

THE EFFECT OF MEAN STRESS ON FATIGUE STRENGTH (TEST PIECE WITH STRESS CONCENTRATION)

1. NOTATION

f_t	tensile strength of material	N/m ²	lbf/in ²
f_p	proof stress of material in tension	N/m ²	lbf/in ²
K'_t	elastic stress concentration factor based on net area		
m	a parameter characterising the shape of the R - M diagram		
N	endurance		
S_a	alternating stress on net area associated with S_m	N/m ²	lbf/in ²
S_{a0}	alternating stress on net area associated with $S_m = 0$	N/m ²	lbf/in ²
S_m	mean stress on net area	N/m ²	lbf/in ²

Both SI and British units are quoted but any coherent system of units may be used.

2. NOTES

This Item illustrates the extension of the data given in Item No. Fat A.00.01 to the case of a notched specimen. It enables the R - M diagram for a notched specimen to be obtained from the R - M diagram for a plain specimen, determined as in Item No. Fat A.00.01 and represented in Figure 1 by the curve MN. It should be noted that "notched specimen" is here used in the common usage of the term to denote a specimen with any type of stress concentration or stress raiser.

In a notched test piece, if elastic conditions could prevail throughout, the R - M curve would be given by GI, where any point F on GI is derived from the corresponding point D on MN in the ratio $OD/OF = K'_t$. However, at the point E on GI, corresponding to C on MN, the maximum stress at the notch in the cycle of loading equals the yield stress, and beyond this point plastic deformation occurs. As it is restricted to only a small region in the vicinity of the notch, it is reasonable to assume that, thereafter, the maximum stress in the cycle is limited to f_p . Once this condition is attained, further increase in mean load (the alternating load remaining constant) does not alter the maximum or the mean stress acting at the notch. The reasons for this are illustrated by Figure 2. If plastic deformation extends over a comparatively large region this model breaks down. However, such high mean stresses are outside the scope of normal design.

These assumptions imply that the shape of the R - M diagram is as shown by GEH on Figure 1, and if the stress-strain curve for the material has a flat yield at the proof stress, EH is a straight line at constant alternating stress. Greater precision in the shape of the part EH is possible when sufficient test data are available.

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If the diagram were based not on the elastic stress concentration factor K'_t , but on experimental values of the fatigue strength reduction factor at zero mean stress, care would have to be taken that fatigue strength was not over-estimated.

3. DERIVATION

1. GUNN, K. Effect of yielding on the fatigue properties of test pieces containing stress concentrations. *Aeronaut. Q.*, Vol. VI, Part 4, pp. 277-294, November 1955.

$$\text{The expression } S_a = \frac{S_{a0}}{K'_t} \left[1 - \left(K'_t \frac{S_m}{f_t} \right)^m \right].$$

4. EXAMPLES

4.1 Example 1

It is required to construct the alternating stress-mean stress diagram for a notched test piece with $K'_t = 2.1$ for an endurance N in a material for which a plain test piece gave $S_{a0} = 0.59 f_t$, $f_p = 0.80 f_t$ and in a repeated axial loading test gave $S_a = S_m = 0.42 f_t$ at an endurance N .

$$\text{At } \frac{S_m}{f_t} = 0.42, \quad \frac{S_a}{S_{a0}} = \frac{S_a}{f_t} \times \frac{f_t}{S_{a0}} = \frac{0.42}{0.59} = 0.71.$$

This point, plotted on Item No. Fat A.00.01, gives m slightly greater than 1.4 and, as it will be sufficiently accurate to work to the nearest full curve below the precise value, $m = 1.4$ will be used. For clarity only the $m = 1.4$ curve is shown in Figure 1.

Using the proof value,

$$\frac{f_p}{f_t} = 0.80 \text{ and } \frac{f_p}{S_{a0}} = \frac{f_p}{f_t} \times \frac{f_t}{S_{a0}} = \frac{0.80}{0.59} = 1.36.$$

These points are laid off on the axis of S_m/f_t and S_a/S_{a0} respectively, and the line AB is drawn, together with OC. The curve GI is then constructed by dividing a number of radials such as OD in the ratio $OD/OF = K'_t$ and completed by drawing EH parallel to the S_m/f_t axis, where E is the intersection of GI and OC.

At H, a point approaching that at which overall yielding will occur, the curve will drop off rapidly to intersect the S_m/f_t axis but, in view of the high mean stresses involved, it is unlikely that this part will be of interest for most practical purposes.

Curve GEH then represents, in non-dimensional form, the relationship between alternating and mean stress at endurance N for a notched test piece with $K'_t = 2.1$ in the material of the example.