/2V$ instead of $\bar{q}\bar{c}/V$ as implied in the Hopkin system. It is also to be noted that a constant datum value of V is employed by Hopkin.

		SI	British
A	aspect ratio, b^2/S		
b	wing span	m	ft
C_L	lift coefficient, $L / \frac{1}{2}\rho V^2 S$		
C_m	pitching moment coefficient, $\mathcal{M} / \frac{1}{2}\rho V^2 S \bar{c}$		
c_r	wing root (centre-line) chord	m	ft
c_t	wing tip chord	m	ft
$\bar{c}$	aerodynamic mean chord	m	ft
d	distance of pitching axis from leading edge of wing root chord, positive aft	m	ft
h_0	ratio of distance of aerodynamic centre aft of leading edge of root chord to aerodynamic mean chord		
L	lift	N	lbf
M	Mach number		
$\mathcal{M}$	pitching moment	N m	lbf ft
M_q	pitching moment derivative due to pitching, $(\partial \mathcal{M} / \partial q) / \frac{1}{2}\rho V S \bar{c}^2$		
M'_q	value of M_q for wing pitching about leading edge of root chord		
q	angular velocity in pitch	radian/s	radian/s
S	wing area	m ²	ft ²
V	forward velocity	m/s	ft/s

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x	distance of leading edge of aerodynamic mean chord aft of leading edge of root chord	m	ft
$\bar{x}$	distance of aerodynamic centre aft of leading edge of aerodynamic mean chord	m	ft
Z_q	lift derivative due to pitching, $-(\partial L/\partial q)/\frac{1}{2}\rho V S \bar{c}$		
Z'_q	value of Z_q for wing pitching about leading edge of root chord		
α	angle of attack	radian	radian
$\Lambda_{1/2}$	sweepback of mid-chord line	degree	degree
λ	ratio c_t/c_r		
ρ	density of air	kg/m ³	slug/ft ³



Sketch 1.1

2. NOTES

In Figures 1a to 1e and 2a to 2e, the derivatives Z'_q/A and M'_q/A are plotted against $A(M^2 - 1)^{1/2} - A \tan \Lambda_{1/2}$ for values of $A \tan \Lambda_{1/2}$ from 0 to 6 and for values of λ from 0 to 1 for straight-tapered wings with streamwise tips. The derivatives Z'_q and M'_q apply to wings pitching about the leading edge of the root chord (*i.e.* the wing apex), and the corresponding derivatives Z_q and M_q for a wing pitching about an axis at a distance d from the wing apex may be obtained from the relations

$$Z_q = Z'_q + (\partial C_L / \partial \alpha) d / \bar{c} \quad (2.1)$$

and

$$M_q = M'_q - Z'_q d / \bar{c} - (\partial C_L / \partial \alpha) [(d / \bar{c}) - h_0] d / \bar{c}. \quad (2.2)$$

In these relations the values of $\partial C_L / \partial \alpha$ and h_0 may be obtained from Item No. 70012.

The parts of the curves drawn as full lines in Figures 1a to 1e and 2a to 2e are based on the linearised theory results given in Derivations 1 to 3 for wing planforms where the Mach lines from the leading edge of the tip chord do not intersect the remote half-wing and where the Mach lines from the trailing edge of the root chord do not intersect the wing leading edge. Linearised theory results are not available for low values of $A(M^2 - 1)^{1/2}$, but the slender-wing theory of Derivation 4 gives results for the special case of $A(M^2 - 1)^{1/2} = 0$, and the curves have been extrapolated to these results by dashed lines for wings with sweptback trailing edges.

As applied in this Item, linearised theory is only applicable to very thin wings in inviscid flow at small angles of attack and with small values of $q\bar{c}/V$. There are no direct experimental data available with which to compare the results given by this Item, but indirect evidence suggests that they are of acceptable accuracy.

3. DERIVATION

The Derivation lists selected sources that have assisted in the preparation of this Item.

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|----|--|--|
| 1. | MALVESTUTO, F.S.
MARGOLIS, K. | Theoretical stability derivatives of thin sweptback wings tapered to a point with sweptback or sweptforward trailing edges for a limited range of supersonic speeds. NACA Rep. 971, 1950. |
| 2. | MALVESTUTO, F.S.
HOOVER, D.M. | Lift and pitching derivatives of thin sweptback tapered wings with stream-wise tips and subsonic leading edges at supersonic speeds. NACA tech. Note 2294, 1951. |
| 3. | MARTIN, J.C.
MARGOLIS, K.
JEFFREYS, I. | Calculation of lift and pitching moments due to angle of attack and steady pitching velocity at supersonic speeds for thin sweptback tapered wings with streamwise tips and supersonic leading and trailing edges. NACA tech. Note 2699, 1952. |
| 4. | MANGLER, K.W. | Calculation of the pressure distribution over a wing at sonic speeds. ARC R&M 2888, 1955. |

4. EXAMPLE

Estimate the theoretical values of the derivatives Z_q and M_q at a Mach number of 2.0 for a wing pitching about an axis which is located at a distance of 50 per cent of the aerodynamic mean chord from the wing apex. The wing has an aspect ratio of 3.0, $\lambda = 0.50$ and $\Lambda_{1/2} = 45^\circ$.

