

# ESDU Controls 01.01.03

## RATE OF CHANGE OF LIFT COEFFICIENT WITH CONTROL DEFLECTION IN INCOMPRESSIBLE TWO-DIMENSIONAL FLOW, $(a_2)_0$

### 1. NOTATION AND UNITS

|              |                                                                                                                                               | SI                   | British              |
|--------------|-----------------------------------------------------------------------------------------------------------------------------------------------|----------------------|----------------------|
| $(a_1)_0$    | slope of lift-coefficient curve with incidence for two-dimensional aerofoil in incompressible flow, $\partial C_L / \partial \alpha$          | radian <sup>-1</sup> | radian <sup>-1</sup> |
| $(a_1)_{0T}$ | theoretical slope of lift-coefficient curve with incidence for two-dimensional aerofoil in inviscid, incompressible flow                      | radian <sup>-1</sup> | radian <sup>-1</sup> |
| $(a_2)_0$    | slope of lift-coefficient curve with control deflection for two-dimensional aerofoil in incompressible flow, $\partial C_L / \partial \delta$ | radian <sup>-1</sup> | radian <sup>-1</sup> |
| $(a_2)_{0T}$ | theoretical slope of lift-coefficient curve with control deflection for a two-dimensional aerofoil in inviscid, incompressible flow           | radian <sup>-1</sup> | radian <sup>-1</sup> |
| $C_L$        | lift coefficient                                                                                                                              |                      |                      |
| $c$          | chord of aerofoil                                                                                                                             | m                    | ft                   |
| $c_f$        | control chord, aft of hinge line                                                                                                              | m                    | ft                   |
| $t$          | maximum thickness of aerofoil                                                                                                                 | m                    | ft                   |
| $\alpha$     | angle of incidence measured from no-lift angle with control undeflected                                                                       | radian               | radian               |
| $\delta$     | control deflection angle, positive downwards                                                                                                  | radian               | radian               |
| $\tau$       | trailing-edge angle                                                                                                                           | degree               | degree               |

### 2. NOTES

The theoretical rate of change of lift coefficient with control deflection  $(a_2)_{0T}$  is plotted against  $c_f/c$  for various values of  $t/c$  in Figure 1. In Figure 2  $(a_2)_0/(a_2)_{0T}$  is plotted against  $c_f/c$  for various values of  $(a_1)_0/(a_1)_{0T}$ . Values of  $(a_1)_0/(a_1)_{0T}$  for a given aerofoil section may be obtained from Item No. Aero W.01.01.05.

Within the linear range of the lift-incidence curve and over the range of control deflection for which the increment of lift is linear with control deflection, the lift coefficient  $C_L$  of a two-dimensional aerofoil at an angle of incidence  $\alpha$  and a control surface deflection  $\delta$  is given by

$$C_L = (a_1)_0 \alpha + (a_2)_0 \delta. \quad (2.1)$$

# ESDU Controls 01.01.03

For small angles of incidence this should apply within a range of  $\delta$  of about  $\pm 15^\circ$ . The accuracy of this Item is assessed to be within  $\pm 5$  per cent.

It may be noted that Equation (2.1) may be written as

$$C_L = (a_1)_0 [\alpha + \delta(a_2)_0 / (a_1)_0] \quad (2.2)$$

and the expression  $[\alpha + \delta(a_2)_0 / (a_1)_0]$  may therefore be regarded as the equivalent angle of incidence for the two-dimensional aerofoil with the control deflected. For calculations in which this concept is used,  $(a_2)_0$  may be obtained from this Item and  $(a_1)_0$  from Item No. Aero W.01.01.05.

The data apply to controls with the gap sealed; for plain and balanced controls the effect of unsealing the gap is given in Item No. Aero C.01.01.04.

For controls of finite aspect ratio  $(a_2)_0$  must be corrected for induced effects and use may be made of the method given in Item No. 74011.

### 3. DERIVATION

1. WOODS, L.C. The theory of aerofoils with hinged flaps in two-dimensional compressible flow. ARC CP 138, 1952.
2. GARNER, H.C. Charts for low-speed characteristics of two-dimensional trailing-edge flaps. ARC R & M 3174, 1957.

### 4. EXAMPLE

Find the lift coefficient increment due to a control deflection of  $10^\circ$  for a two-dimensional aerofoil in incompressible flow for which  $c_f/c = 0.35$  and  $t/c = 0.10$ . The profile of the aerofoil from 80 per cent of the chord to the trailing edge is generated by two straight lines such that  $\tan \frac{1}{2}\tau = 0.12$ . The Reynolds number based on the chord  $c$  is  $1 \times 10^7$  and transition may be assumed to occur at mid-chord.

From Item No. Aero W.01.01.05, for  $\tan \frac{1}{2}\tau = 0.12$  and  $R = 1 \times 10^7$  and with transition at mid-chord

$$(a_1)_0 / (a_1)_{0T} = 0.880.$$

From Figure 1, with  $t/c = 0.10$  and  $c_f/c = 0.35$ ,

$$(a_2)_{0T} = 4.80 \text{ rad}^{-1}.$$

From Figure 2, with  $(a_1)_0 / (a_1)_{0T} = 0.880$  and  $c_f/c = 0.35$ ,

$$(a_2)_0 / (a_2)_{0T} = 0.83.$$

Hence  $(a_2)_0 = 0.83 \times 4.80 = 3.98 \text{ rad}^{-1}$ .