From the given information

$$A \tan \Lambda_{1/2} = 3.0 \tan 45^\circ = 3.0$$

and $A(M^2 - 1)^{1/2} = 3.0(2^2 - 1)^{1/2} = 5.20,$

so $A(M^2 - 1)^{1/2} - A \tan \Lambda_{1/2} = 2.20.$

From Item No. 70012, $(1/A) \partial C_L / \partial \alpha = 0.820.$

From Figure 1c, $Z'_q / A = -0.965.$

Hence, from Equation (2.1) with $d/\bar{c} = 0.5$,

$$Z_q/A = -0.965 + 0.820 \times 0.5 = -0.555$$

and $Z_q = -0.555 \times 3 = -1.665$.

From Item No. 70012, $\bar{x}/\bar{c} = 0.512$ and $x/\bar{c} = 0.786$

whence $h_0 = x/\bar{c} + \bar{x}/\bar{c} = 1.298$.

From Figure 2c, $M'_q/A = -1.415$, whence, from Equation (2.2) with $d/\bar{c} = 0.5$,

$$\begin{aligned} M_q/A &= -1.415 + 0.965 \times 0.5 - 0.820(0.5 - 1.298) \times 0.5 \\ &= -1.415 + 0.4825 + 0.3279 \\ &= -0.605. \end{aligned}$$

Therefore, $M_q = -0.605 \times 3 = -1.815$.

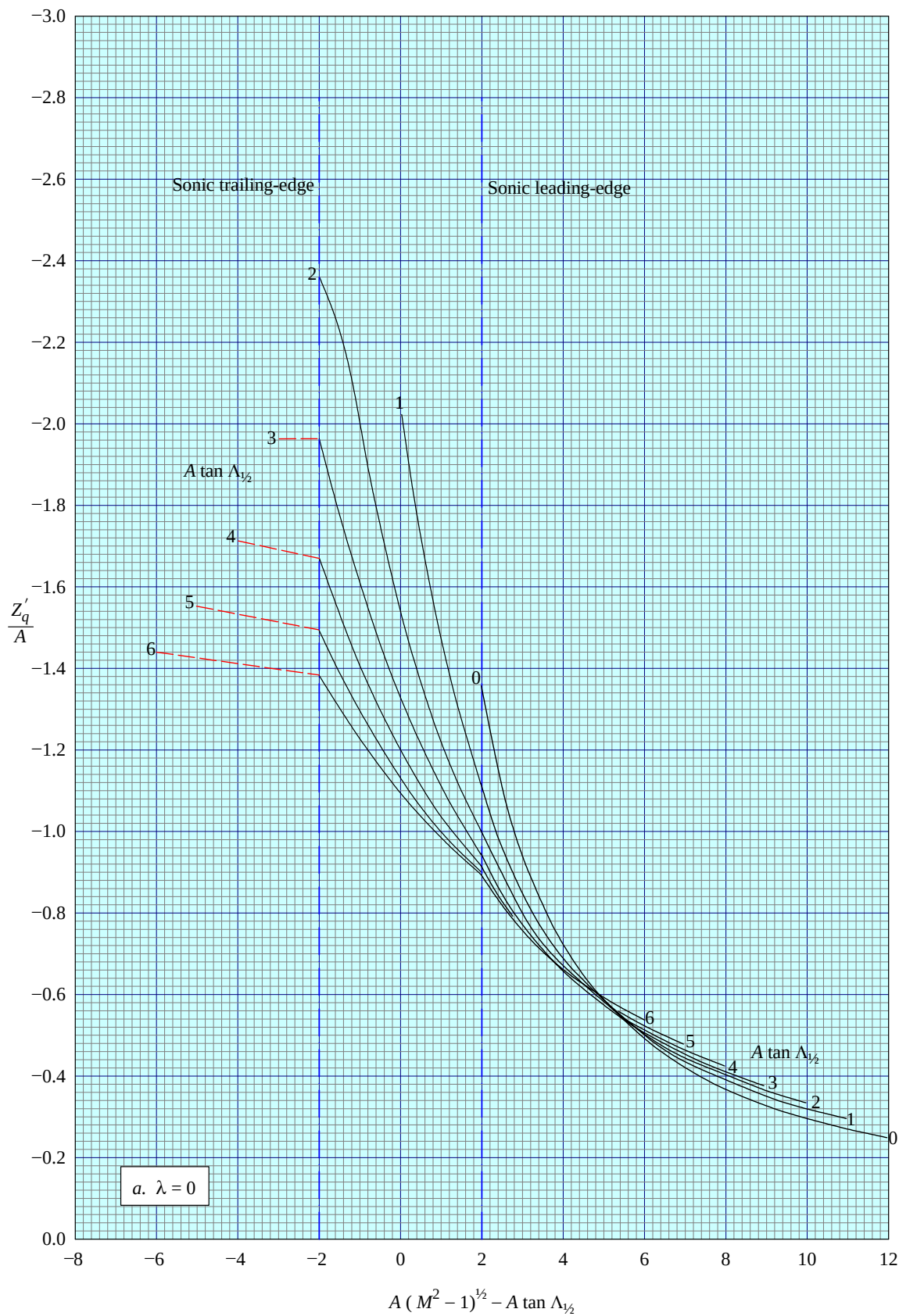


FIGURE 1a